Adaptive Spacetime Discontinuous Galerkin Model for Wave Propagation in Layered Composite Plates with Defects

Reza Abedi
University of Tennessee Space Institute

Robert B. Haber
University of Illinois at Urbana-Champaign

12th U.S. National Congress for Computational Mechanics
Raleigh, North Carolina – 22 July 2013

Funded in part by: NSF Grant OCI-0948393 and NCSA Industrial Affiliates Program
Application: Structural Health Monitoring in Layered Plate Systems

- Flaw detection and identification
  - Holes, through-cracks, delamination, bulk damage, etc.
  - Many layers with strong anisotropic response

- Detect and simulate changes in wave propagation
  - Precise physical excitation and sensing
  - Inverse problem to identify flaws (type, location, size, shape)
    - Requires many high-resolution forward simulations
    - Modeling and numerical errors must be smaller than change in physical response
Spacetime discontinuous Galerkin finite element methods

- Direct spacetime discretization
  - Replaces time integration

ALE+
- Continuous remeshing w/o projections preserves high-order accuracy
- Unstructured grids graded in both space and time
- No tangling in moving-boundary problems

Asynchronous solver
- $O(N)$ for hyperbolic systems
Causal Spacetime Mesh and $O(N)$ Advancing-Front Solution Strategy
Tent Pitcher: causal spacetime meshing

Given a space mesh, Tent Pitcher constructs a spacetime mesh such that every facet on sequence of advancing fronts is spacelike (patch height bounded by causality constraint)

Similar to CFL condition, except entirely local and not related to stability (required for $O(N)$ solution)
Tent Pitcher: patch–by–patch meshing & solution

- Patches (‘tents’) of tetrahedra; solve immediately for $O(N)$ method with rich parallel structure
- Maintain “space mesh” as advancing, space-like front with non-uniform time coordinates
Spacetime adaptive meshing operations

New spacetime adaptive meshing operations:

- Vertex deletion (coarsening); Edge flip; Inclined tent poles (ALE, smoothing, tracking and repositioning)

- Spacetime format eliminates projection error

- Preserves high-order accuracy during remeshing

(a) Vertex deletion (degree 5)  (b) Edge flip  (c) Inclined tent pole
Crack-tip Wave Scattering
Crack-tip Wave Scattering

click to play movie
Crack-tip Wave Scattering

click to play movie
Crack-tip Wave Scattering
Dynamic fracture with damage-delay *interfacial* failure model
Dynamic fracture with modified damage-delay cohesive model
Square-Plate Impact Example

click to play movie
Spacetime fields

\[0, l, d, \text{ and } (d+1)\text{-forms} \]

- Displacement (0-form): \( u \)

- Strain-velocity (1-form): \( \varepsilon := E + v \)
  - Linearized strain + velocity

- Spacetime Momentum Flux (\(d\)-form): \( M := p - S \)
  - Linear momentum density - stress

- Body force density (\((d + 1)\)-form): \( b \)
3-field SDG formulation

**Problem** (Weighted residual form). For each $Q \in \mathcal{P}$, find $(u, \varepsilon) \in \mathcal{V}_u \times \mathcal{V}_\varepsilon$ such that for every $Q \in \mathcal{P}$

\[
\int_Q \left[ \mathbf{i} \hat{\varepsilon} \wedge (dM - \rho b) + d\varepsilon \wedge \mathbf{i} \hat{M} + (du - v) \wedge \hat{f} \right] \\
+ \int_{\partial Q} \left[ \mathbf{i} \hat{\varepsilon} \wedge (M^* - M) + (\varepsilon^* - \varepsilon) \wedge \mathbf{i} \hat{M} + (u^* - u) \wedge \hat{f} \right] = 0
\]

\[\forall (\hat{u}, \hat{\varepsilon}) \in \mathcal{V}_u \times \mathcal{V}_\varepsilon\]

in which $\hat{f} = k^Q \mathbf{1}(\hat{u}) \times \text{d}t$.

