Tent Pitching Meshes for Asynchronous Spacetime Discontinuous Galerkin Solvers in 3d×time

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Abstract

The asynchronous Spacetime Discontinuous Galerkin (aSDG) method [1] is a powerful solution scheme for hyperbolic systems. In lieu of traditional implicit or explicit time-marching procedures, aSDG solvers implement a locally implicit solution scheme on fully unstructured and asynchronous spacetime grids. The locality property, critical to the computational efficiency of aSDG solvers, derives from discontinuous spacetime basis functions defined on spacetime meshes that satisfy a so-called causality constraint. The resulting aSDG solution schemes feature unconditional stability, conservation over every spacetime cell, linear computational complexity, support for arbitrarily high-order elements, a rich structure for parallel implementations, and dynamic adaptive spacetime meshing.

The Tent Pitcher algorithm with adaptive extensions [2] is the key technology for generating causal spacetime meshes for aSDG solvers. Let d be the spatial dimension of the analysis domain. Tent Pitcher's main procedure advances a front mesh of d-simplices through the spacetime analysis domain. In addition to spatial coordinates, each front-mesh vertex is assigned a private time coordinate that is initialized to zero. Tent Pitcher advances the front locally and asynchronously by incrementing the time coordinate of one vertex at a time. The causality constraint limits the vertex's time increment to ensure that the updated front remains space-like with respect to the characteristics of the target hyperbolic system. We generate a new spacetime patch, comprised of a small set of (d+1)-simplices that covers the spacetime volume between the old and new fronts, every time a vertex advances.

Previously, robust adaptive meshing for aSDG solvers has only been available for meshes in up to 2d×time. However, robust meshing in up to 3d×time is essential for the aSDG method to achieve its full potential. This presentation describes recent progress toward a new meshing implementation that meets that requirement. A new dimension-independent version of the basic Tent Pitcher algorithm is enhanced with dimension-dependent h-adaptive extensions. Examples will demonstrate meshing and solution capabilities in 3d×time; directions for continuing development will be discussed.

References:

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