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A damage-based cohesive model in an adaptive spacetime discontinuous Galerkin method

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Abstract

We describe a new method for modeling elastodynamic fracture using a spacetime discontinuous Galerkin (SDG) finite element method and a novel, damage-based cohesive model. The underlying SDG formulation features powerful h-adaptive meshing capabilities, exact balance of linear and angular momentum on every spacetime element, good shock-capturing properties and scalable performance with linear complexity in the number of elements. Enhanced adaptive meshing capabilities provide a flexible framework for extending cohesive interfaces to track solution-dependent crack paths. Rather than use a traditional traction-separation law, we propose a damage-based cohesive model that properly accounts for the change in the structure of the interface conditions between the undamaged and fully fractured states. In so doing, the model incorporates favorable aspects from both intrinsic and extrinsic fracture models.

1. Introduction

This work describes a new method for modeling elastodynamic brittle fracture using a spacetime discontinuous Galerkin (SDG) finite element method and a novel, damage-based cohesive model. In previous work [1,2,3], we proposed an h-adaptive SDG model for elastodynamic fracture that delivers exact balance of linear and angular momentum on every spacetime element, superior shock-capturing properties and scalable, linear complexity in the number of elements. Independent error indicators, for energy dissipation and cohesive work of separation, drive the adaptive meshing process. The resulting high-precision solutions led to the discovery of quasi-singular velocity response in elastodynamic fracture and the first transient studies of the nonlinear relation between crack-tip velocity and process-zone size. Our previous implementation was restricted to problems where the potential crack paths can be determined *a priori*. Here, we introduce an improved method that supports predictions of crack propagation along trajectories determined during the solution process. The new advancing-front solution method, in which patch-wise finite element solutions are interleaved with unstructured spacetime mesh generation, supports unrestricted evolution of crack geometry while maintaining the quality of the spacetime mesh.

Most cohesive models use traction-separation laws to model the transition of material from the undamaged state to the fully separated condition. *Intrinsic* cohesive models use a large initial cohesive stiffness to approximate the compatibility constraint for undamaged material; this approximation can be problematic, especially in simulations of dynamic brittle fracture. *Extrinsic*, initially-rigid cohesive models have so far proved more suitable for brittle fracture, but these models do not describe the gradual loss of stiffness prior to reaching the cohesive strength and are generally not differentiable. The latter property is not a practical concern in explicit time integration schemes, but it does present a serious problem in the implicit patch-wise solutions used in the SDG approach. We propose a new class of cohesive models that uses a damage parameter to smoothly transition from the continuum jump conditions that describe compatibility and momentum balance in the undamaged state to the jump conditions that describe the traction-free condition in the fully separated state. This model is differentiable, describes compatibility in the undamaged state to within the accuracy of the discretization, and includes the gradual loss of stiffness prior to reaching the cohesive strength.

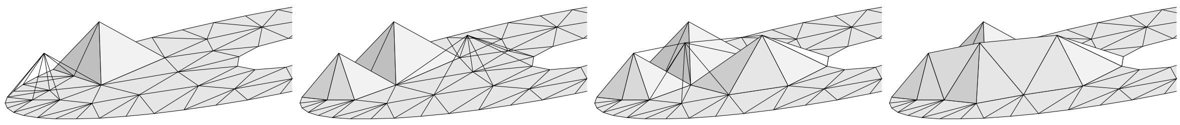

Figure 1. Pitching tents (patches) in spacetime. Local causality constraint limits patch duration; time-axis is vertical.

2. Adaptive SDG method for elastodynamics

2.1. SDG finite element formulation for elastodynamics

Our method inherits the favarable properties of the SDG method for elastodynamics [1]. It uses basis functions defined on fully unstructured spacetime meshes to describe displacement solutions that admit jumps across all inter-element boundaries. This discontinuous solution structure leads to exact balance of linear and angular momentum on every spacetime element and superior shock-capturing properties. When implemented on suitable spacetime grids, the SDG method exhibits linear complexity in the number of elements. The SDG formulation easily incorporates cohesive damage models. Displacement jumps are intrinsic to the model, so the only modification is the use of the cohesive traction model to define the target momentum flux on cohesive interfaces. There is no need for cohesive elements or other special data structures.

