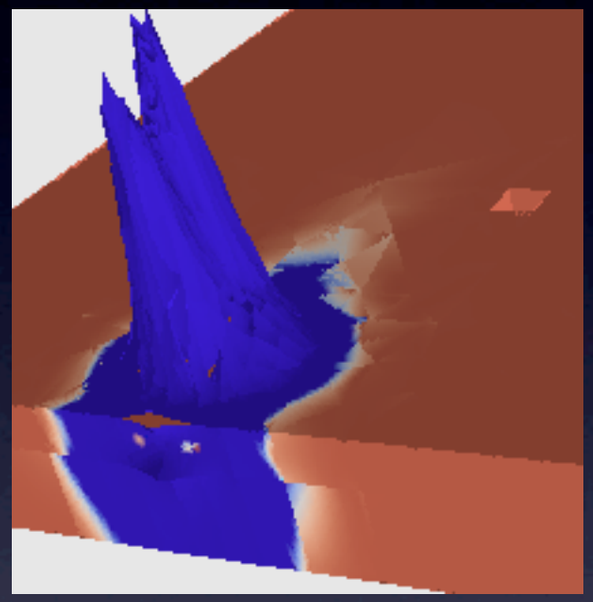
Adaptive Spacetime Discontinuous Galerkin Model for Wave Propagation in Layered Composite Plates with Defects



Reza Abedi

University of Tennessee Space Institute

Robert B. Haber

University of Illinois at Urbana-Champaign

12th U.S. National Congress for Computational Mechanics

Raleigh, North Carolina – 22 July 2013

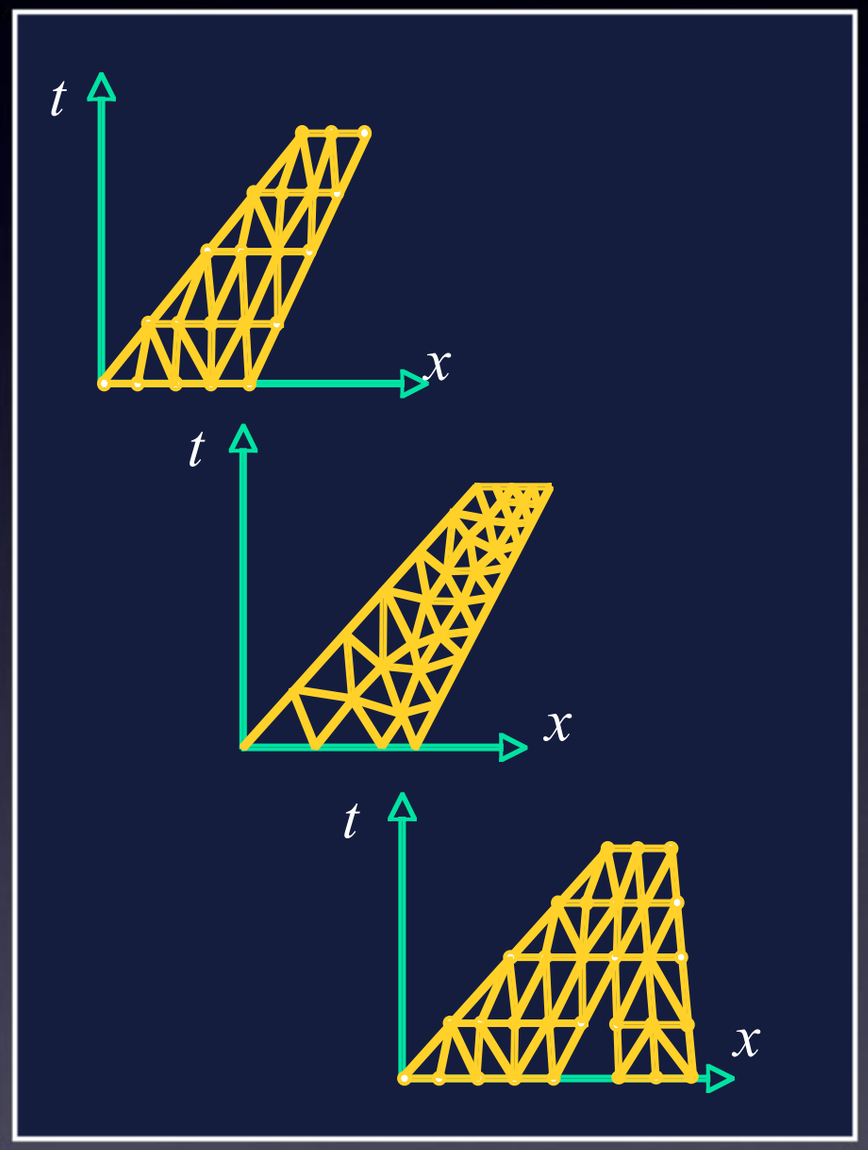
Funded in part by: NSF Grant OCI-0948393 and NCSA Industrial Affiliates Program

