Riemann Conditions for Elastodynamic Contact and Rate-Dependent Interfacial-Damage Fracture Model

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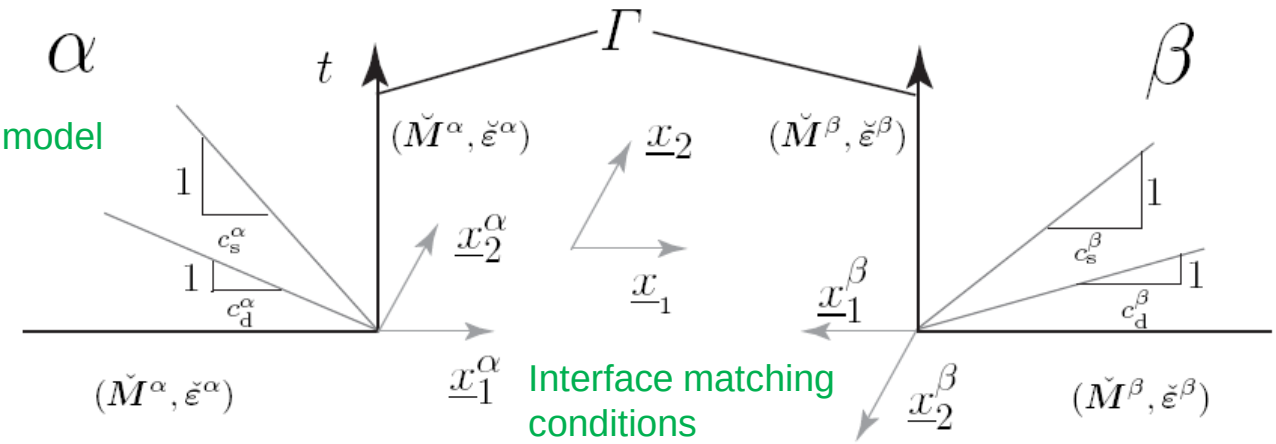
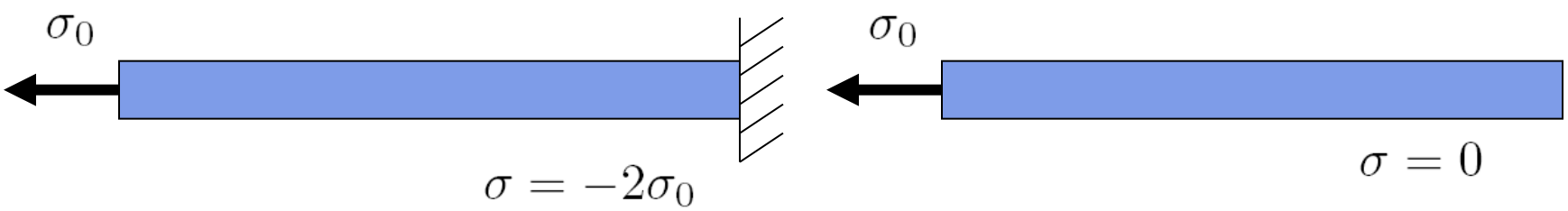
Riemann Solution

Bulk material model



Interface matching conditions

- Direct solution of the Riemann problem from momentum balance and compatibility condition jump parts
- Unified expressions for normal and tangential directions
- Matching conditions at material interface $\Rightarrow$ different contact/separation solutions
- The characteristic values $\underline{w}^{(j)} := \underline{s}^j + \underline{c}^{(j)}\underline{p}^j$ are preserved



$\sigma = -2\sigma_0$

$\sigma = 0$

$w = 2\sigma_0$

$v = 0$

$v = 2\dfrac{\sigma_0}{c\rho}$

Interface Matching condition: Bonded and contact-stick

- Matching conditions
  - Momentum: $[\![\check{\mathbf{s}}]\!] = 0$
  - Compatibility: $[\![\check{\mathbf{v}}]\!] = 0$
- Riemann Solution
  - $\underline{s}^{j^I} = [\![\underline{w}^{(j)}/\rho\underline{c}^{(j)}]\!]/[\![(\rho\underline{c}^{(j)})^{-1}]\!]$
  - $\underline{v}_j^I = [\![\underline{w}^{(j)}]\!]/[\![(\rho\underline{c}^{(j)})^{-1}]\!]$
- Contact-stick and bonded solutions are identical
- Contact-stick and separation solutions are physically distinct:
  - Contact-stick
  - Separation

$$\Delta\mathbf{u} = 0 \qquad \Delta\mathbf{u} = ?$$

$$\mathbf{s} = ? \qquad \mathbf{s} = \mathbf{0} \quad or \quad \mathbf{s} = f(\Delta\mathbf{u})(TSL)$$

Interface Matching condition: Contact-slip

- Matching conditions
  - Momentum: $[\![\check{\mathrm{s}}]\!] = 0$
  - Compatibility: $[\![\underline{\check{v}}_1]\!] = 0$
  - Coulomb's law: $\underline{s}^{j\,II} = k\langle -\underline{s}^{1\,I}\rangle_+ (e_{\check{\mathbf{v}}})^j \; j \neq 1$
- Riemann Solution

$$\underline{s}^{j\,II} = \begin{cases} \underline{s}^{1\,I} & j = 1 \\ k\langle -\underline{s}^{1\,I}\rangle_+ (e_{\boldsymbol{\tau}^I})^j & j \neq 1 \end{cases}$$

$$\underline{v}_j^{II} = \begin{cases} \underline{v}_1^I & j = 1, \\ \underline{w}^{(j)}/\rho\underline{c}^{(j)} - \underline{s}^{j\,II}/\rho\underline{c}^{(j)} = \underline{\check{v}}_j + \left(\underline{\check{s}}^j - \underline{s}^{j\,II}\right)/\rho\underline{c}^{(j)} & j \neq 1 \end{cases}$$

- Slip velocity $[\![\check{\mathrm{v}}]\!] \to \mathbf{0}$ at stick-slip transition $\Rightarrow$ Discontinuous response from $(e_{\check{\mathbf{v}}})^j$
- many regularizations proposed to solve the problem (Karnopp, Mostaghel, Quinn)
- $\boldsymbol{\tau}^I > 0$ at stick-slip transition $\Rightarrow$ Continuous response from $\boldsymbol{\tau}^I > 0$

Interface Matching condition: Separation

- Matching conditions
  - Momentum and constitutive equation: $\check{\mathbf{s}} = \mathbf{0}$
  - No compatibility condition
- Riemann Solution
  - $\mathbf{s}^{III} = \mathbf{0}$
  - $\underline{v}_j^{III} = \frac{\underline{w}^{(j)}}{\rho \underline{c}^{(j)}}$

Contact modes numerical regularization

- Separation to Contact transition may introduce shocks in tractions and velocities

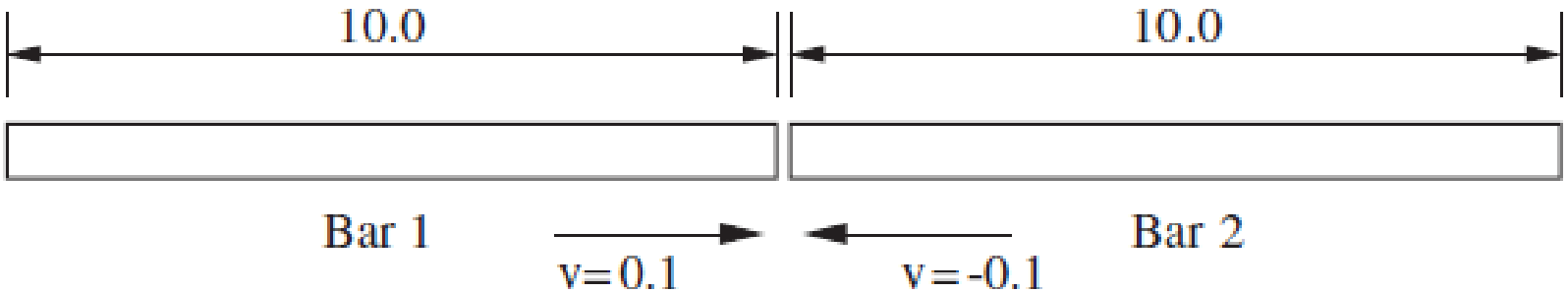
$$w^{\alpha} = -c\rho\bar{v} \qquad w^{\beta} = -c\rho\bar{v}$$

Separation	$s^{\alpha,\beta} = 0,$	$v^{\alpha,\beta} = \bar{v}$
Contact	$s^{\alpha,\beta} = -c\rho\bar{v},$	$v^{\alpha,\beta} = 0$

- Regularization is based on separation mode induced displacement jump ($\delta_s : \quad \dot{\delta}_s = 2\bar{v}$)
- A minimum separation value of $\bar{r}_c$ is ensured in the regularized model
- Continuous stick to slip transitions $\Rightarrow$ no stick-slip regularization

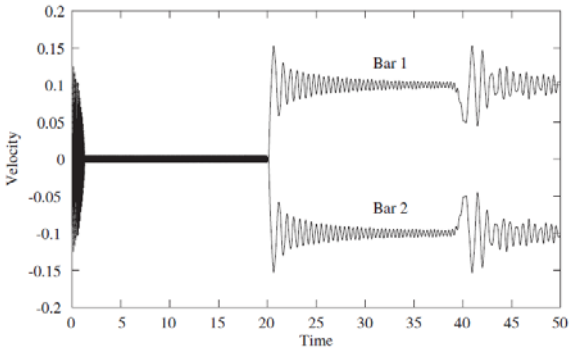
Numerical verification: Identical bars

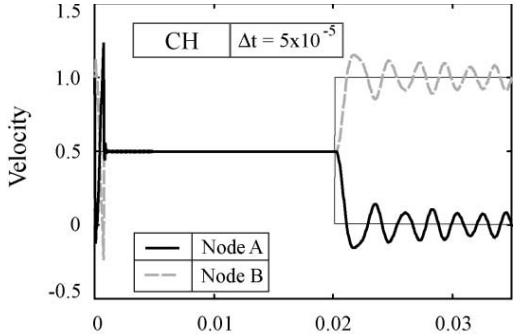
- Bench mark problem studied by T.J.R. Hughes *et al.* (1976); T. A. Laursen, V. Chawla (1997), A. Czekanski, S.A. Meguid (2001); F. Cirak, M. West (2005), ...

