Spacetime discontinuous Galerkin method for hyperbolic advection–diffusion with a non-negativity constraint

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Abstract

Applications where the diffusive and advective time scales are of similar order give rise to advection--diffusion phenomena that are inconsistent with the predictions of parabolic Fickian diffusion models. Non-Fickian diffusion relations can capture these phenomena and remedy the paradox of infinite propagation speeds in Fickian models. In this work, we implement a modified, frame-invariant form of Cattaneo's hyperbolic diffusion relation within a spacetime discontinuous Galerkin advection--diffusion model. An *h*-adaptive spacetime meshing procedure supports an asynchronous, patch-by-patch solution method with linear computational complexity in the number of spacetime elements. This localized solver enables the selective application of optimization algorithms in only those patches that require inequality constraints to ensure a non-negative concentration solution. In contrast to some previous methods, we do not modify the numerical fluxes to enforce non-negative concentrations. Thus, the element-wise conservation properties that are intrinsic to discontinuous Galerkin models are defined with respect to physically meaningful Riemann fluxes on the element boundaries. We present numerical examples that demonstrate the effectiveness of the proposed model, and we explore the distinct features of hyperbolic advection--diffusion response in subcritical and supercritical flows.