

# Riemann contact conditions and a rate-dependent interfacial damage model for elastodynamics

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We present the Riemann solutions for bonded, contact-stick, frictional contact-slip, and separation modes for linearized elastodynamics. The Riemann values are the physically correct fluxes that preserve the characteristic structure of the elastodynamic problem. Consequently, and in contrast to most existing numerical methods, no artificial regularization is required for stick-to-slip transitions, for which the underlying physical process does not involve shocks. We incorporate these Riemann solutions in our *spacetime discontinuous Galerkin* (SDG) finite element method [1]. Verification simulations not only demonstrate convergence to the exact solution, but also are free of numerical oscillations observed in many of the existing methods. We also present simulations modeling stick-slip, stick-slip-separation, and slip-separation oscillations observed on brake pads.

In lieu of a traditional traction-separation relation, we derive a two-scale, rate-dependent, *interfacial* damage model to represent the crack propagation through the processes of void nucleation, growth, and coalescence in nominally brittle materials. The three distinct states of contact-stick, contact-slip, and separation modes may occur on the damaged area fraction, while the bonded Riemann values govern the behavior on the remainder of the interface. We invoke the contact Riemann solutions in SDG models in high-resolution studies of crack propagation, closure, and frictional sliding; see Figure 1(a).

A probabilistic criterion, representing the random distribution of material defects, governs the nucleation of new cohesive interfaces where damage can accumulate. The SDG solver's powerful adaptive meshing capabilities permit the cohesive surfaces to extend in any direction dictated by the model, while continuously refining and smoothing the surrounding mesh to ensure solution accuracy and good element quality. As seen in Figure 1(b), probabilistic nucleation and free crack extension suffice to capture phenomena such as a crack branching, and microcracking. The influence of activating the crack-closure contact model on the pattern of branching is seen in Figure 1(c).

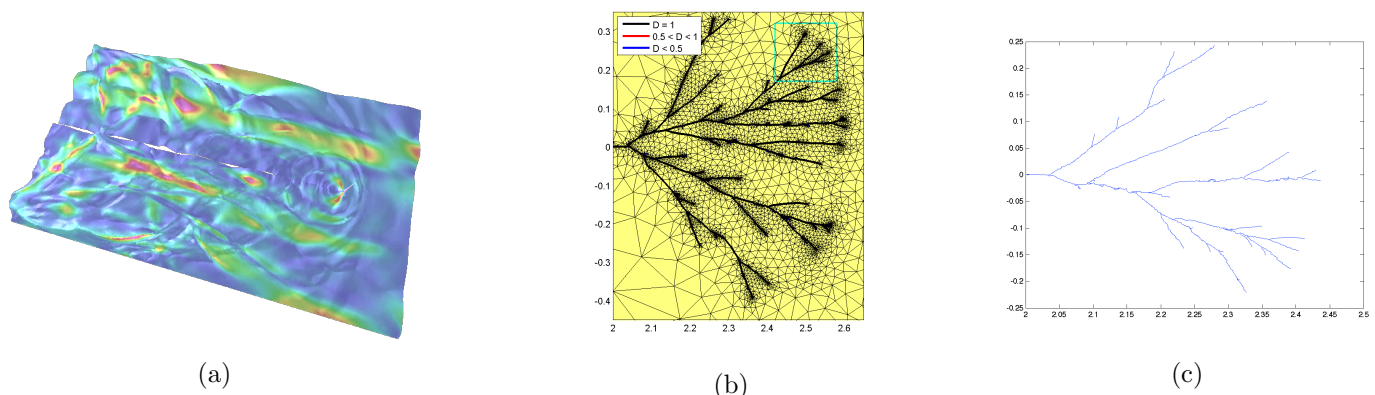


Figure 1: Dynamic contact and fracture: (a) Distinct Riemann conditions are enforced for contact-stick, frictional contact-slip and separation modes; (b) Adaptive meshing supports mesh-independent crack growth; (c) Crack-closure contact model dramatically alters the crack pattern.

## References

- [1] Reza Abedi, Robert B. Haber, and Boris Petracovici. A spacetime discontinuous Galerkin method for elastodynamics with element-level balance of linear momentum. *Computer Methods in Applied Mechanics and Engineering*, 195:3247–3273, 2006.