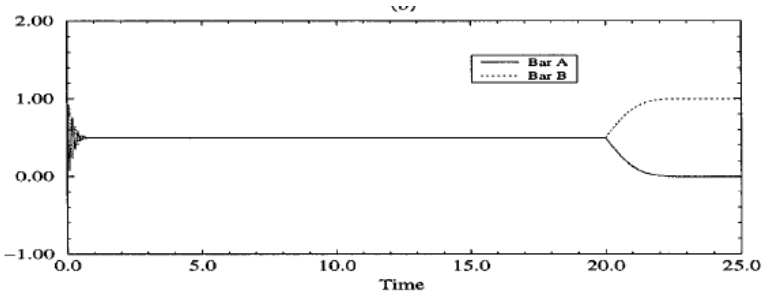


F. Cirak, M. West, Int. J. Numer. Meth. Engng 2005; 64:1078-1110

$$E_1 = E_2 = 100$$

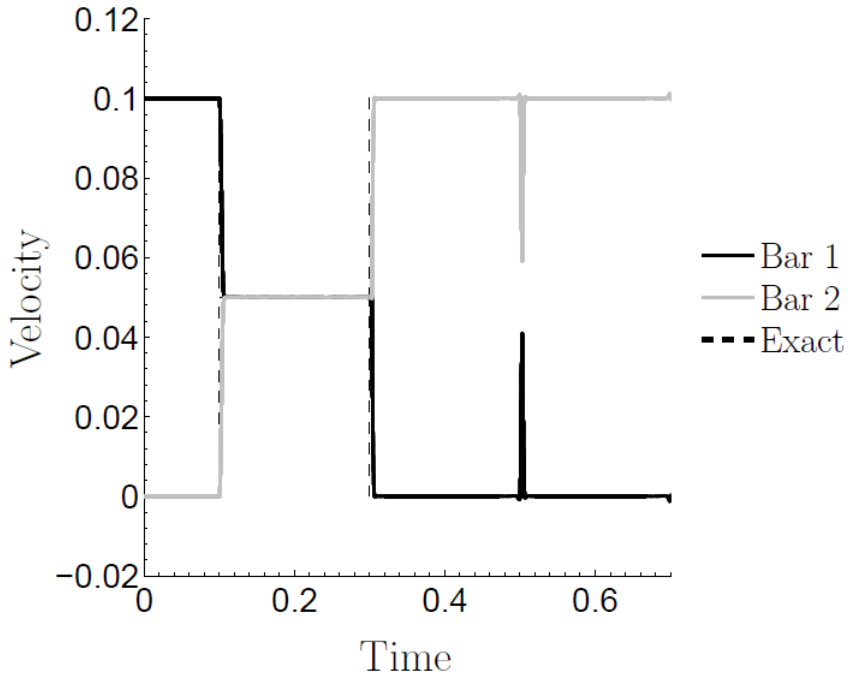


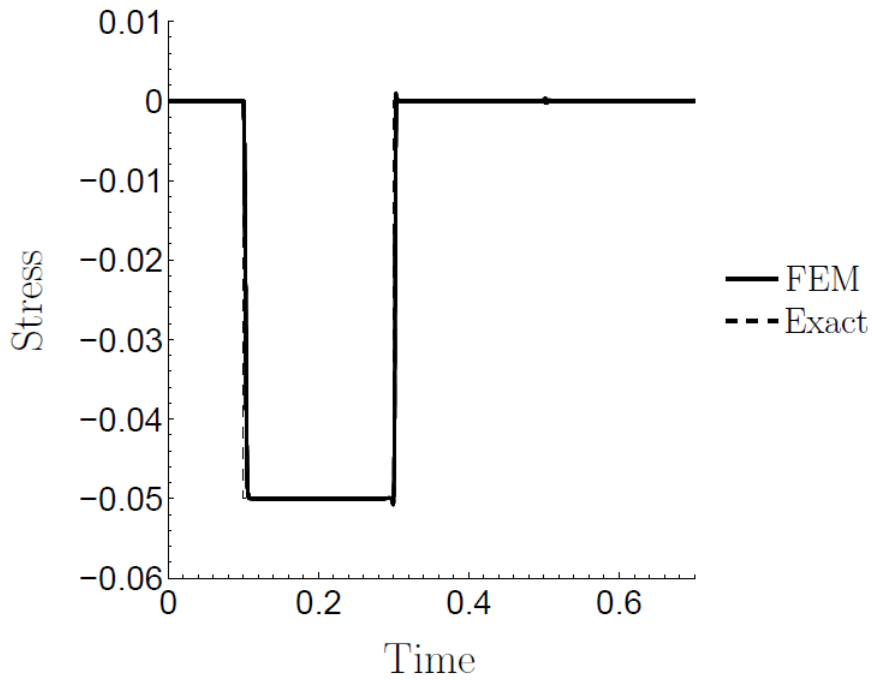




Identical bars
Riemann contact values

- Numerical results obtained by incorporating the Riemann target values in the Spacetime Discontinuous Galerkin (SDG) finite element method

