Riemann Conditions for Elastodynamic Contact and Rate-Dependent Interfacial-Damage Fracture Model

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11th U.S. National Congress on Computational Mechanics  
University of Minnesota  
July 25-28, 2011

Center for Process Simulation & Design  
NSF ITR/AP DMR 01-21695  
DOE

Center for Simulation of Advanced Rockets
Riemann Solution

Bulk material model

- Direct solution of the Riemann problem from momentum balance and compatibility condition jump parts

- Unified expressions for normal and tangential directions

- Matching conditions at material interface ⇒ different contact/separation solutions

- The characteristic values $w^{(j)} := s^j + e^{(j)} p^j$ are preserved

\[ \sigma = \begin{cases} -2\sigma_0 & \text{in } \alpha \\ \sigma = 0 & \text{in } \beta \end{cases} \]

\[ w = 2\sigma_0 \]
Interface Matching condition: 
Bonded and contact-stick

- **Matching conditions**
  - Momentum: $\llbracket \dot{s} \rrbracket = 0$
  - Compatibility: $\llbracket \tilde{\nu} \rrbracket = 0$

- **Riemann Solution**
  - $s^I_j = \left[ \frac{w^{(j)}/\rho c^{(j)}}{\left[ (\rho c^{(j)})^{-1} \right]} \right]$
  - $v^I_j = \left[ \frac{w^{(j)}}{\left[ (\rho c^{(j)})^{-1} \right]} \right]$

- Contact-stick and bonded solutions are identical

- Contact-stick and separation solutions are physically distinct:
  - Contact-stick
    - $\Delta u = 0$
    - $s = ?$
  - Separation
    - $\Delta u = ?$
    - $s = 0 \text{ or } s = f(\Delta u)(TSL)$
Interface Matching condition: Contact-slip

- Matching conditions
  - Momentum: $[\vec{s}] = 0$
  - Compatibility: $[[\vec{u}_1]] = 0$
  - Coulomb’s law: $g^{j\, II} = k\langle -s^{1\, I}\rangle_+ (e_\varphi)^j \ j \neq 1$

- Riemann Solution

$$g^{j\, II} = \begin{cases} 
  s^{1\, I} & j = 1 \\
  k\langle -s^{1\, I}\rangle_+ (e_\tau)^j & j \neq 1 
\end{cases}$$

$$v_j^{\, II} = \begin{cases} 
  v_1^{\, I} & j = 1, \\
  \frac{w^{(j)}}{\rho_c^{(j)}} - s^{j\, II}/\rho_c^{(j)} = \tilde{v}_j + \left(\tilde{s}_j - s^{j\, II}\right)/\rho_c^{(j)} & j \neq 1
\end{cases}$$

- Slip velocity $[[\vec{v}]] \to 0$ at stick-slip transition ⇒ Discontinuous response from $(e_\varphi)^j$
- many regularizations proposed to solve the problem (Karnopp, Mostaghel, Quinn)
- $\tau^I > 0$ at stick-slip transition ⇒ Continuous response from $\tau^I > 0$
Interface Matching condition: Separation

- **Matching conditions**
  
  - Momentum and constitutive equation: $\ddot{s} = 0$
  
  - No compatibility condition

- **Riemann Solution**
  
  - $s^{III} = 0$
  
  - $v_j^{III} = \frac{w(j)}{\rho_e(j)}$
Contact modes numerical regularization

- Separation to Contact transition may introduce shocks in tractions and velocities:
  \[ w^\alpha = -c \rho \ddot{u} \quad w^\beta = -c \rho \ddot{u} \]

  ![Diagram showing separation and contact conditions](image)

  - Separation: \( s^{\alpha,\beta} = 0 \), \( \nu^{\alpha,\beta} = -c \rho \ddot{u} \)
  - Contact: \( s^{\alpha,\beta} = -c \rho \ddot{u} \), \( \nu^{\alpha,\beta} = 0 \)

- Regularization is based on separation mode induced displacement jump (\( \delta_s : \dot{\delta}_s = 2\ddot{u} \))

- A minimum separation value of \( \bar{r}_c \) is ensured in the regularized model

- Continuous stick to slip transitions \( \Rightarrow \) no stick-slip regularization
Numerical verification: Identical bars