Application: Structural Health Monitoring in Layered Plate Systems

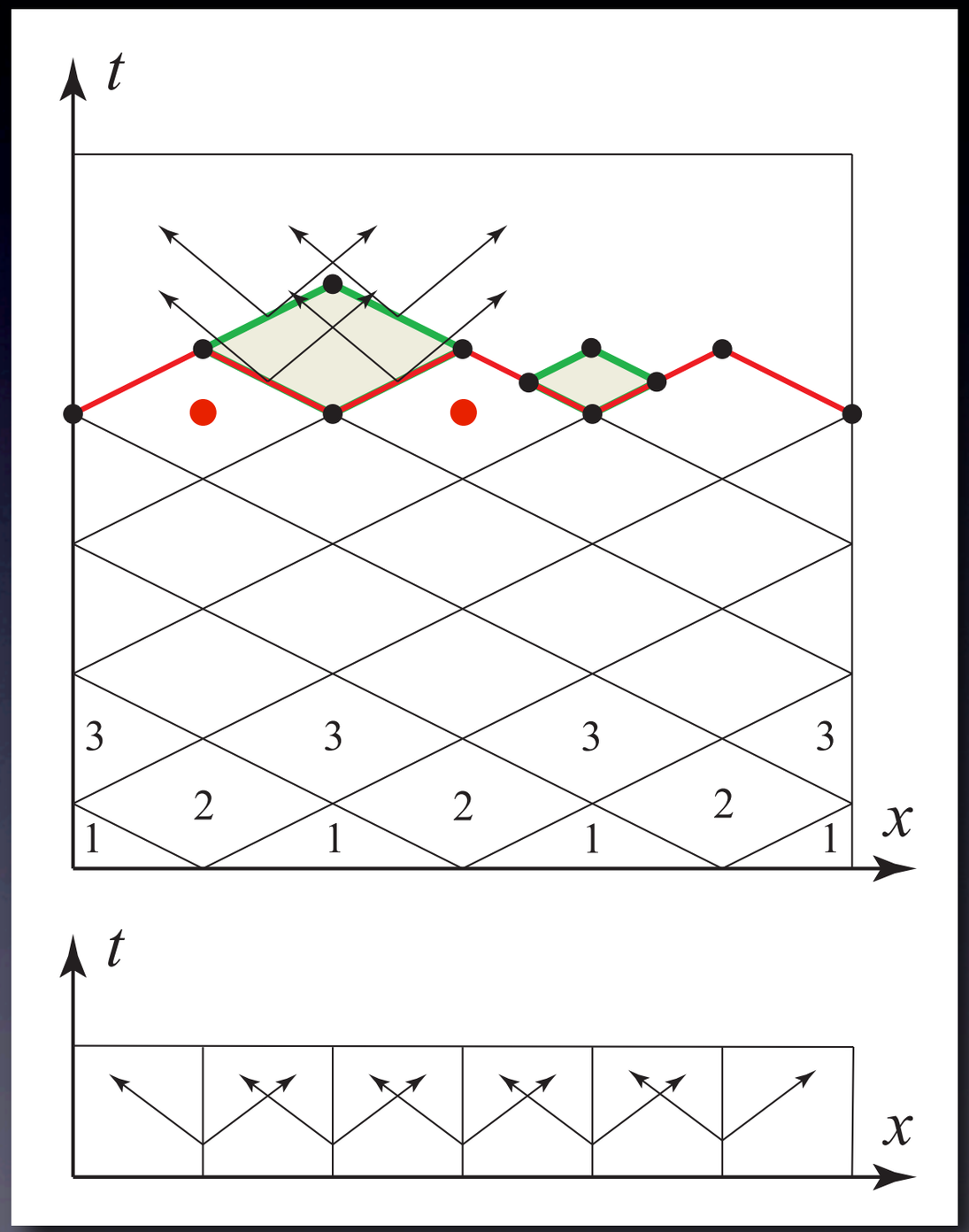
- Flaw detection and identification
  - ✦ Holes, through-cracks, delamination, bulk damage, etc.
  - ✦ Many layers with strong anisotropic response
- Detect and simulate changes in wave propagation
  - ✦ Precise physical excitation and sensing
  - ✦ Inverse problem to identify flaws (type, location, size, shape)
    - Requires many high-resolution forward simulations
    - Modeling and numerical errors must be smaller than change in physical response