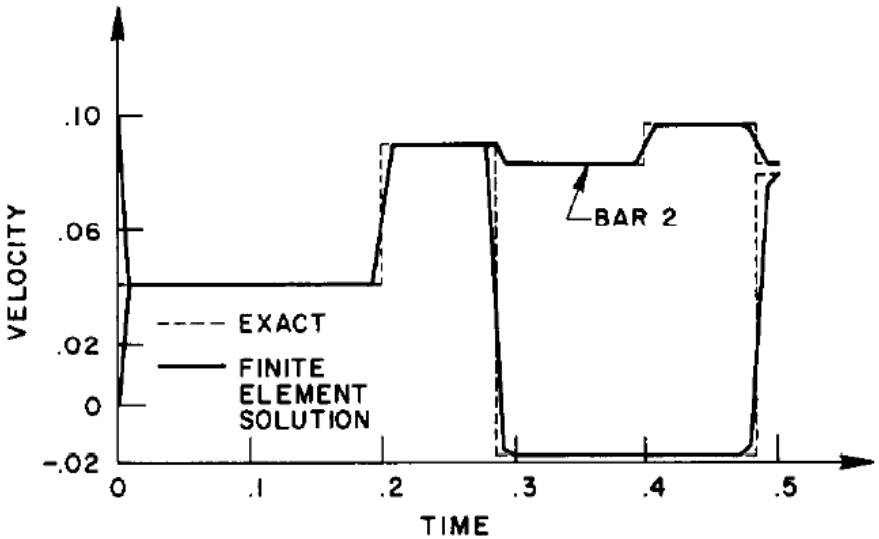




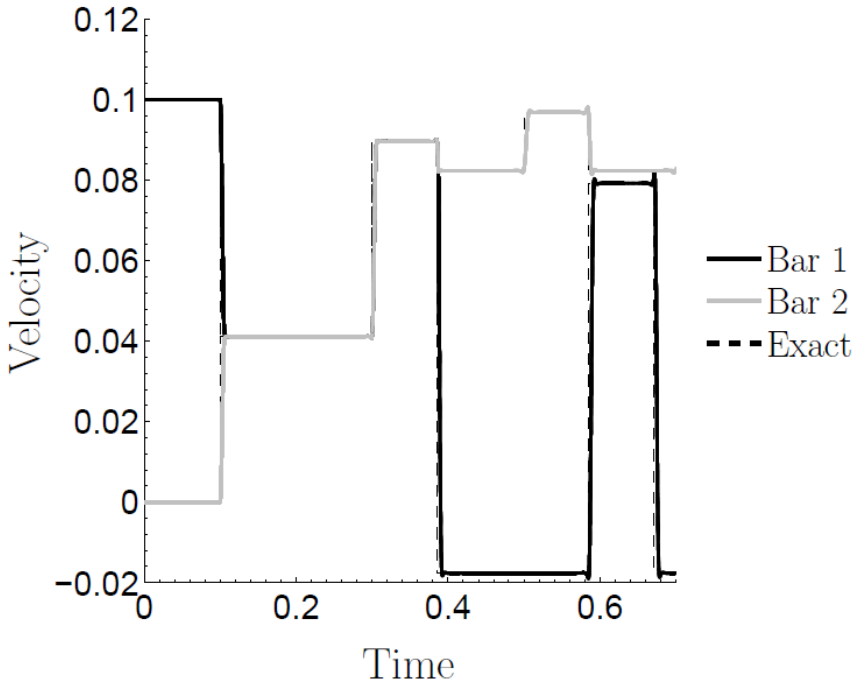
Numerical verification: Dissimilar bars

- Same as previous example except $E_1 = 49, E_2 = 100$

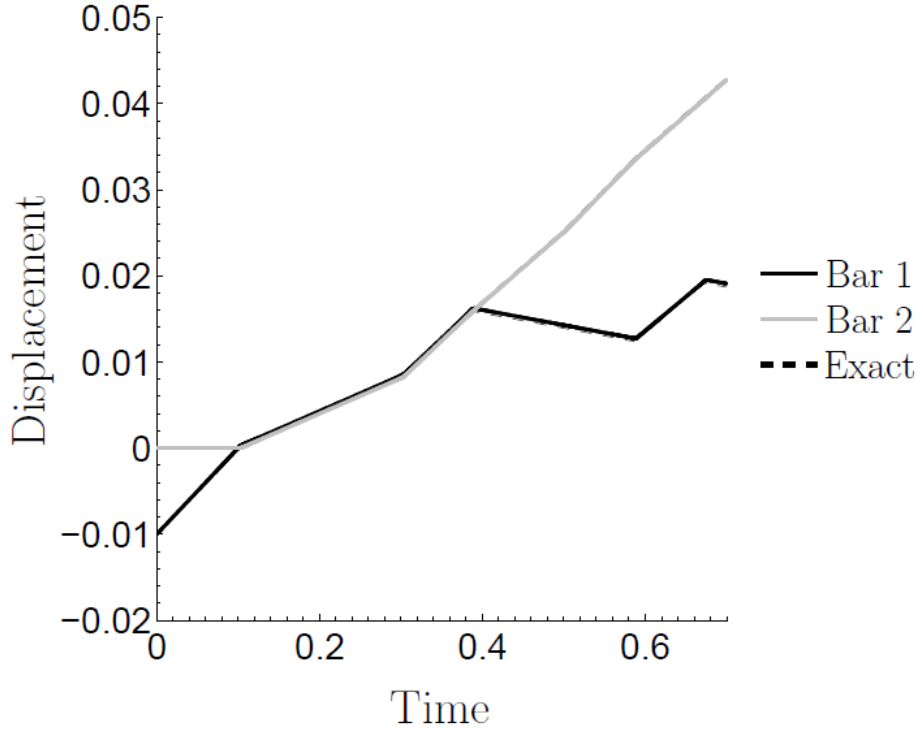
T.J.R. HUGHES, *et al.*, Comput. Methods Appl. Mech. Engrg., 8 (1976) 249-276 (varying numerical dissipation for different time intervals)

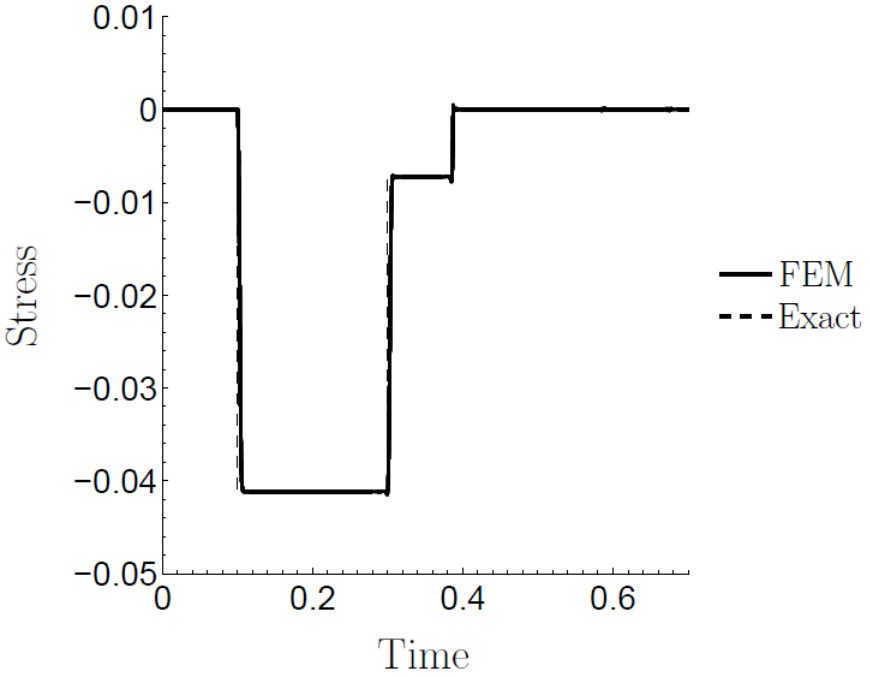


SDG with Riemann contact solutions

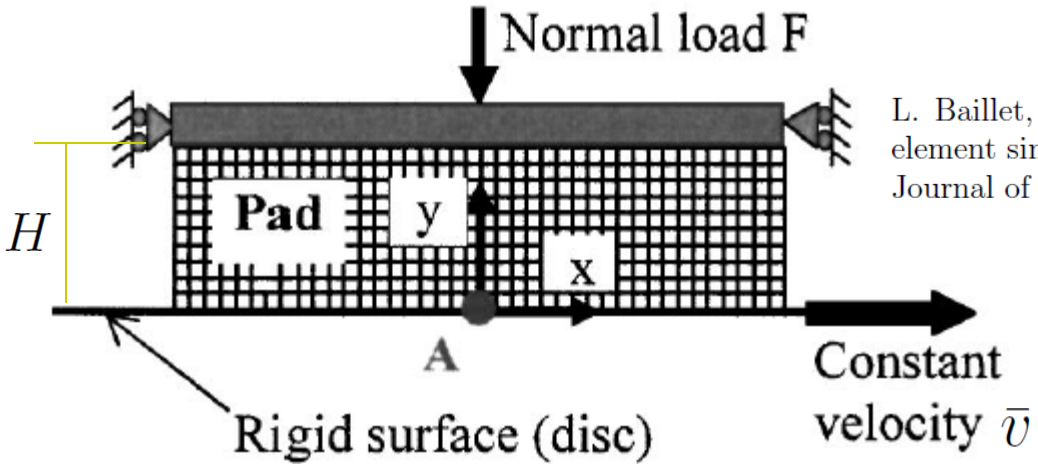


Dissimilar bars





Brake simulation
Contact mode transitions

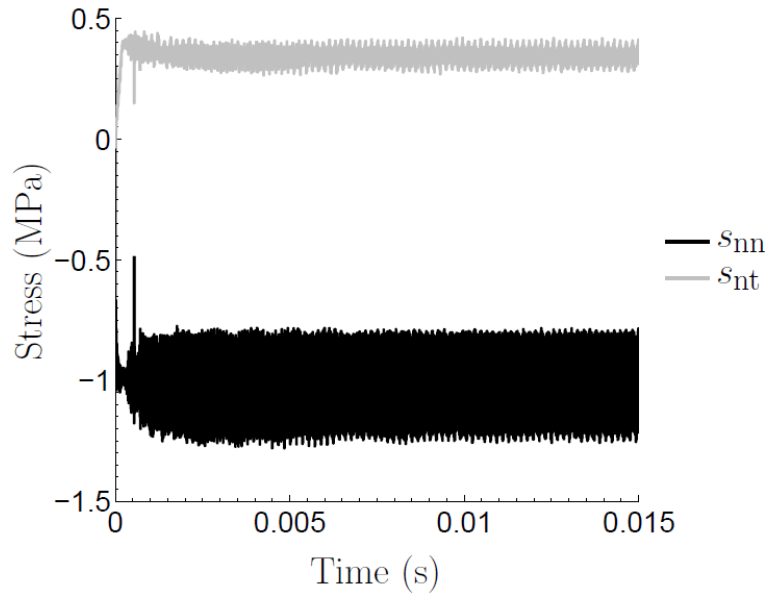


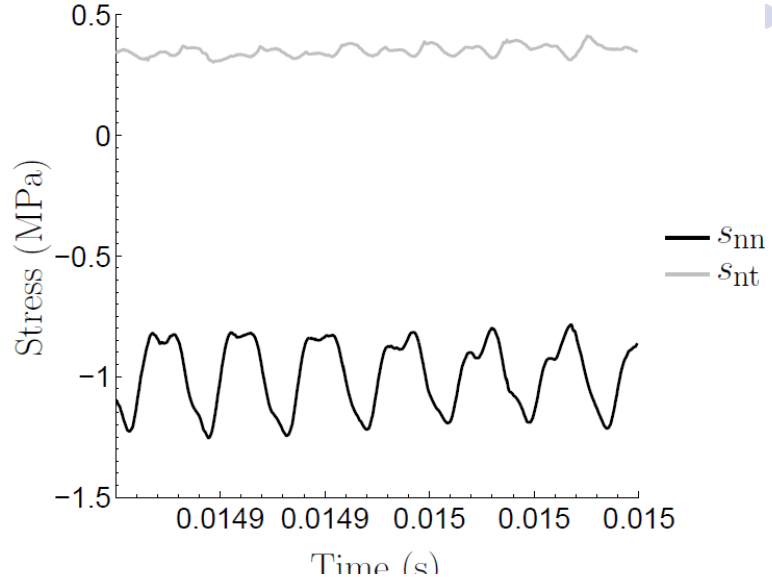
L. Baillet, V. Linck, S. D'Errico, B. Laulagnet, Y. Berthier, Finite element simulation of dynamic instabilities in frictional sliding contact, Journal of Tribology 127 (2005) 652 – 657.

