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Spacetime Discontinuous Galerkin Finite Element Method for Time Domain Electromagnetics

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Most problems in modern electromagnetics are defined on complex geometries, have complicated solution features and are inherently multiscale, all calling for very accurate numerical methods. Time domain (TD) solution methods are preferred over frequency domain (FD) ones when solutions are directly sought in TD. Even if frequency response is sought, TD methods can still be more efficient as the solution to a single broad-band signal can generate the entire frequency spectrum and in contrast to FD methods some TD methods have linear solution scaling versus the number of unknowns.

Discontinuous Galerkin (DG) methods have been widely used for the solution of a variety of PDEs. They do not suffer from high dispersive and dissipation errors often encountered in finite volume and finite difference schemes. The main advantages of the DG methods over continuous finite element (FEM) methods are better performance in problems with shocks and discontinuities, higher efficiencies implied by their linear solution scaling for explicit time integration, more flexible mesh adaptive schemes, and better parallel computing performance.

We present a spacetime discontinuous Galerkin (SDG) formulation for TD analysis of electromagnetics problem. The main difference of the SDG method with other TD DG methods is the direct discretization of the spacetime using unstructured grids that satisfy a special causality constraint, posed by the hyperbolic structure of electromagnetics equations. The direct and unstructured discretization of spacetime results in major advantages over TD DG and TD FEM methods that advance the solution in time by a time marching approach. Some examples are arbitrarily and per element adjustment of temporal order of accuracy, excellent performance in multiscale meshes (high ratio of largest to smallest element size), and excellent performance in parallel and adaptive simulations due to its local and asynchronous solution structure.

We present formulations for all Maxwell's equations and DG finite element discrete statements using the elegant mathematical framework of differential form in spacetime. We present mathematical proof of the method's energy stability and show numerical studies that verify the method's expected analytical convergence rates for harmonic solutions and those with sharp moving wave fronts. We will also present numerical results that demonstrate the efficiency and accuracy of the method in solving some TD benchmark problems, including cavity problems.