Adaptive Space-Time Discontinuous Galerkin Method for Unsteady Elliptic and Parabolic PDEs with First-Order Hyperbolic System Approach

Reza Abedi[1], Alireza Mazaheri[2], Robert B. Haber[3]
[1]University of Tennessee Space Institute
[2]NASA Langley Research Center
[3]University of Illinois at Urbana-Champaign

We present a pseudo-time approach to relax elliptic and parabolic PDEs to hyperbolic equations. When an elliptic PDE is solved, the pseudo-time plays the role of physical time for a hyperbolic equation. However, for parabolic equation pseudo-time is added to the physical time and for each physical time advance the problem is solved in pseudo-time until it reaches steady state in pseudo-time. The relaxation times, target solutions for the hyperbolized discontinuous Galerkin method, and other aspects of the pseudo-time formulation are carefully chosen to optimize the convergence of the hyperbolized system to its steady state limit at each pseudo-time step [1]. In [1], this hyperbolic system is solved in pseudo-time using an implicit time marching scheme. Our main contribution will be to demonstrate, for the first time, an adaptive space-time discontinuous Galerkin for first-order system of hyperbolic equations. For this purpose, the spatial discretization is extruded in physical time to form implicit spacetime finite elements. We will also verify the order of accuracy of the solution and solution gradients for time-dependent problems. More specifically, we will demonstrate that the spatial order of accuracy of both solution and solution gradients is (k+1) for Pk polynomial. When an elliptic problem is hyperbolized by the pseudo-time approach, the resulting system can also be solved with other spacetime discontinuous Galerkin formulations such as [2]. Therein, adaptivity in spacetime [3] can expedite the convergence of solution in pseudo-time. Thus, quality, order of accuracy, and efficiency of the proposed approach will be compared with those presented in [2] and [3].

References:

[1] Alireza Mazaheri and Hiroaki Nishikawa. Efficient high-order discontinuous Galerkin schemes with first-order hyperbolic advection-diffusion system approach. Journal of Computational Physics, 321:729-54, 2016.

[2] R. Abedi, B. Petracovici, and R.B. Haber, “A spacetime discontinuous Galerkin method for linearized elastodynamics with element–wise momentum balance”, Computer Methods in Applied Mechanics and Engineering, 195:3247 – 3273, 2006.

[3] R. Abedi, R. B. Haber, S. Thite, and J. Erickson. An h-adaptive spacetime discontinuous Galerkin method for linearized elastodynamics. European Journal of Computational Mechanics, 15(6):619-42, 2006.