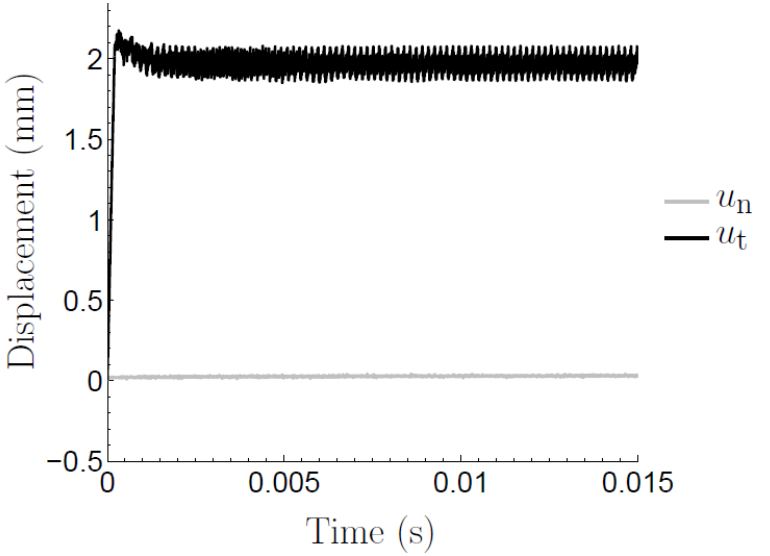
$E = 10$ GPa
$\rho = 2000$ kg/m^3
$\nu = 0.3$
$L \times H = 100$ mm $\times$ 20 mm
$\bar{\sigma} = 1$ MPa
10^{-5} m/s $\leq \bar{v} \leq 2 \times 10^{-3}$ m/s

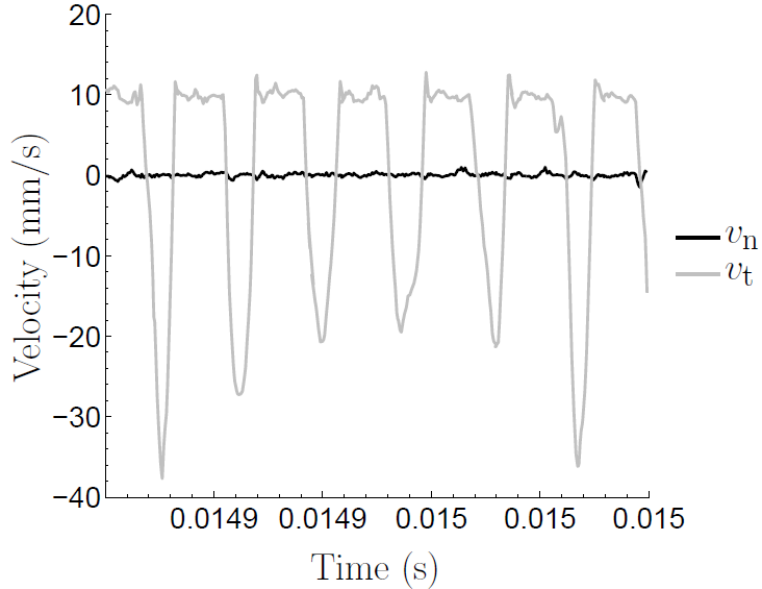
- Variation of $\bar{v}$ produces different modes of contact instabilities (Baillet *et al.*, 2005; Adams, 1995; Oueslati *et al.*, 2003; Linck, 2005; Massi et al., 2007)
- $T \approx 2H/c_1 \approx 15.4\ \mu$s