We use differential forms and the exterior calculus on manifolds to obtain a direct, coordinate-free notation that facilitates our formulation on unstructured spacetime meshes (*cf.* [1] for details). We have *displacement* $\mathbf{u}$, *spacetime momentum flux* $\boldsymbol{M}$, *body force* $\boldsymbol{b}$, *strain–velocity* $\boldsymbol{\varepsilon}$. The discrete weighted residual statement for balance of linear momentum and kinematic compatibility takes the following form: Find $\mathbf{u} \in \mathcal{V}_h$ such that, for all elements Q in the spacetime domain,

$$\int_Q \dot{\hat{\mathbf{u}}} \wedge (\boldsymbol{dM} + \rho \boldsymbol{b}) + \int_{\partial Q} \left\{ \dot{\hat{\mathbf{u}}} \wedge (\boldsymbol{M}^* - \boldsymbol{M}) + (\boldsymbol{\varepsilon}^* - \boldsymbol{\varepsilon}) \wedge \boldsymbol{i}\hat{\boldsymbol{M}} \right\} + \int_{\partial Q^{\mathrm{ti}}} k \hat{\mathbf{u}}_0 \wedge (\mathbf{u}^* - \mathbf{u}) \star \boldsymbol{d}t = 0 \quad \forall \hat{\mathbf{u}} \in \mathcal{V}_h^Q, \tag{1}$$

in which a superposed '$\hat{}$' indicates a weight function, and k is a constant introduced for dimensional consistency. $\mathcal{V}_h$ is the discrete space of discontinuous Galerkin functions over the full space-time domain, and $\mathcal{V}_h^Q$ is the restriction of $\mathcal{V}_h$ to element Q. Items marked with asterisks are target fluxes that are computed from prescribed boundary or initial data, as Godunov values from the solution to a local Riemann problem on interior boundaries, or from cohesive values based on the particular fracture model at hand. The Stokes theorem applied to (1) leads to the discrete weak form that defines our finite element method:

$$\int_Q (-\boldsymbol{d}\dot{\hat{\mathbf{u}}} \wedge \boldsymbol{M} + \dot{\hat{\mathbf{u}}} \wedge \rho \boldsymbol{b}) + \int_{\partial Q} \left\{ \dot{\hat{\mathbf{u}}} \wedge \boldsymbol{M}^* + (\boldsymbol{\varepsilon}^* - \boldsymbol{\varepsilon}) \wedge \boldsymbol{i}\hat{\boldsymbol{M}} \right\} + \int_{\partial Q^{\mathrm{ti}}} k \hat{\mathbf{u}}_0 \wedge (\mathbf{u}^* - \mathbf{u}) \star \boldsymbol{d}t = 0 \quad \forall \hat{\mathbf{u}} \in \mathcal{V}_h^Q. \tag{2}$$

It is easily shown that the discrete solution to (2) exactly satisfies the integral forms of balance of linear momentum and balance of angular momentum over every spacetime element Q [1].

2.2. Spacetime meshing

We use an advancing-front meshing/solution procedure in which the *Tent Pitcher* algorithm [4] generates unstructured spacetime meshes that obey a *causality constraint* based on the characteristics of the governing partial differential equations; see Figure 1. The causality constraint and discontinuous basis ensure that the solution on each new patch from Tent Pitcher depends exclusively on prescribed initial/boundary data and outflow data from previously-solved patches. This structure enables a scalable, patch-by-patch solution procedure with $\mathcal{O}(N)$ complexity (N is the number of spacetime elements), in which we immediately compute the local finite element solution on each new patch as soon as it is generated. The causality constraint limits the duration of each patch, but the durations of individual patches vary as there is no global time-step constraint.