Spacetime discontinuous Galerkin finite element methods

- Direct spacetime discretization
  - Replaces time integration
- ALE+
  - Continuous remeshing w/o projections preserves high-order accuracy
  - Unstructured grids graded in both space and time
  - No tangling in moving–boundary problems
- Asynchronous solver
  - *O(N)* for hyperbolic systems

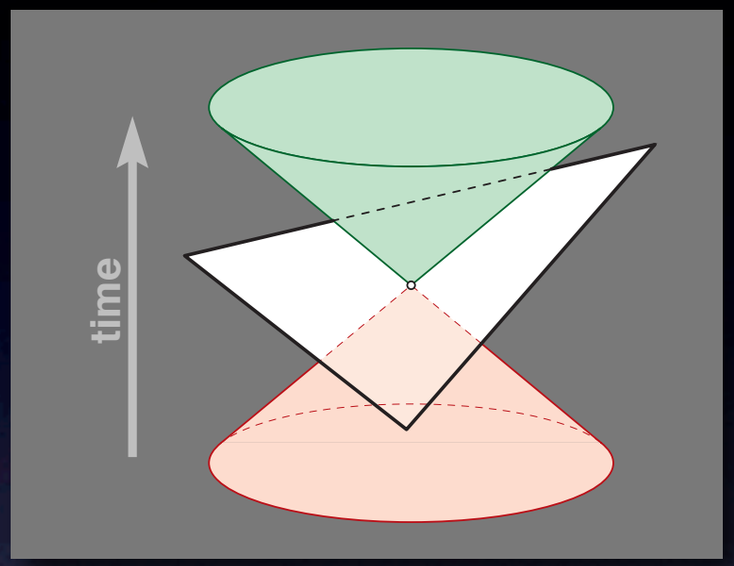


Causal Spacetime Mesh and O(*N*) Advancing-Front Solution Strategy

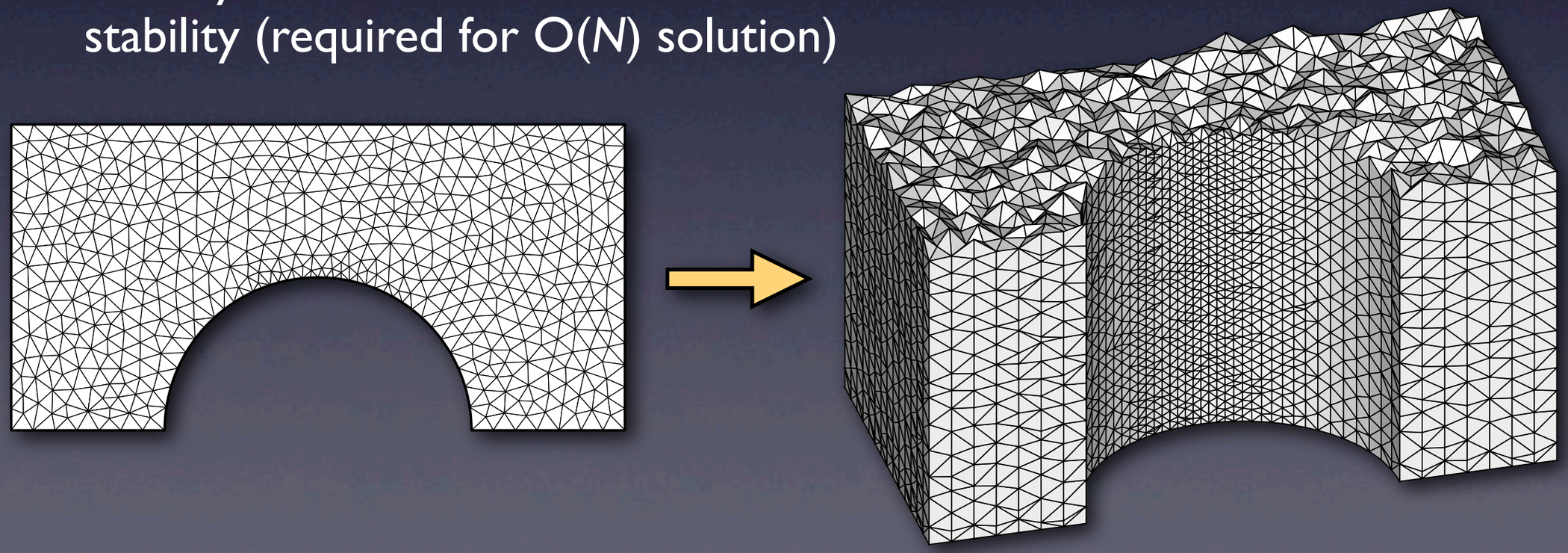


Tent Pitcher: causal spacetime meshing

- Given a space mesh, Tent Pitcher constructs a spacetime mesh such that every facet on sequence of advancing fronts is spacelike (patch height bounded by *causality constraint*)
- Similar to CFL condition, except entirely *local* and not related to stability (required for O(*N*) solution)