Brake simulation low amplitude load

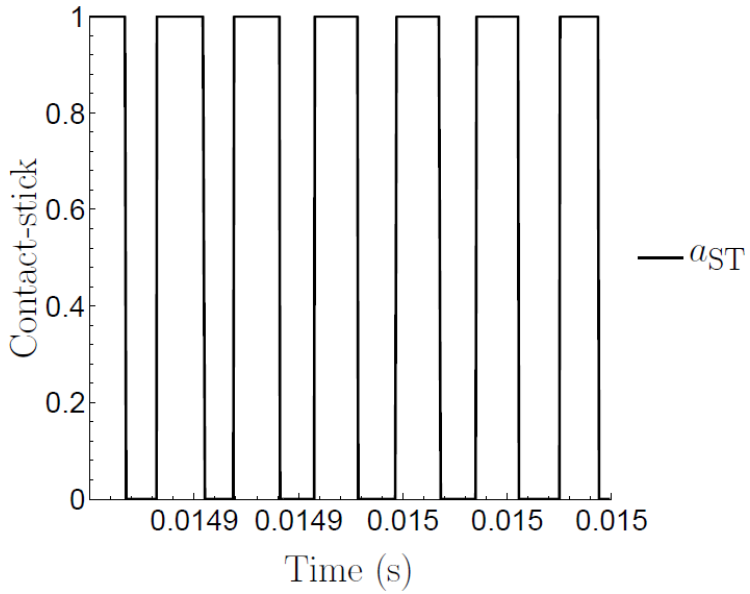


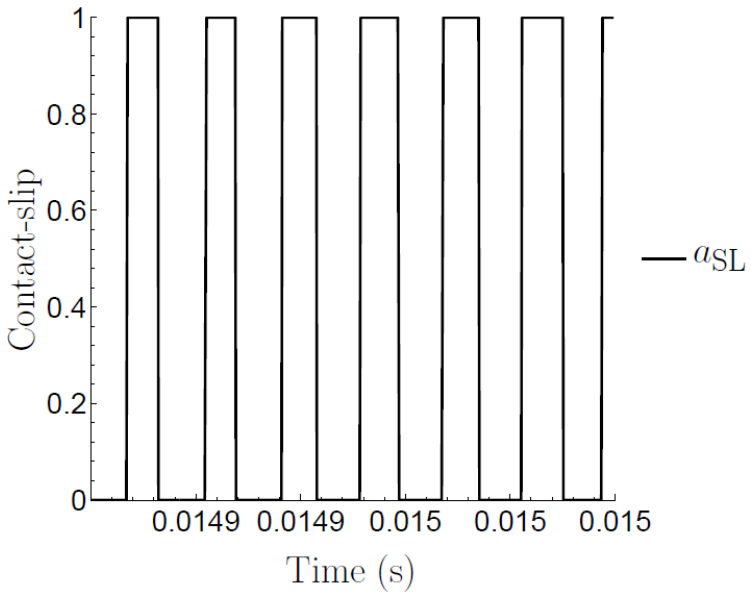


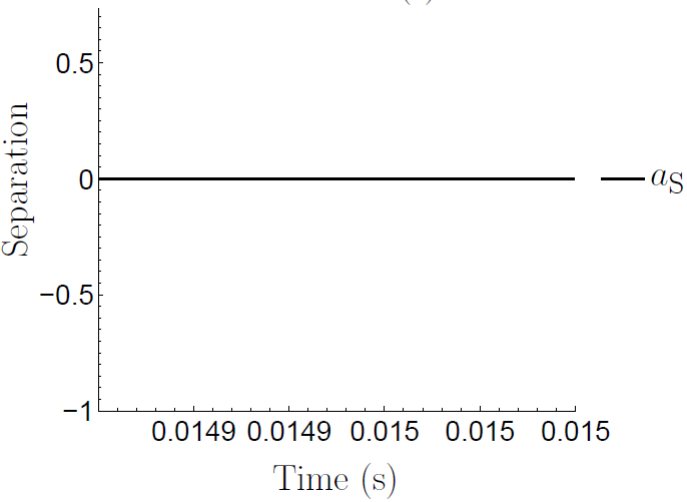




Brake simulation
low amplitude load







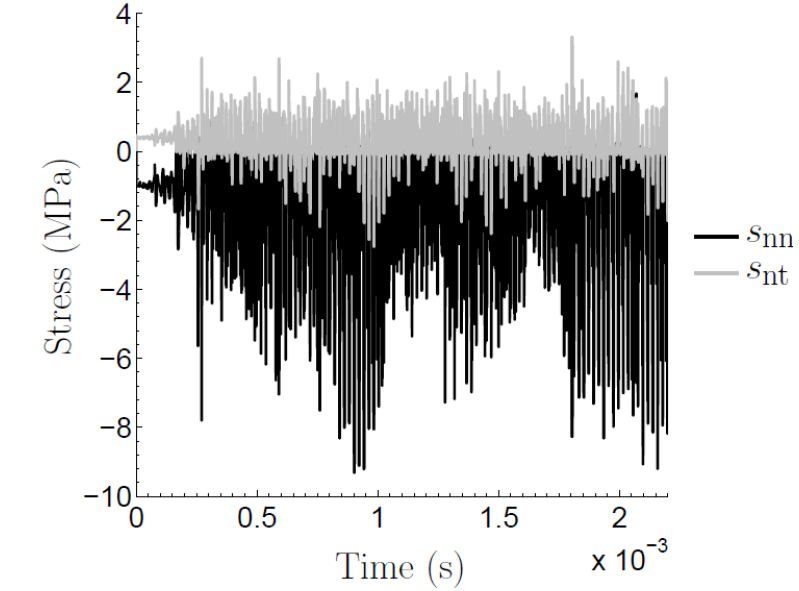
stick-slip transitions

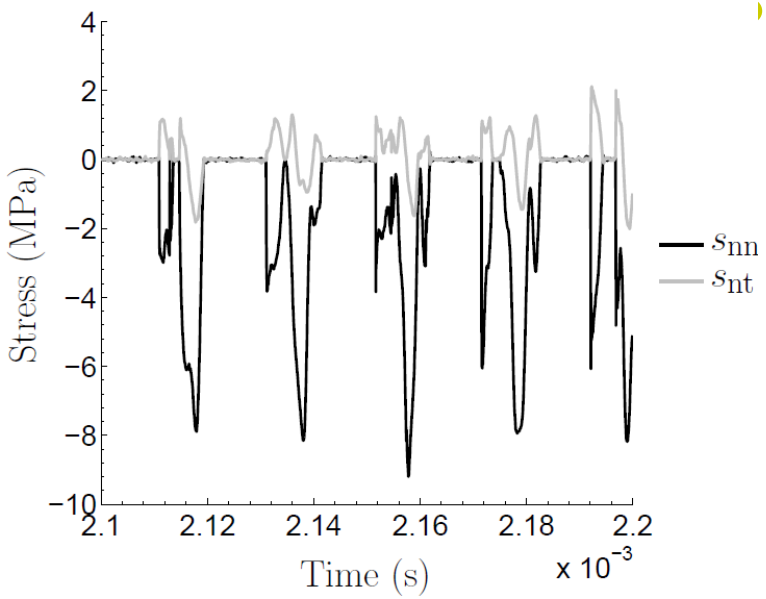
Point A

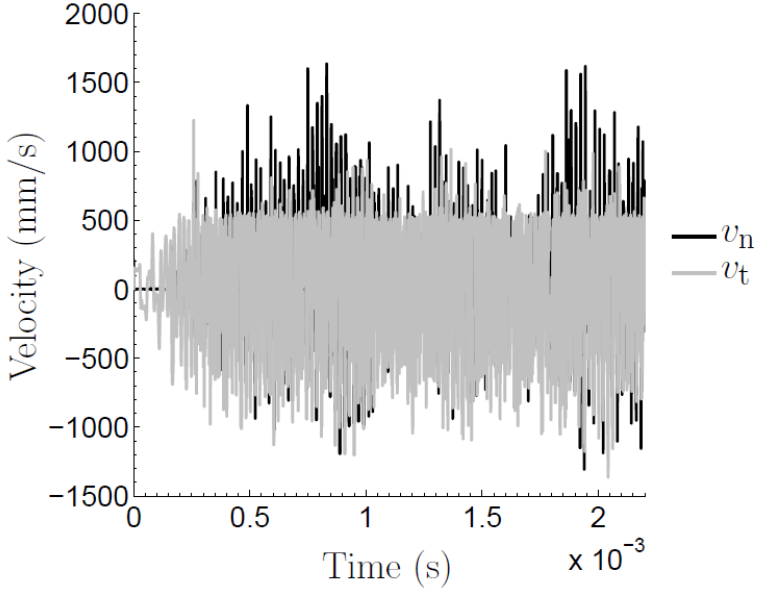
low amplitude load simulation

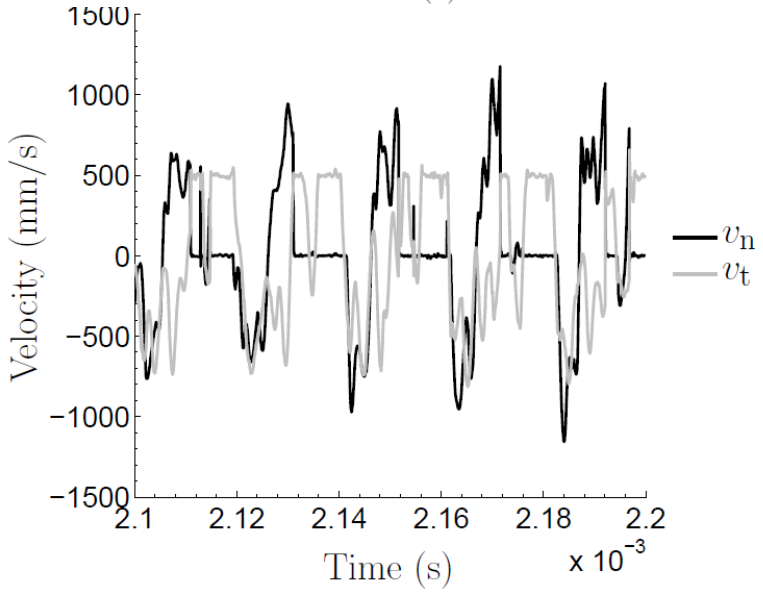
click to play movie

Brake simulation medium amplitude load

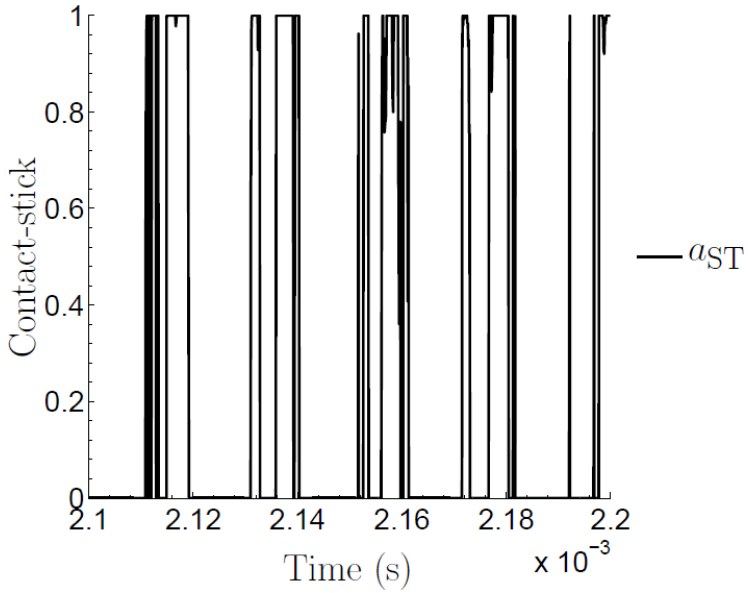


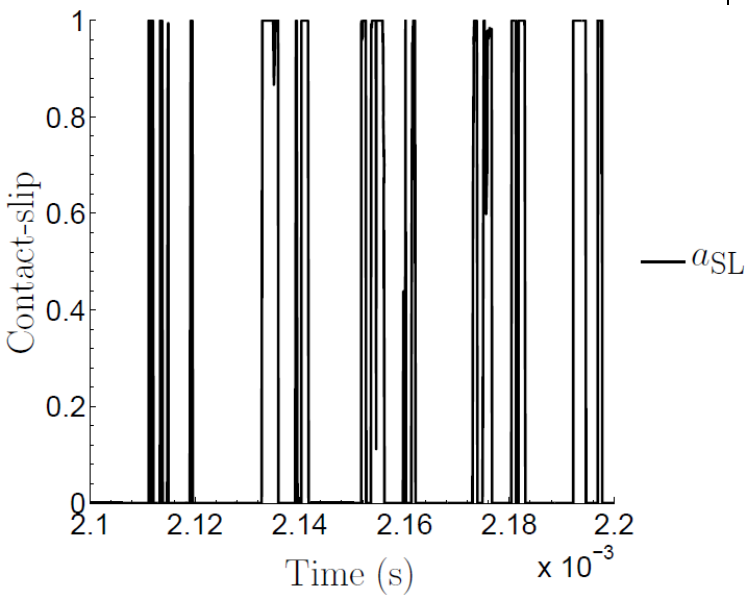


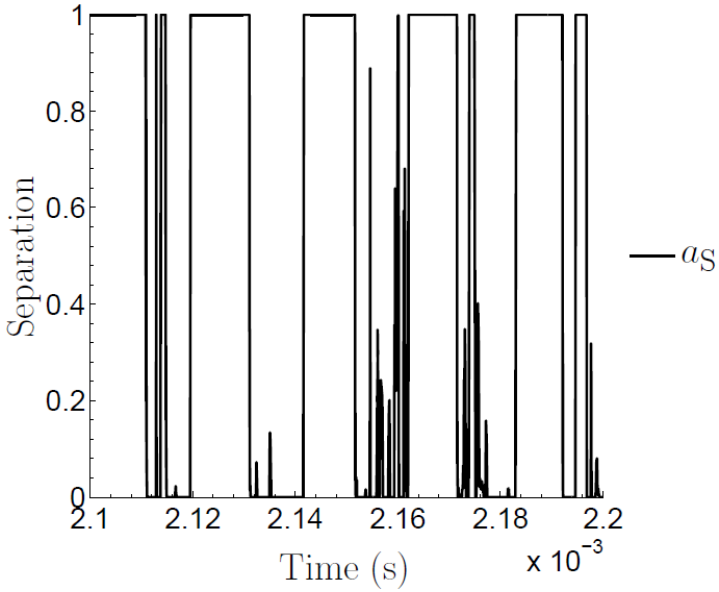




