

# Coupling Interfacial and Bulk Response Using Riemann Solutions in an Adaptive Spacetime Model for Elastodynamic Contact

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## ABSTRACT

Elastodynamic contact presents special problems in numerical simulation, including spurious oscillations, contact-mode chatter, and nonconvergence. Discontinuous response and sharp wave fronts in the bulk response pose additional challenges. Many numerical models address these problems with heuristic methods. We present an alternative approach in which an improved continuum formulation eliminates several of these problems. A regularization and implementation within an adaptive spacetime discontinuous Galerkin model resolves the remaining issues.

Most numerical models for elastodynamic contact are quasistatic — they enforce the impenetrability constraint and a friction relation, but not compatibility of characteristic structure between the contact surface and the bulk. Our previous experience [1] shows that enforcing Riemann jump conditions across element interfaces eliminates spurious ringing and generally improves the quality of numerical elastodynamic solutions. We present continuum Riemann jump conditions for the complete set of contact modes to extend this approach to address linear elastodynamic contact subject to a Coulomb friction relation. The Riemann jump conditions directly enforce the impenetrability constraint, the friction relation, and proper dynamic coupling with the bulk response.

The standard Coulomb friction relation is mathematically discontinuous at stick–slip transitions where the direction of the slip-mode friction becomes ill-defined as the sliding velocity vanishes. This discontinuity has no physical basis and can cause contact-mode chatter and convergence problems in numerical solutions. We reformulate the Coulomb friction relation to eliminate this problem. However, a legitimate discontinuity remains at separation-to-contact transitions where we expect discontinuous response with weak shocks emitted from the contact interface. We regularize these events to control numerical overshoot and undershoot, but still preserve characteristic structure.

We discretize our continuum model with the adaptive spacetime discontinuous Galerkin finite element method described in [1]. In addition to cell-wise momentum balance, good shock capturing, and powerful adaptive capabilities, this model provides a natural means to weakly enforce the Riemann contact jump conditions without resorting to auxiliary constraint methods. We report numerical results for a 1d benchmark problem, a model brake-pad system, and crack closure in elastodynamic fracture. Our solutions compare favourably with results from other methods.

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## REFERENCES

- [1] R. Abedi, R. B. Haber and B. Petracovici, “A spacetime discontinuous Galerkin method for elastodynamics with element-level balance of linear momentum,” *Comp. Methods Appl. Mech. Engrng.*, Vol. 195, pp. 3247–3273, (2006).