A SPACETIME DISCONTINUOUS GALERKIN METHOD FOR ELASTODYNAMICS WITH ELEMENT–LEVEL MOMENTUM BALANCE[1]

R. B. Haber[a], **B. Petracovici**[b], **R. Abedi**[a] and **R. Jerrard**[b]

[a]Department of Theoretical and Applied Mechanics
[b]Department of Mathematics

Center for Process Simulation and Design
University of Illinois at Urbana–Champaign
Urbana, IL 61801, USA
{r-haber, petracov, rabedi, rjerrard}@uiuc.edu

We present a new spacetime discontinuous Galerkin (SDG) formulation for linearized elastodynamics. The new formulation improves on the one presented by Yin et al. [1, 2], in that low-order elements are supported and balance of both linear and angular momentum is satisfied exactly on every spacetime finite element. These improvements are due to two modifications of the formulation in [2]. First, we always enforce balance with respect to the physically correct Godunov fluxes as defined by the local Riemann problem; we do not introduce averaged numerical fluxes. Second, we use a projection onto the subspace of zero–energy spacetime displacement fields to define the kinematic compatibility constraint. This restores exact momentum balance on every element and supports the use of low–order ($p \geq 1$) elements for problems in all spatial dimensions.

We weakly enforce momentum balance and kinematic compatibility via a continuum weighted-residual statement based on a solution space of displacement functions with derivatives in $\mathrm{BV}(\Omega)$, where Ω is the spacetime analysis domain. In the discrete setting, we approximate this space with piecewise–continuous polynomial functions. Although we use a straightforward Bubnov-Galerkin weighting without stabilization, we show that the method is naturally dissipative and high–order stable. That is, solutions are stable and free of spurious oscillations for $p \geq 1$.

The method can be used with either structured or unstructured spacetime grids that need not be conforming. Direct element–by–element or patch–by–patch solution procedures with $\mathcal{O}(N)$ computational complexity are possible when the mesh satisfies a causality cone constraint [3]. We present examples that demonstrate the SDG formulation's high–order convergence properties as well as a spacetime h–adaptive analysis capability.

References

[1] L. Yin, A. Acharya, N. Sobh, R. Haber, D. Tortorelli, "A space–time discontinuous Galerkin method for elastodynamic analysis," in B. Cockburn, G. E. Karniadakis, and C. -W. Shu, eds., *Discontinuous Galerkin Methods: Theory, Computation and Applications*, pp. 459-464, Springer, (2000).

[2] L. Yin, "A New Spacetime Discontinuous Galerkin Finite Element Method for Elastodynamic Analysis," Thesis submitted in partial fulfillment of the requirements of the degree of Doctor of Philosophy in Theoretical and Applied Mechanics, Graduate College, University of Illinois at Urbana-Champaign, 2002.

[3] J. Erickson, D. Guoy, A. Üngör, and J. Sullivan, "Building space–time meshes over arbitrary spatial domains," *Proc. 11th Int. Meshing Roundtable*, pp. 391–402, 2002.

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