We exploit this flexibility in h-adaptive analysis methods that simultaneously refine in space and time to achieve significant performance gains, especially in hyperbolic solutions with sharp wavefronts, as seen in Figure 2 (Left). The SDG scheme supports higher-order bases on fixed stencils and features an asynchronous parallel structure that facilitates high-performance implementations. In fracture applications, adaptive spacetime meshing ensures accurate resolution of sharp wavefronts and sufficient refinement in the active fracture process zone to

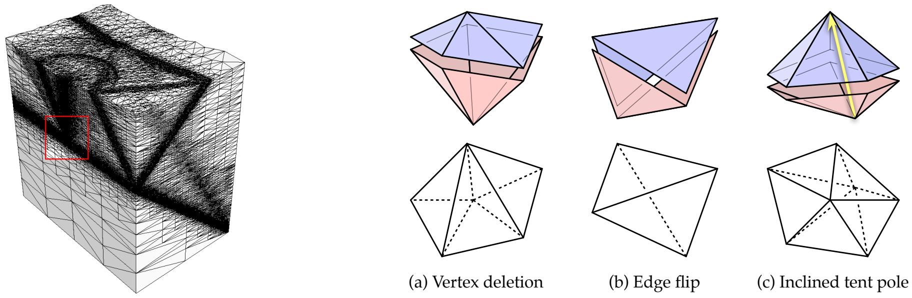

Figure 2. Left: Spacetime mesh for a crack-tip simulation with shock loads. Right: Spacetime adaptive meshing operations.

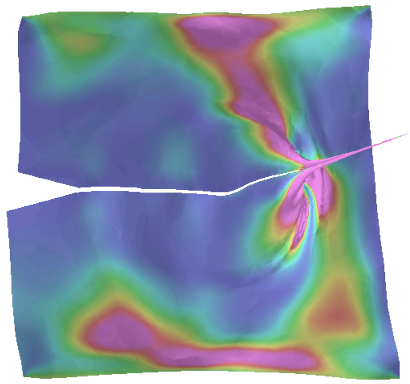

Figure 3. Dynamic fracture under mixed-mode loading showing quasi-singular velocity field

automatically ensure numerical stability and an accurate rendering of cohesive traction–separation laws. Two independent error indicators drive the adaptive procedure: a dissipation-based indicator that limits numerical energy dissipation throughout the solution domain and one that controls the discrepancy between the works of separation predicted by the trace of the finite element stress field and the cohesive traction model. The resulting high-precision solutions led to the discovery of quasi-singular velocity response in the neighborhood of the process zone and the first transient studies of the nonlinear relation between crack velocity and process-zone size.

In new work reported here, we extend the adaptive meshing capabilities to support solution-dependent nucleation and extension of cohesive surfaces. We have extended Tent Pitcher to implement common adaptive remeshing operations as special spacetime patches that incur zero projection error; Figure 2 (Right). We introduce a new set of spacetime adaptive meshing operations where each operation is implemented as a special spacetime patch rather than as a discrete operation in space. The inflow faces of the special patches conform to the outflow faces of previously solved elements, so there is no need to project the old solution onto a new mesh. This eliminates the projection errors incurred by conventional adaptive remeshing procedures and preserves the full convergence rates of high-order elements. Patches with *inclined tent poles* reposition vertices in the space mesh; we use these to continuously smooth the space mesh to maintain and improve its quality and to track moving interfaces, such as cohesive surfaces. Special single-tetrahedron patches perform *edge-flip operations* to improve the quality of the spatial triangulation. *Coarsening patches* remove a vertex from the space mesh. Mesh refinement involves a nested subdivision of the space mesh that incurs zero projection error. We use these operations in combination to nucleate cohesive interfaces at arbitrary locations and to extend existing interfaces in any direction, as indicated by the physics of the solution. Element quality is maintained throughout the procedure, and there are no restrictions on the direction of crack propagation. Figure 3 shows an example of dynamic crack propagation under mixed-mode loading conditions.

2.3. Cohesive model

Initially-rigid cohesive models are often used to simulate dynamic fracture, especially in combination with explicit time integration. These models transition abruptly from an undamaged state to the regime of a traction-separation law, and are, therefore, non-differentiable. This makes them unsuitable for our patch-wise-implicit SDG solution scheme. We propose a new initially-rigid cohesive model that uses a damage parameter to transition smoothly between enforcing the flux conditions for undamaged material and the traction-free conditions for a crack.

