Efficiency of High Order Methods in Space and Time: Study of Elastodynamics Problem Using Spacetime Discontinuous Galerkin Finite Element Method

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Abstract

Generally, higher order methods refer to third order or higher [1]. Two general beliefs that have resulted in more widespread use of lower order methods in academia and especially in industry are [1]: 1) Higher order methods are more expensive; 2) They are not needed for engineering accuracies. Referring to the former, efficiency is a better measure than solution cost. To investigate a method's efficiency both error convergence rate and solution cost scaling versus number of elements and element order should be well-understood. While the former generally has a simple power form, the solution cost scaling is much more complicated and highly dependent on particular formulation of a finite element method (FEM). For example as a result of the global coupling of conventional continuous FEMs, [2] reports that even for 0.1% relative accuracies—which is tenfold the commonly used 1% range in many engineering applications—linear elements are more efficient than higher order elements for realistic 3D applications.

We investigate when higher order methods are more efficient in the context of a highly advanced spacetime discontinuous Galerkin (SDG) finite element method [3] where a novel use of characteristic structure of the wave equation in discretization, yields a method with linear solution complexity. Some aspects of our study are: 1) Unlike continuous FEMs the very local solution nature of the SDG method may favor higher order elements. This clearly advocates the use of higher order elements for realistic 3D applications; 2) In many dynamic applications, *e.g.* wave propagation, temporal order is as important as spatial order. We comment on general challenges in achieving arbitrary high temporal orders—particularly when more efficient non-spatially uniform orders are demanded—and discuss how the SDG method gracefully addresses these issues; 3) We demonstrate multi-field formulations can in fact be more efficient than single-field formulations; 4) An FEM's efficiency is greatly affected by the cost scaling of assembly and the solution of global matrices. We demonstrate this concept through comparison between single-cell and a new multi-cell element formulation.

References:

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