Adaptive Space-Time Discontinuous Galerkin Methods for Solutions of Steady and Transient Partial Differential Equations

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The asynchronous Spacetime Discontinuous Galerkin (aSDG) method [1] can be used directly, without relaxation, to solve hyperbolic problems. It has proven very effective in this role due to its local conservation properties, linear computational complexity, unconditional stability, powerful adaptive meshing capabilities, and other favorable properties. Here we focus on extensions of aSDG solution technology to address elliptic and parabolic PDEs using hyperbolic pseudo-time relaxations.

In the case of elliptic PDEs, the role of pseudo-time in the relaxed system is analogous to physical time in a conventional hyperbolic system, and the steady limit of the relaxed problem corresponds to the solution of the original elliptic system. We compare three pseudo-time solution schemes for the relaxation: (i) the scheme presented in [2], where we use a discontinuous Galerkin (DG) discretization in space and a finite difference scheme in pseudo-time; (ii) an implicit space-time DG method in which a spatial mesh is extruded in time to form the space-time mesh; and (iii) the aSDG method [1] in which a special asynchronous space-time meshing procedure constructs unstructured spacetime meshes that satisfy a special causality constraint defined in terms of the pseudo wave speed in the relaxed system. The aSDG pseudo-time solver exploits powerful adaptive meshing procedures [3] to ensure accuracy and expedite convergence to the steady limit. For parabolic problems, the relaxed problem comprises a system of PDEs in space, physical time, and pseudo-time that we cast in first-order form. For each increment in physical time, we seek the steady solution of a transient hyperbolic problem in pseudo-time.

In contrast to time-marching schemes that use finite difference operators in time, we can easily introduce arbitrarily high-order space-time polynomial bases to balance and simultaneously improve the aSDG method's order of accuracy with respect to element spatial diameter and temporal duration. We present numerical results for problems involving nonlinear advection-diffusion and the wave equation including studies of asymptotic convergence rates obtained with aSDG and pseudo-time aSDG methods.

References:

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