Brake simulation medium amplitude load

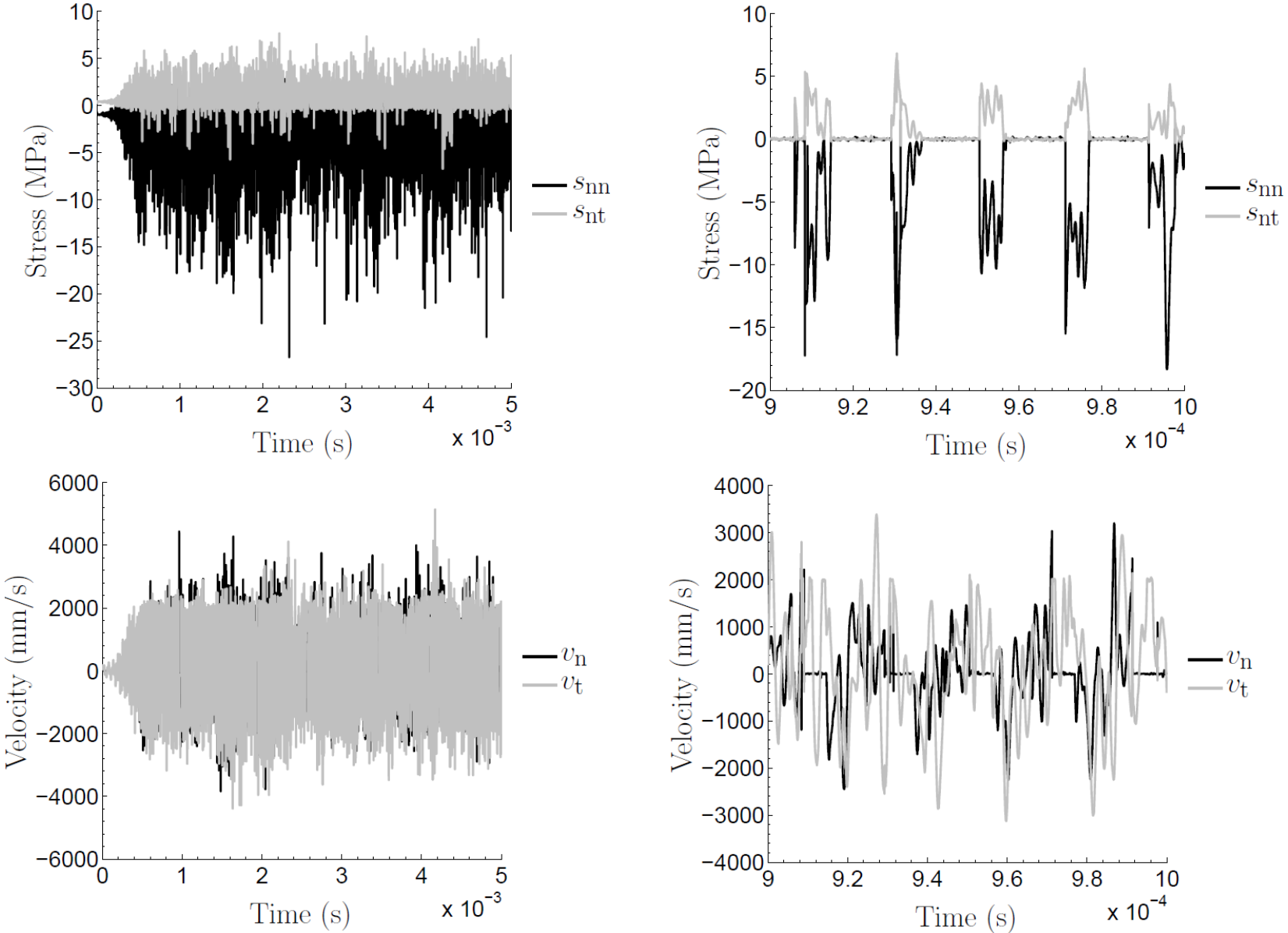




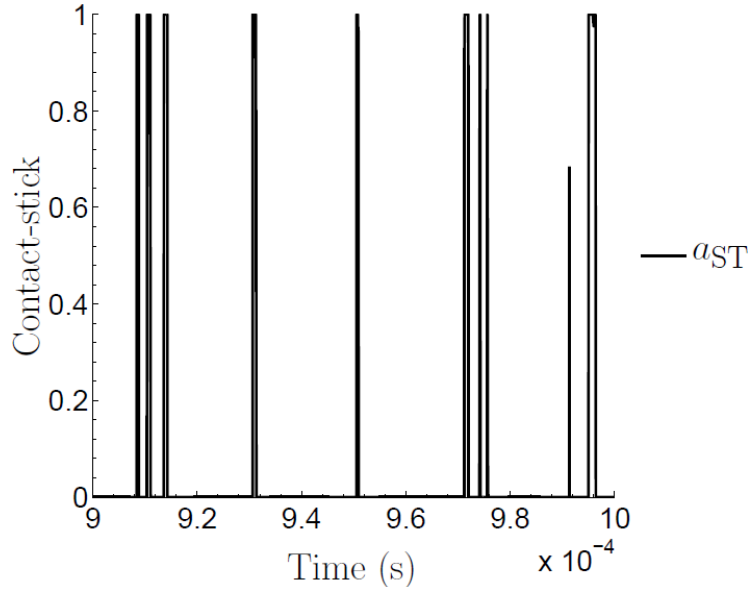


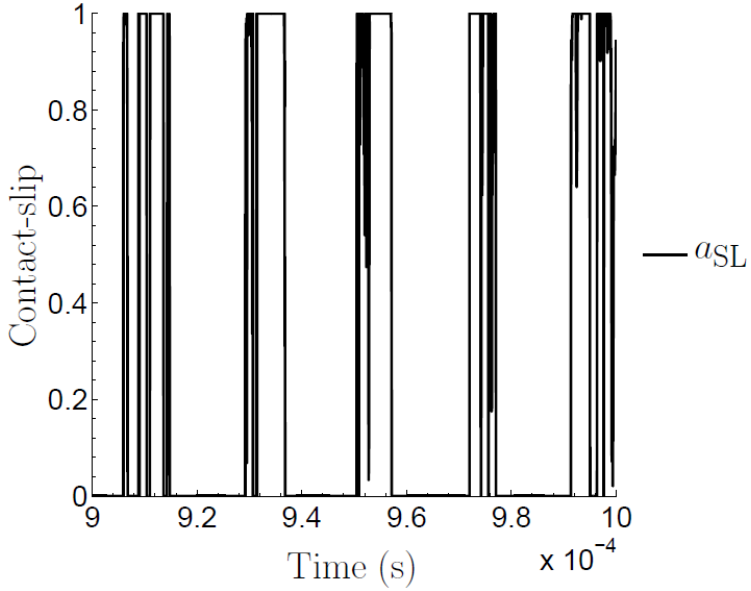
stick-slip-separation transitions

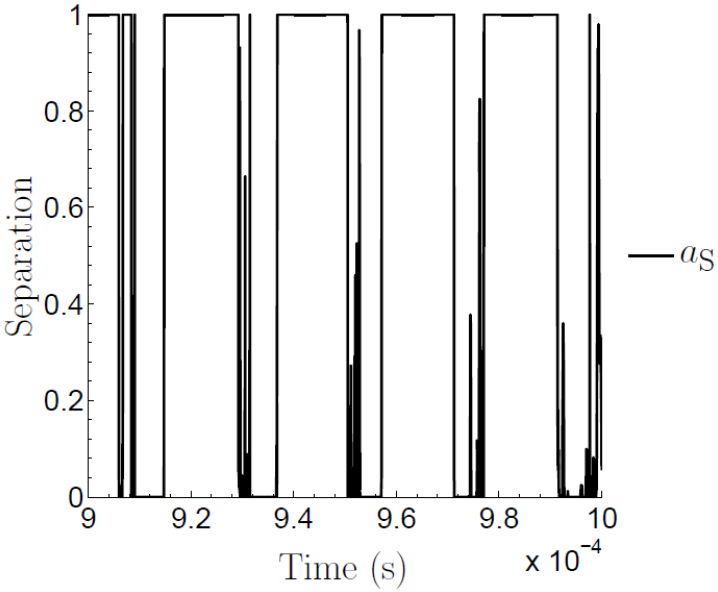
Brake simulation high amplitude load



Brake simulation high amplitude load



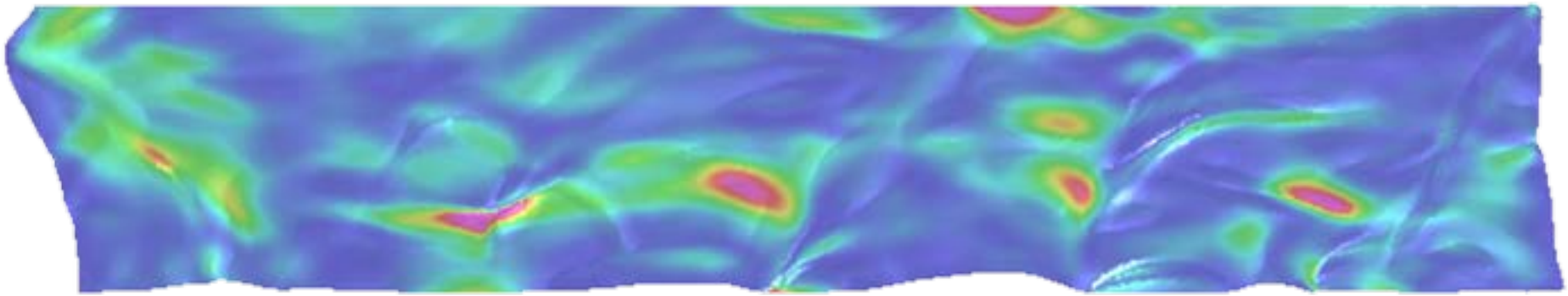




slip-separation transitions

high amplitude load simulation

click to play movie



Interfacial damage models

Conical (parabolic) marking on the crack surface

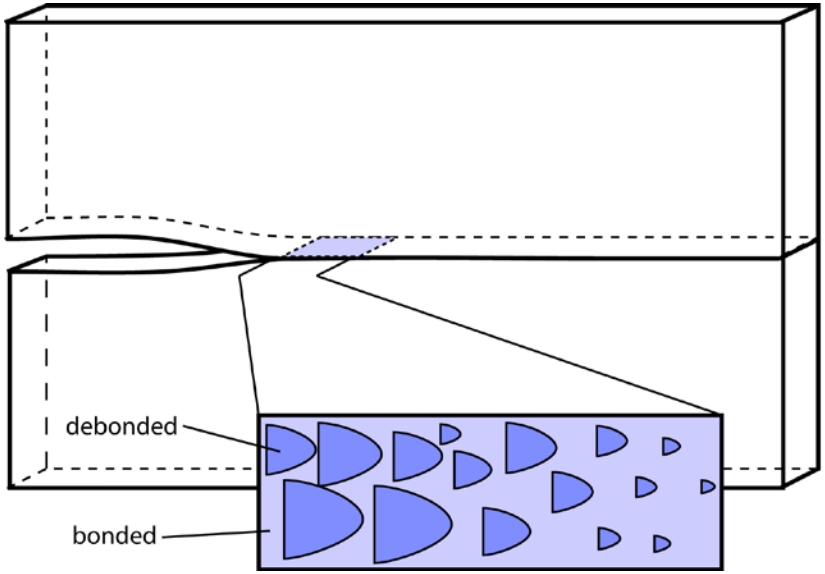
Ravi-Chandar, Knauss 1984

∂Q^d

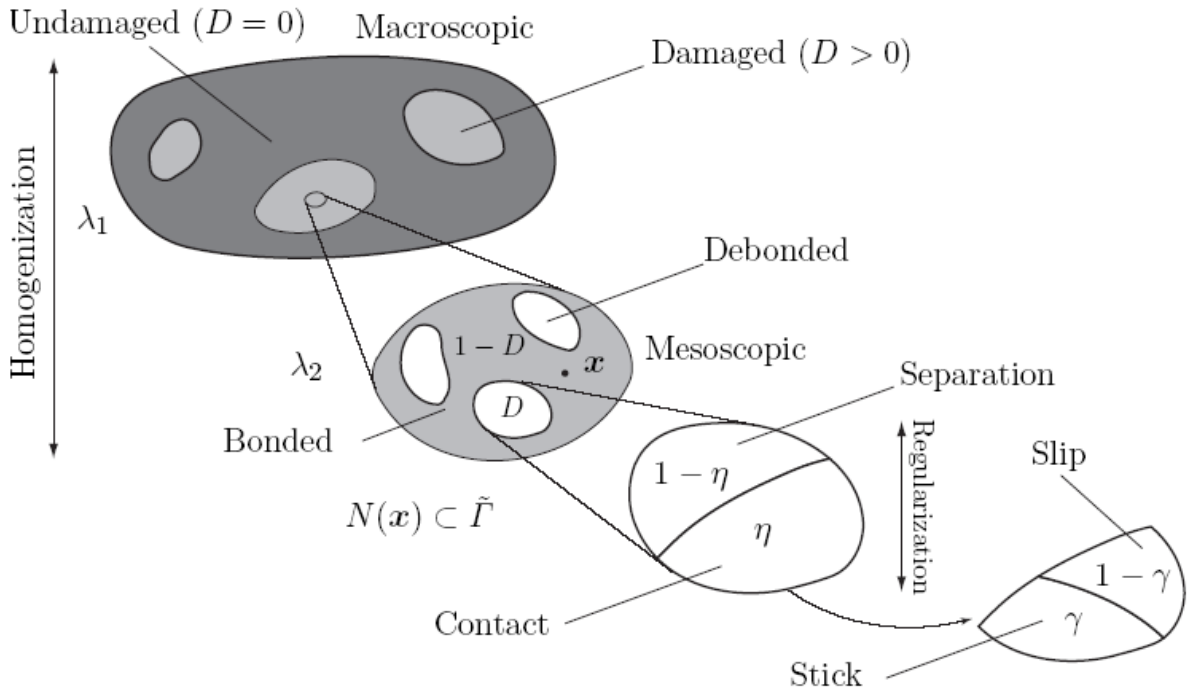
$\text{area fraction} = D$