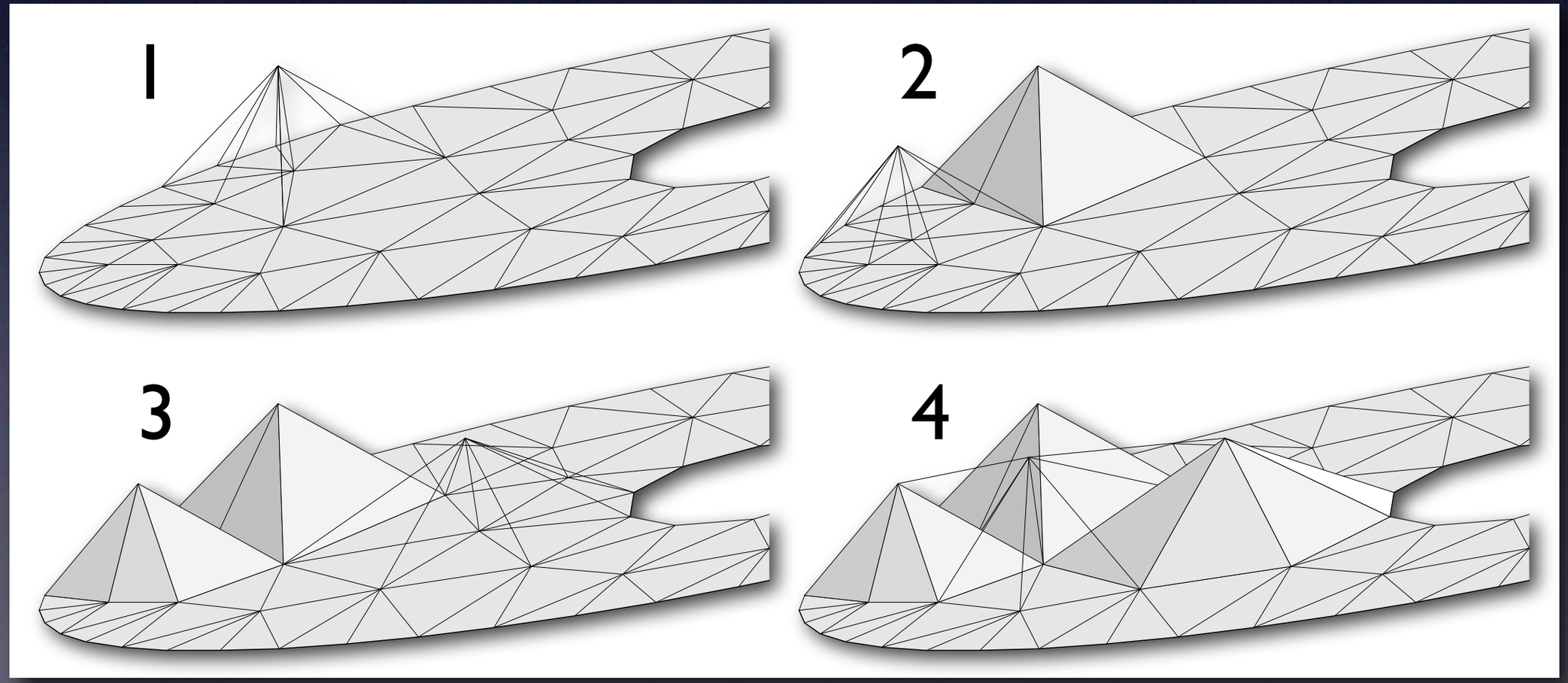


causality constraint