2.3.1. Modification of SDG formulation to incorporate a cohesive model

It is relatively easy to extend the SDG formulation for elastodynamics to incorporate a cohesive model. The SDG bases naturally support discontinuous displacements, velocity and stress across element boundaries. So the only modification required is to compute values for $\boldsymbol{M}^*$ and $\boldsymbol{\varepsilon}^*$ consistent with the cohesive model on cohesive faces. Cohesive interfaces are material interfaces, and this simplifies the formulation: $\boldsymbol{M}^*$ and $\boldsymbol{\varepsilon}^*$ simplify to $\boldsymbol{\sigma}^*$ and $\boldsymbol{v}^*$, the *stress/traction form* and the *velocity form*, respectively. All that is required to implement a traditional cohesive traction separation law is (1) to set $\boldsymbol{v}^* = \boldsymbol{v}$ (the trace of the element velocity field on the cohesive face) to relax the velocity jump condition across the cohesive surface, and (2) set $\boldsymbol{\sigma}^* = \boldsymbol{\sigma}^{\mathrm{C}}$, where $\boldsymbol{\sigma}^{\mathrm{C}}$ is the cohesive traction predicted by the TSL.

In developing a new damage-based cohesive model, we attempt to combine the best properties of extrinsic and intrinsic cohesive models in a single formulation. Consider a perfectly brittle fracture process at the microstructure level, and let D be the area fraction of the cohesive surface that has fractured due to micro-crack formation. Then we have microscopic target values $\bar{\boldsymbol{\sigma}}^* = \mathbf{0}$ and $\bar{\boldsymbol{v}}^* = \boldsymbol{v}$ on the damaged area fraction D, *vs.* $\bar{\boldsymbol{\sigma}}^* = \boldsymbol{\sigma}^{\mathrm{G}}$ and $\bar{\boldsymbol{v}}^* = \boldsymbol{v}^{\mathrm{G}}$ on the intact area fraction $1 - D$. Applying these conditions in the SDG weighted residual expression at the microstructural level and integrating to get a simple homogenization, we obtain the macroscopic target values,

$$\boldsymbol{\sigma}^* = (1 - D)\boldsymbol{\sigma}^{\mathrm{G}} \tag{3}$$

$$\boldsymbol{v}^* = (1 - D)\boldsymbol{v}^{\mathrm{G}} + D\boldsymbol{v}. \tag{4}$$

in which a superscript 'G' refers to a Godunov value obtained from the local Riemann problem for intact material. Quantities with no superscript are traces from the interiors of elements adjacent to the cohesive interface. A single dimensionless damage parameter D evolves according to a rule of the form,

$$\dot{D} = f(\boldsymbol{\sigma}^{\pm}, \boldsymbol{v}^{\pm}, \mathbf{u}^{\pm}, D) \tag{5}$$

in which a superscript '$\pm$' denotes the traces from both sides of the interface. We report on evolution models similar in structure to those proposed in [5], except we do not introduce an interface stiffness. Thus, we avoid a key disadvantage of intrinsic models that modify the bulk material properties whenever cohesive surfaces are introduced, especially as their spacing approaches zero. Provided that $D = 0$ initially, cohesive surfaces can be introduced at any density without affecting the bulk properties in the present model. In contrast to many extrinsic models, the cohesive surfaces can sustain increasing loads, so that crack nucleation and propagation can be determined intrinsically by the eventual damage evolution.

2.3.2. Propagation of cohesize surfaces

We nucleate and propagate cohesive surfaces at locations where an effective stress exceeds a critical value. Existing cohesive surfaces extend in the direction that maximizes the trace of the effective stress at the current tip of the cohesive zone, provided a specified critical value is exceeded. The flexibility of our adaptive spacetime meshing scheme allows complete freedom to follow this criterion wherever it is active and in any direction for extension.

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