

## Spacetime Damage-Delay Cohesive Model for Elastodynamic Fracture with Riemann Contact Conditions

R. Abedi<sup>1</sup>, R. B. Haber<sup>1</sup>, O. Allix<sup>2</sup>

<sup>1</sup> University of Illinois at Urbana-Champaign, Urbana, Illinois, USA {rabedi,r-haber}@illinois.edu

<sup>2</sup> LMT-Cachan, ENS Cachan/UPMC/CNRS/PRES UniverSud Paris, France, {allix,neron}@lmt.ens-cachan.fr

Dynamic material failure is important in a number of scientific and engineering applications and a variety of numerical methods for its modeling have been proposed. We present a new interfacial-damage, cohesive-fracture model, including frictional contact during crack closure, for dynamic failure of brittle materials. The model is implemented within a spacetime discontinuous Galerkin (SDG) finite element method, and extends previous work reported in [1]. An adaptive meshing procedure generates spacetime grids that satisfy a special *causality constraint* to enable an efficient patch-by-patch, advancing-front solution scheme with  $O(N)$  computational complexity. Per-element balance properties, local adaptive operations, and the use of Riemann fluxes provide to the SDG method the high accuracy and efficiency required to solve multiscale fracture problems.

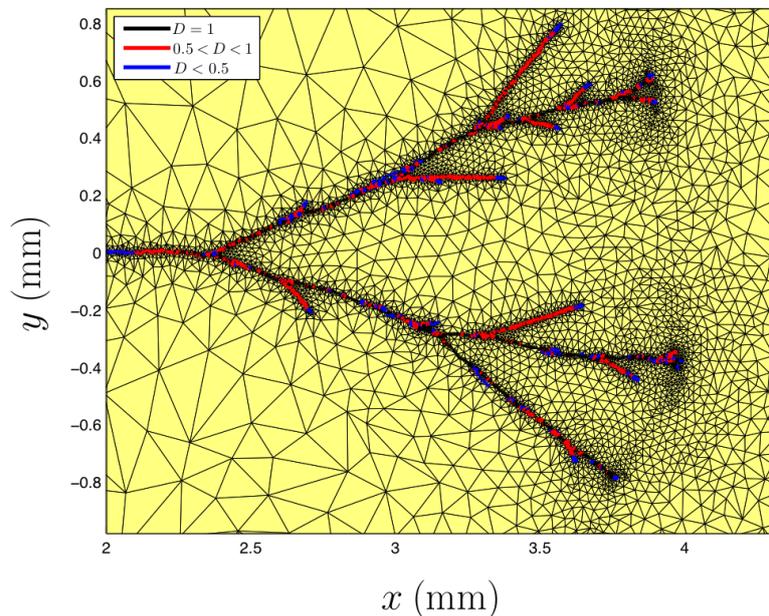


Figure 1: Detail of space mesh showing adaptive mesh refinement around multiple propagating crack tips, modeled by nucleation and extension of cohesive interfaces. There are no mesh-dependent constraints on the crack path because the spacetime mesh is continually refined, coarsened and smoothed to ensure mesh quality and solution accuracy while maintaining and extending the crack geometry. Micro-cracks and crack-branching are evident in the solution.

A new two-scale cohesive fracture model replaces the usual traction-separation law with a delay-damage model [2] that represents, for example, mesoscale processes of void growth and coalescence. The evolution of a single damage parameter  $D$ , the debonded area fraction on a cohesive interface, is

governed by an irreversible, time-delay evolution law characterized by a cohesive strength and a relaxation time  $\tau$  that determines the maximum damage rate. Riemann fluxes for the fully-bonded condition are enforced in the undamaged area fraction  $(1 - D)$  of the cohesive interface, while the Riemann fluxes for the contact–stick, contact–slip or separation conditions determine the fluxes in the debonded area fraction. These mesoscale Riemann values are homogenized to obtain the macroscopic cohesive response. The Riemann solution for the contact-slip case is apparently an original contribution. Overall, the use of the Riemann contact fluxes is essential to preserving the characteristic structure of the elastodynamic problem. The use of quasi-static contact conditions will inevitably introduce error.

Beyond ensuring solution accuracy, the model uses the SDG scheme’s adaptive meshing capabilities [3, 4] to freely nucleate and extend cohesive interfaces to capture solution-dependent crack paths. The SDG adaptive meshing aligns the boundaries of spacetime elements with crack-path trajectories having arbitrary position and orientation, and two adaptive error indicators ensure the accurate rendering of both the cohesive model and the bulk solution. Thus, the present model does not suffer the limited resolution and mesh-dependent effects encountered in most other numerical fracture models. Numerical results obtained with the proposed model demonstrate crack propagation, microcrack formation and crack branching phenomena.

## References

- [1] Reza Abedi, Morgan A. Hawker, Robert B. Haber, and Karel Matouš. An adaptive spacetime discontinuous Galerkin method for cohesive models of elastodynamic fracture. *International Journal for Numerical Methods in Engineering*, 1:1–42, 2009.
- [2] O. Allix, P. Feissel, and P. Thevenet. A delay damage mesomodel of laminates under dynamic loading: basic aspects and identification issues. *Computers and Structures*, 81(12):1177 – 1191, 2003.
- [3] R. Abedi, S. H. Chung, J. Erickson, Y. Fan, M. Garland, D. Guoy, R. B. Haber, J. Sullivan, S. White, and Y. Zhou. Space-time meshing with adaptive refinement and coarsening. In *Proceedings 20th Annual ACM Symposium on Computational Geometry*, pages 300–309. Association for Computing Machinery, 2004.
- [4] R. Abedi, R. B. Haber, S. Thite, and J. Erickson. An  $h$ -adaptive spacetime–discontinuous Galerkin method for linearized elastodynamics. *Revue Européenne de Mécanique Numérique (European Journal of Computational Mechanics)*, 15(6):619–642, 2006.