∂Q^b

$\text{area fraction} = 1 - D$



Mesoscopic interface subdivisions

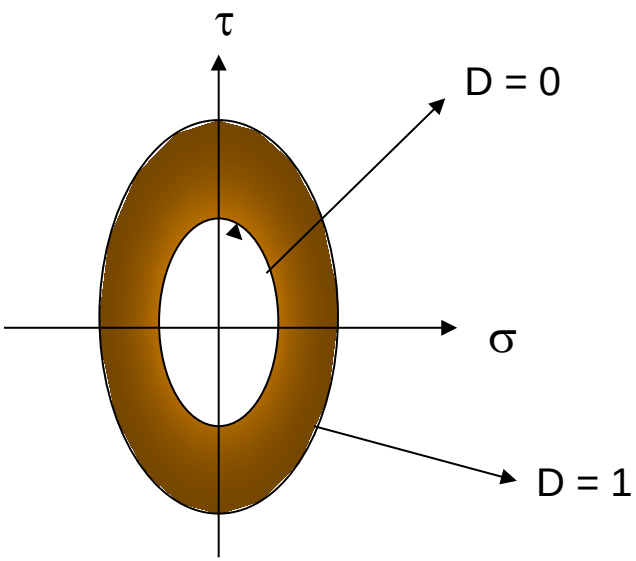


Problem set up

- Target Riemann values on individual subdivisions
- Area fractions of subdivisions

Damage Evolution law

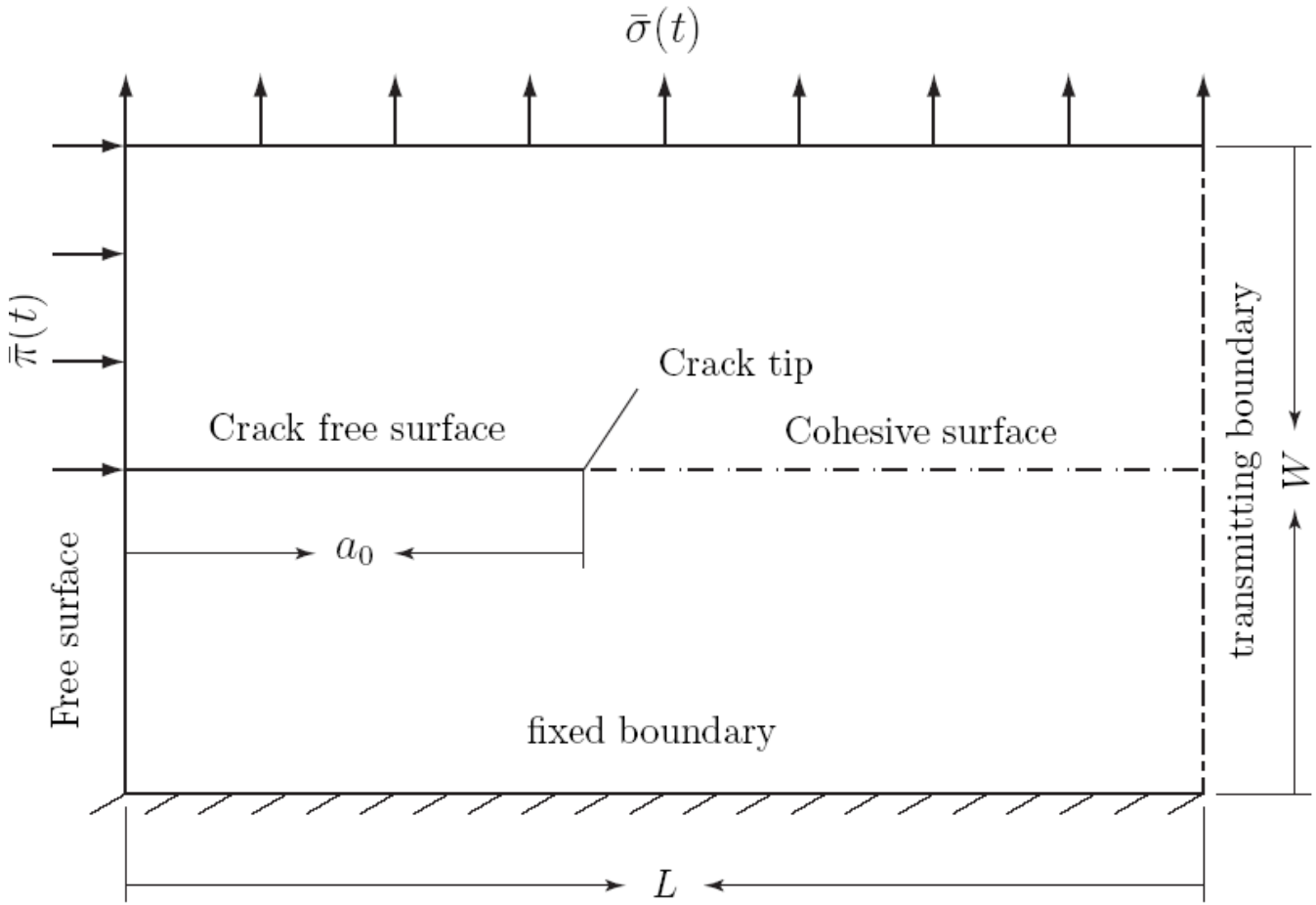
$$\dot{D} = \begin{cases} \frac{1}{\tilde{\tau}}[1 - H(\langle f(y) - D \rangle_+)] & D < 1 \\ 0 & D = 1 \end{cases} \qquad H(x) = \exp(-a.x)$$



Properties of the evolution law

- a maximum damage rate, exists.
- Target damage value, $f(y)$, is a function of stress.
- The rate is a function of the difference between damage and its target value.