**Problem** (Weak form). Find $(u, \varepsilon) \in \mathcal{V}_u \times \mathcal{V}_\varepsilon$ such that for every $Q \in \mathcal{P}$ such that

\[
-\int_Q \left[ d\mathbf{i} \hat{\varepsilon} \wedge M + \mathbf{i} \hat{\varepsilon} \wedge \rho b - \varepsilon \wedge d\mathbf{i} \hat{M} + u \wedge d\hat{f} + v \wedge \hat{f} \right] \\
+ \int_{\partial Q} \left[ \mathbf{i} \hat{\varepsilon} \wedge M^* + \varepsilon^* \wedge \mathbf{i} \hat{M} + u^* \wedge \hat{f} \right] = 0
\]

\[\forall (\hat{u}, \hat{\varepsilon}) \in \mathcal{V}_u \times \mathcal{V}_\varepsilon\]
New-Generation SDG FEM framework

• Old model: patches + (elements = integration cells)

• New model: patches + elements + integration cells
Generation of multi-ply spacetime meshes (thick beam example)

- Generate lower-dimension spacetime mesh
- Extrude in thickness direction to get stack of cylindrical spacetime cells
- Elements may consist of one or more cells
Application to multi-ply plates
(only front mesh shown below)

- Extrude 2d spatial geometry in transverse ‘thickness’ direction

- Product bases: (in-plane) x (transverse) x (time)
  - Independent selections for \( (u,v) \) and \( (w) \) d.o.f.
  - Elements with multiple int. cells for simple layered plate model
    - First and higher-order shear-deformation-theory models
    - ‘Zig-zag’ layered plate model; \( C^0 \) piecewise-linear basis
  - Multi-element model: one (element/int. cell) per layer
    - SDG models with per-ply dofs
    - Interface models for delamination
  - Higher-order and \( p \)-adaptive models
    - avoid shear correction factors; transitions to full 3d x time models
First-order shear deformation theory (FSDT) plate model

- Start with general three-dimensional theory; configure elements and restrict basis functions to implement desired plate theory

- FSDT
  - Treat whole stack as one element with multiple integration cells
  - Support of basis functions spans entire multi-cell stack

\[ P_{mn} := P_m(x, y, t) \times P^n(z) \]

\[ u_1, u_2 \in P^{m1} \]
\[ u_3 \in P^{m0} \]
SDG-zigzag model

- Single-cell elements at each layer
- Basis functions have layer-wise support (per-layer d.o.f.)
- SDG jump conditions weakly enforce continuity at layer boundaries

\[ \mathcal{P}^{mn} := \mathcal{P}^{m}(x, y, t) \times \mathcal{P}^{n}(z) \]
\[ u_1, u_2 \in \mathcal{P}^{m1} \]
\[ u_3 \in \mathcal{P}^{m0} \]
2-ply plate with flaw

sensor

hole in top ply

sinusoidal impulse load

fixed support

0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6

0 0.2 0.4 0.6 0.8 1

| orthotropic layer |
| isotropic layer |
2-ply plate with flaw

No hole  

With hole
2-ply plate with flaw
Effect of flaw on velocity at sensor position
2-ply plate with flaw
initial mesh for $h$-adaptive solutions
2-ply plate with flaw

$h$-adaptive solution: $\varepsilon_D = 10^{-9.5}$

click to play movie
2-ply plate with flaw
Adaptive vs. non-adaptive response
Conclusions

- Still higher resolution is needed
- More horsepower is on the way
  - $hp$-adaptivity (already beats $h$-adaptive)
  - Code optimization and vectorization
  - Parallel meshing and patch solution
    - asynchronous
    - hierarchical
    - heterogeneous
    - dynamic load balancing at same granularity of meshing/solution
- Plate models vs. full 3d?
  - Initial response to surface loading and scattering by flaws require 3d
  - Let $hp$-adaptive solver decide what is needed