Spacetime Analyses of Elastodynamic Fracture: Exterior Calculus, Dimensional Analysis, and Numerical Simulation

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ABSTRACT

For over a decade, we have been exploring new mechanics formulations using the exterior calculus applied to spacetime manifolds. This presentation reviews results from applications of this approach to continuum and numerical analyses of dynamic fracture. We begin with continuum formulations of linearized elastodynamics using differential forms to describe balance laws and compatibility conditions over spacetime control volumes. These lead to powerful spacetime discontinuous Galerkin (SDG) methods that are easily extended to incorporate traditional cohesive fracture models.

In addition to circumventing inherent limitations of tensor notation in classical spacetime and providing an elegant dynamics formulation, the differential forms notation simplifies the dimensional analysis of elastodynamics and dynamic fracture. We present a *fundamental set of nondimensional parameters* that identifies families of dynamic cohesive fracture models that share self-similar solutions. Using cohesive fracture mechanics as a simple model for nonlinear material response, we use our dimensional analysis results to investigate the validity of the small-scale-yielding (SSY) assumption under dynamic conditions to predict when LEFM theory is applicable to dynamic fracture as well as the onset of the quasi-singular velocity response observed in our high-resolution numerical simulations.

We close with a review of our latest numerical models of dynamic fracture using adaptive SDG models. These use dynamic adaptive meshing techniques based on special spacetime configurations that do not require solution projections from old to new meshes. These adaptive capabilities provide high-resolution solutions of crack-tip fields, even at high crack-tip velocities, and eliminate mesh-dependence in determining the crack path. We describe a new rate-dependent interfacial damage model that replaces traditional traction-separation relations and incorporates new dynamic Riemann solutions that determine the correct response for each of the contact modes during crack closure. Adding a probabilistic nucleation model provides a robust fracture simulation capability, Figure 1.

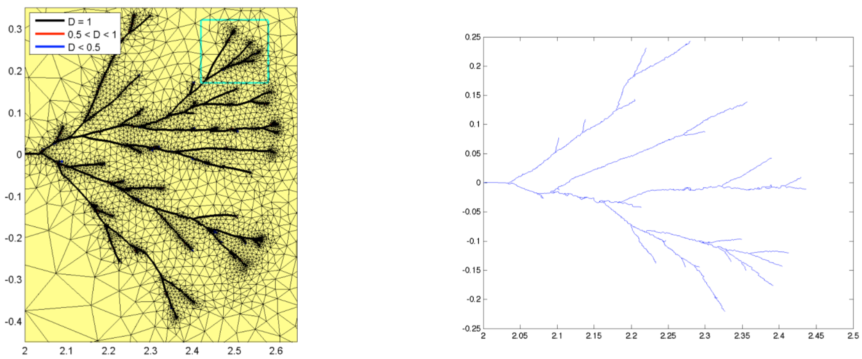

Figure 1 Adaptive SDG analyses of dynamic fracture. Left: Probabilistic nucleation model and mesh-independent extension capture crack branching and microcracking. Right: Adding crack-closure model changes the fracture pattern.

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