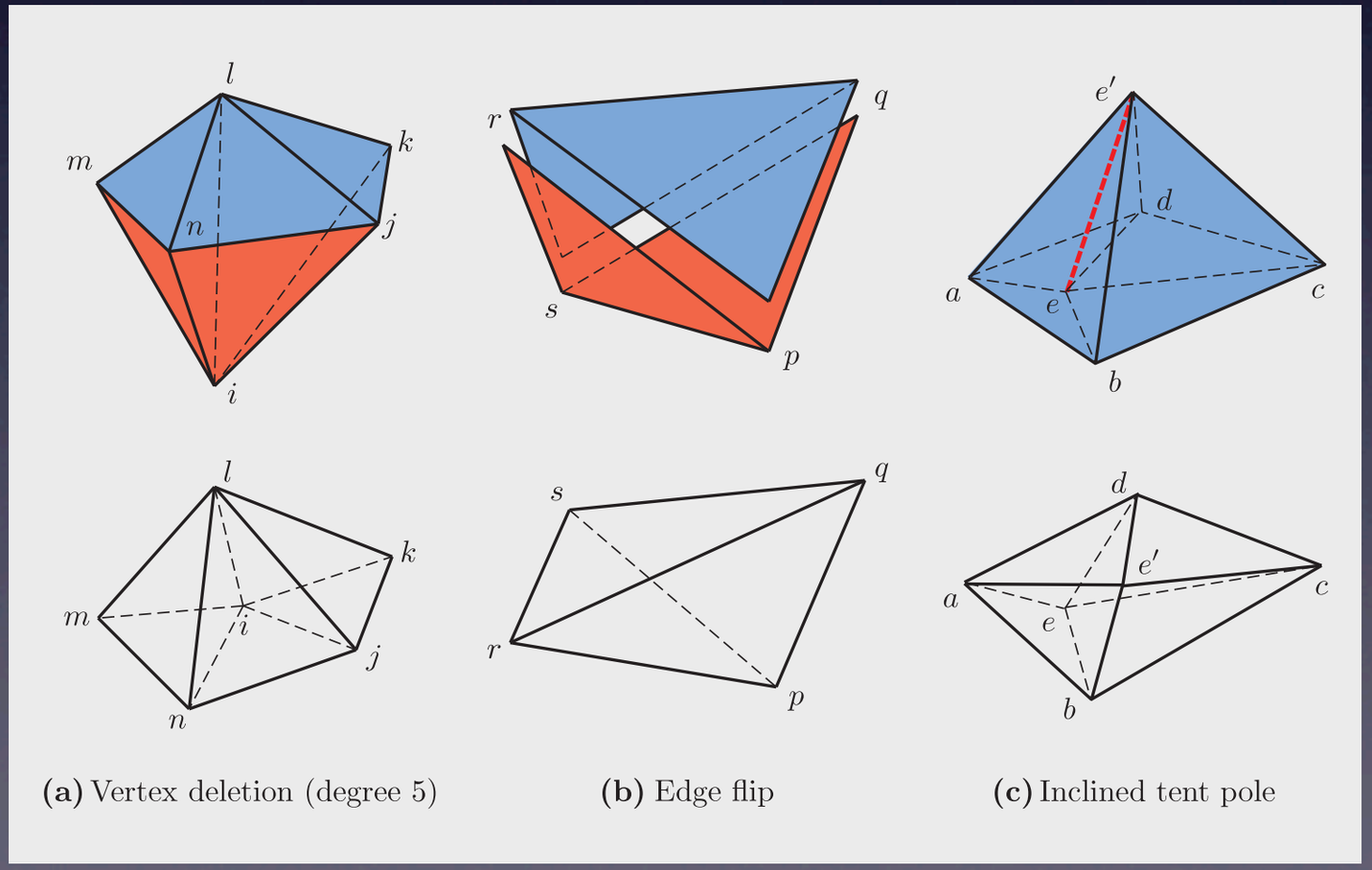
Tent Pitcher: patch–by–patch meshing & solution