Numerical Examples: All modes



Crack closure: cyclic, dynamic loading

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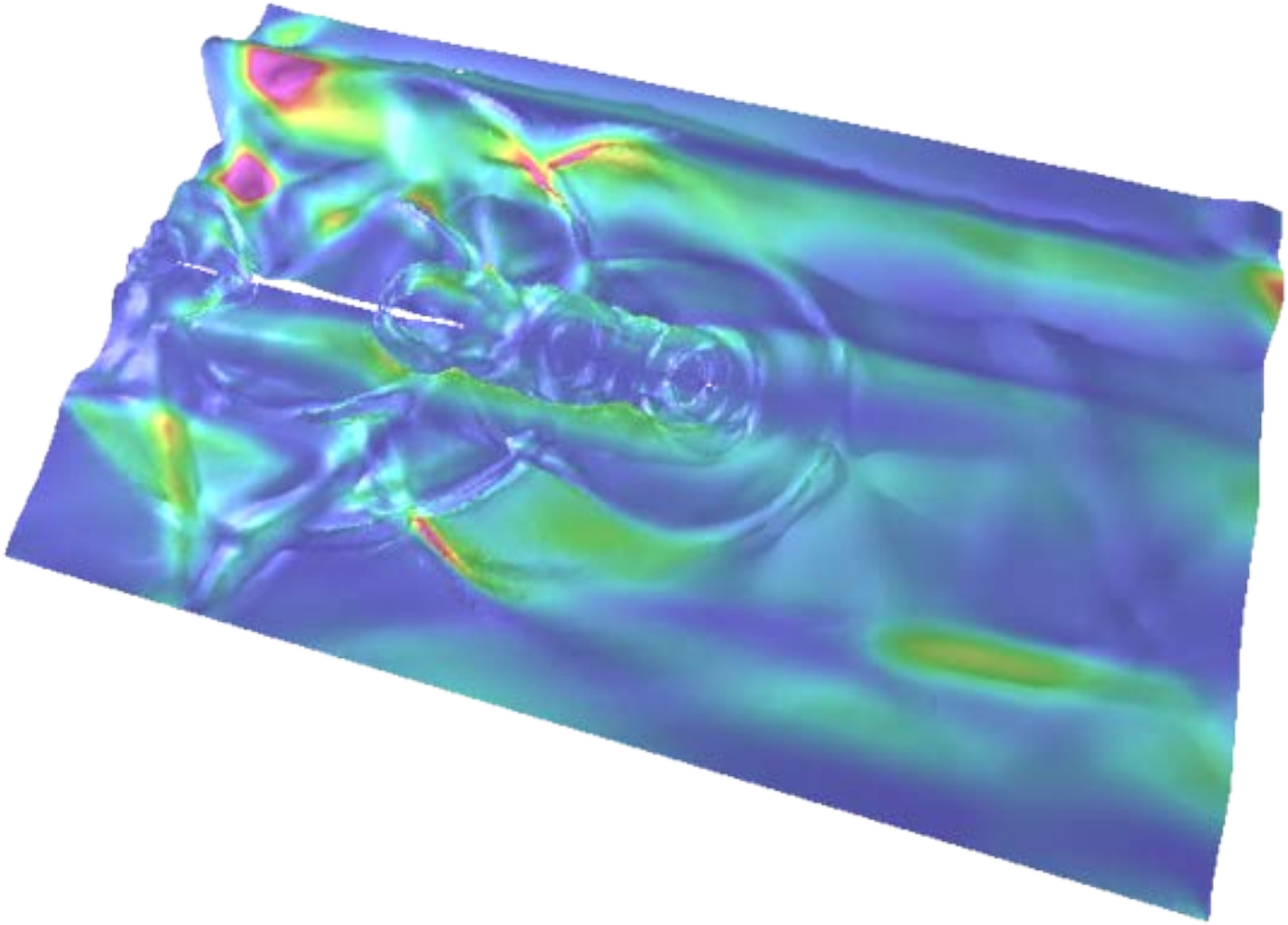


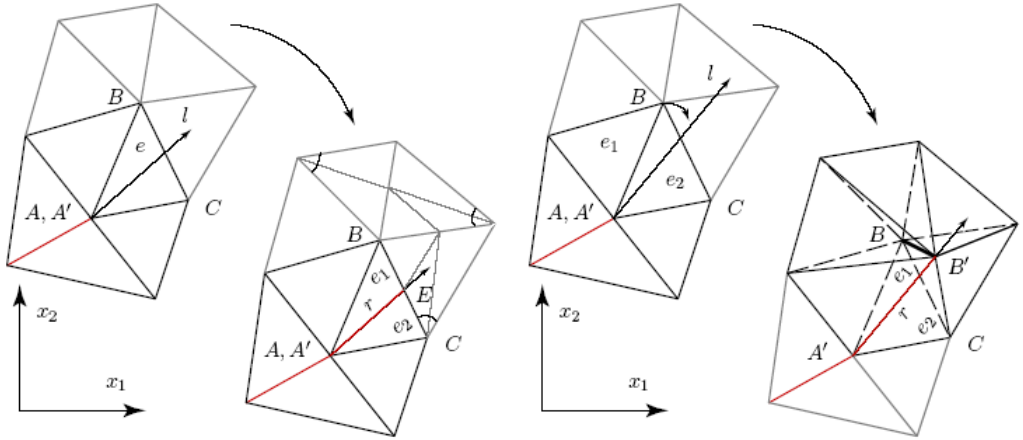
Solution-dependent crack path

Propabilistic crack nucleation

- Crack nucleates from defects with random distribution of strength

Crack growth

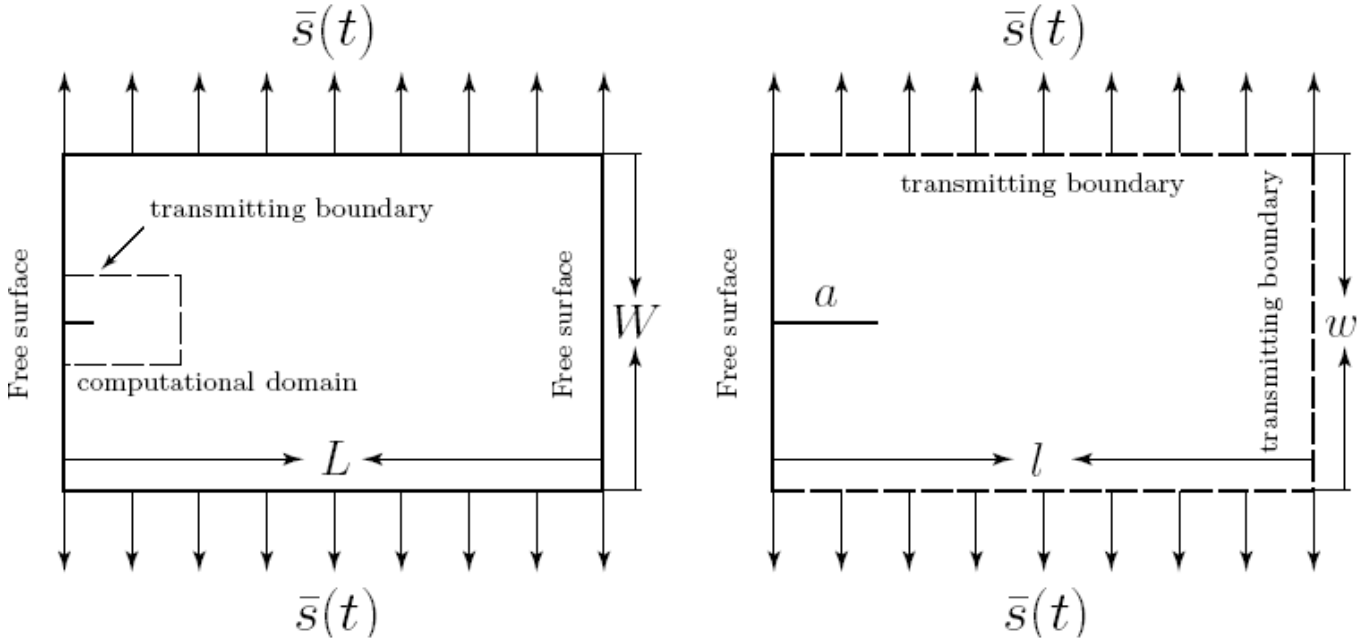
- Crack propages along the direction(s) of maximum effective stress
- Element boundaries are aligned with arbitrary propagation directions
- No discontinuous features are introduced within the finite elements
- No additional criteria used for branching



(a) CZT insertion on the opposite edge of the CZT by a refinement operation.

(b) The spatial transition of a vertex by a tent pitching operation.

Solution-dependent crack propagation: Geometry and loading

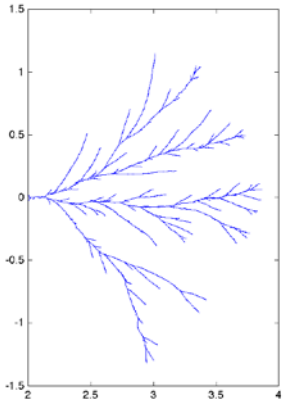


(a) The experimental setting. (b) The computational domain.

PMMA:

- $E = 3.24GPa.$
- $\rho = 1190\frac{Kg}{m^3}.$
- $w = 0.020MPa.$
- $\tau = 10^{-2}\mu s.$

based on Sharon and Fineberg experiment (1996)



Solution-dependent crack propagation

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