- Benchmark problem studied by T.J.R. Hughes et al. (1976); T. A. Laursen, V. Chawla (1997), A. Czekanski, S.A. Meguid (2001); F. Cirak, M. West (2005), ...

\[ E_1 = E_2 = 100 \]
Identical bars
Riemann contact values

- Numerical results obtained by incorporating the Riemann target values in the Spacetime Discontinuous Galerkin (SDG) finite element method
Numerical verification: Dissimilar bars

- Same as previous example except $E_1 = 49, E_2 = 100$

(varying numerical dissipation for different time intervals)
Dissimilar bars
Brake simulation
Contact mode transitions


\[ E = 10 \text{ GPa} \]
\[ \rho = 2000 \text{ kg/m}^3 \]
\[ \nu = 0.3 \]
\[ L \times H = 100 \text{ mm} \times 20 \text{ mm} \]
\[ \bar{\sigma} = 1 \text{ MPa} \]
\[ 10^{-5} \text{ m/s} \leq \bar{v} \leq 2 \times 10^{-3} \text{ m/s} \]

- Variation of \( \bar{v} \) produces different modes of contact instabilities (Baillet et al., 2005; Adams, 1995; Oueslati et al., 2003; Linck, 2005; Massi et al., 2007)

- \( T \approx 2H/c_1 \approx 15.4 \mu s \)
Brake simulation
low amplitude load
Brake simulation
low amplitude load

stick-slip transitions

Point A
low amplitude load simulation

click to play movie
Brake simulation
medium amplitude load

Stress (MPa)

Time (s) $\times 10^{-3}$

$S_{nn}$ $S_{nt}$

Velocity (mm/s)

Time (s) $\times 10^{-3}$

$v_n$ $v_t$
Brake simulation
medium amplitude load

stick-slip-separation transitions
Brake simulation
high amplitude load

![Graphs showing stress and velocity over time for brake simulation.](image)
Brake simulation
high amplitude load

Contact-stick

Time (s) x 10^{-4}

-- $a_{ST}$

Separation

Time (s) x 10^{-4}

-- $a_{S}$

Contact-slip

Time (s) x 10^{-4}

-- $a_{SL}$

slip-separation transitions
high amplitude load simulation

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Interfacial damage models

Conical (parabolic) marking on the crack surface

Ravi-Chandar, Knauss 1984

\[ \partial Q^d \]
area fraction = \( D \)

\[ \partial Q^b \]
area fraction = \( 1 - D \)
Mesoscopic interface subdivisions

Problem set up

- Target Riemann values on individual subdivisions
- Area fractions of subdivisions
Damage Evolution law

\[
\dot{D} = \begin{cases} 
\frac{1}{\tau}[1 - H(\langle f(y) - D \rangle_+)] & D < 1 \\
0 & D = 1 
\end{cases}
\]

\[H(x) = \exp(-a.x)\]

Properties of the evolution law

- a maximum damage rate, exists.
- Target damage value, \( f(y) \), is a function of stress.
- The rate is a function of the difference between damage and its target value.
Numerical Examples: All modes

\[ \tilde{\sigma}(t) \]

- Free surface
- Crack free surface
- Cohesive surface
- Crack tip
- Fixed boundary
- Transmitting boundary
- \( \alpha_0 \)
- \( L \)
Crack closure: cyclic, dynamic loading

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Solution-dependent crack path

Propabilistic crack nucleation

- Crack nucleates from defects with random distribution of strength

Crack growth

- Crack propagates along the direction(s) of maximum effective stress
- Element boundaries are aligned with arbitrary propagation directions
- No discontinuous features are introduced within the finite elements
- No additional criteria used for branching

(a) CZT insertion on the opposite edge of (b) The spatial transition of a vertex by a the CZT by a refinement operation. tent pitching operation.
Solution-dependent crack propagation: Geometry and loading

PMMA:

- \( E = 3.24 \text{GPa} \).
- \( \rho = 1190 \frac{Kg}{m^3} \).
- \( w = 0.020 \text{MPa} \).
- \( \tau = 10^{-2} \mu \text{s} \).

based on Sharon and Fineberg experiment (1996)
Solution-dependent crack propagation

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