- Patches ('tents') of tetrahedra; solve immediately for O(N) method with rich parallel structure
- Maintain "space mesh" as advancing, space-like front with non-uniform time coordinates



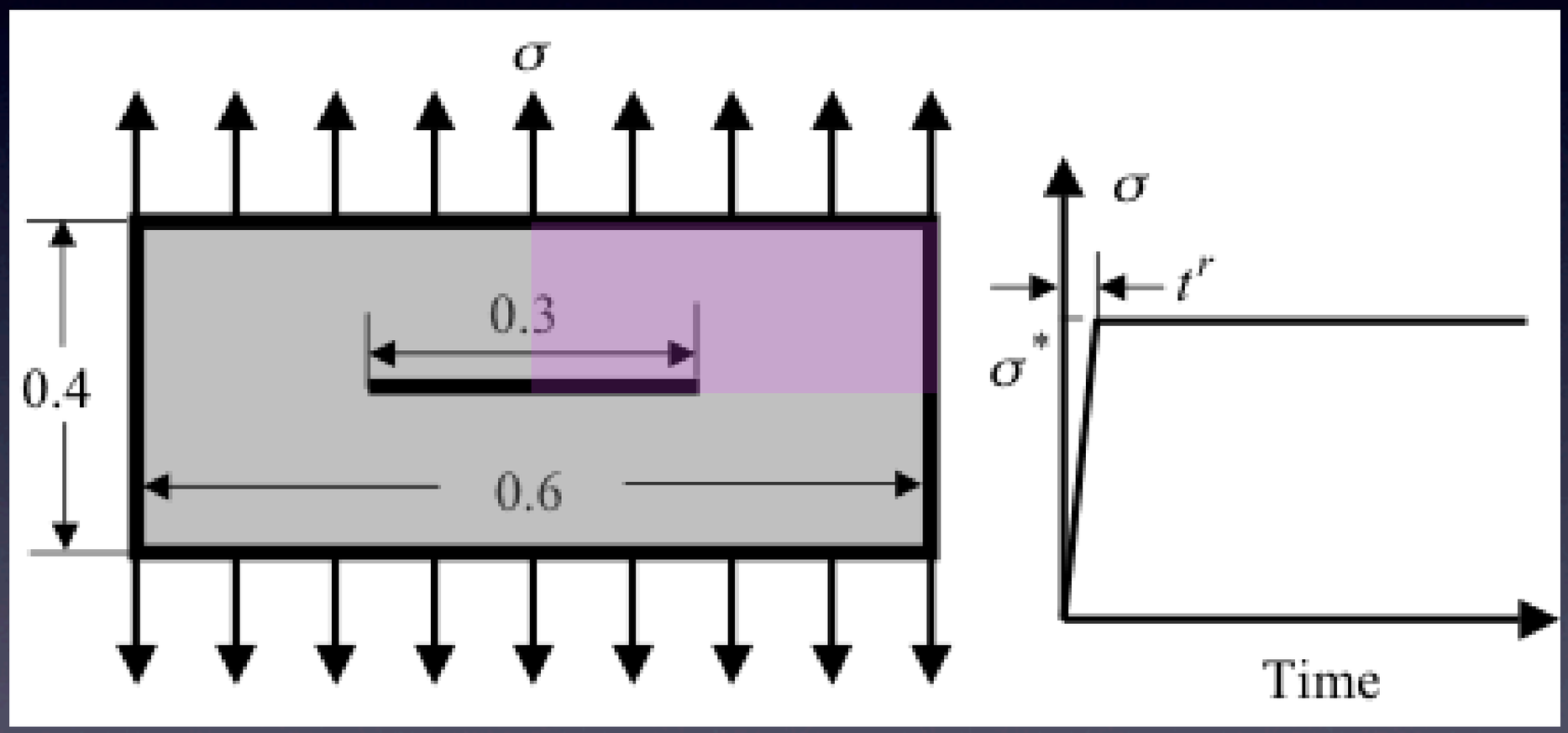
Spacetime adaptive meshing operations

- New spacetime adaptive meshing operations:
  - Vertex deletion (coarsening); Edge flip; Inclined tent poles (ALE, smoothing, tracking and repositioning)
  - Spacetime format eliminates projection error
  - Preserves high-order accuracy during remeshing



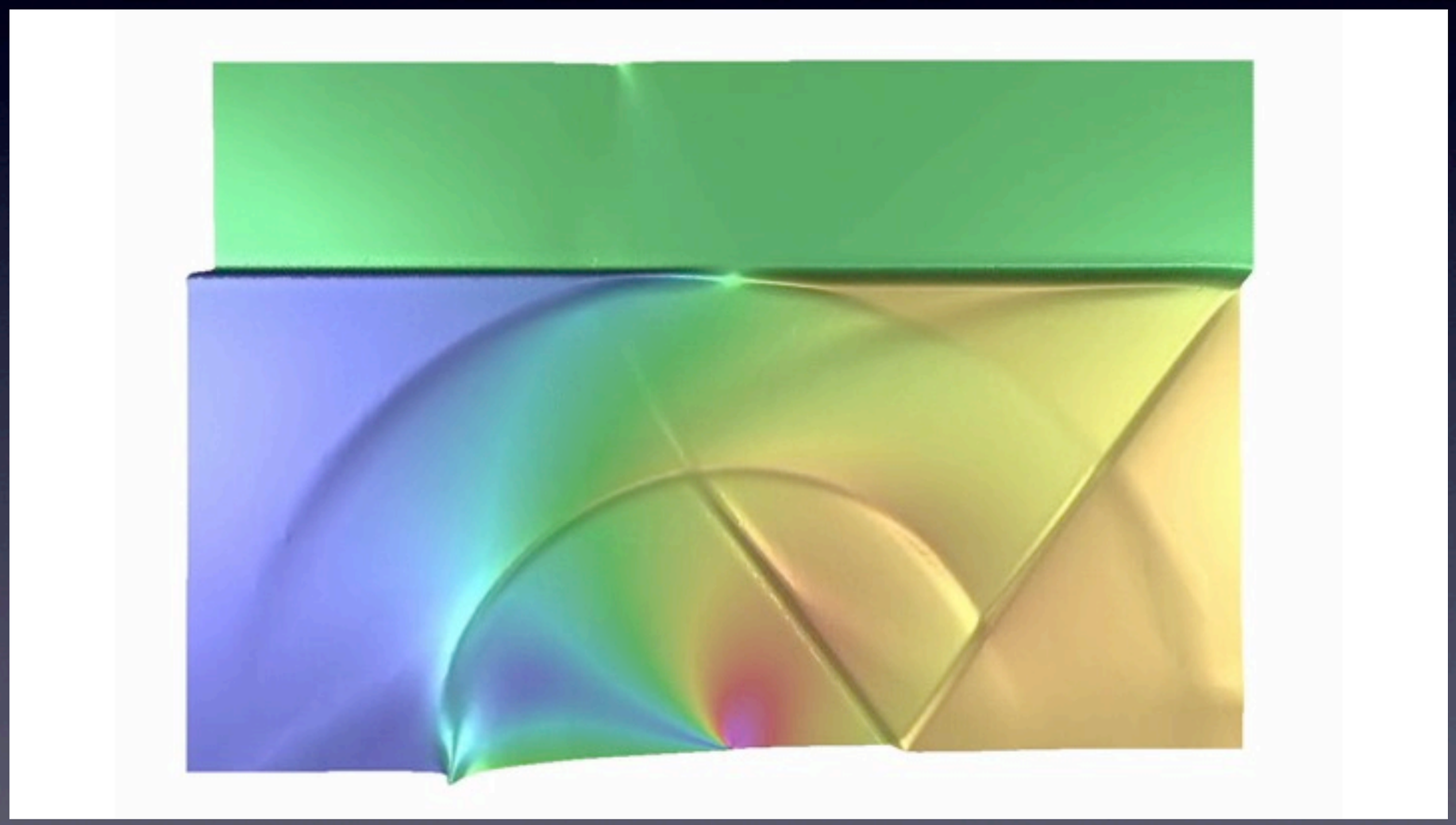
(a) Vertex deletion (degree 5) **(b)** Edge flip **(c)** Inclined tent pole

Crack-tip Wave Scattering



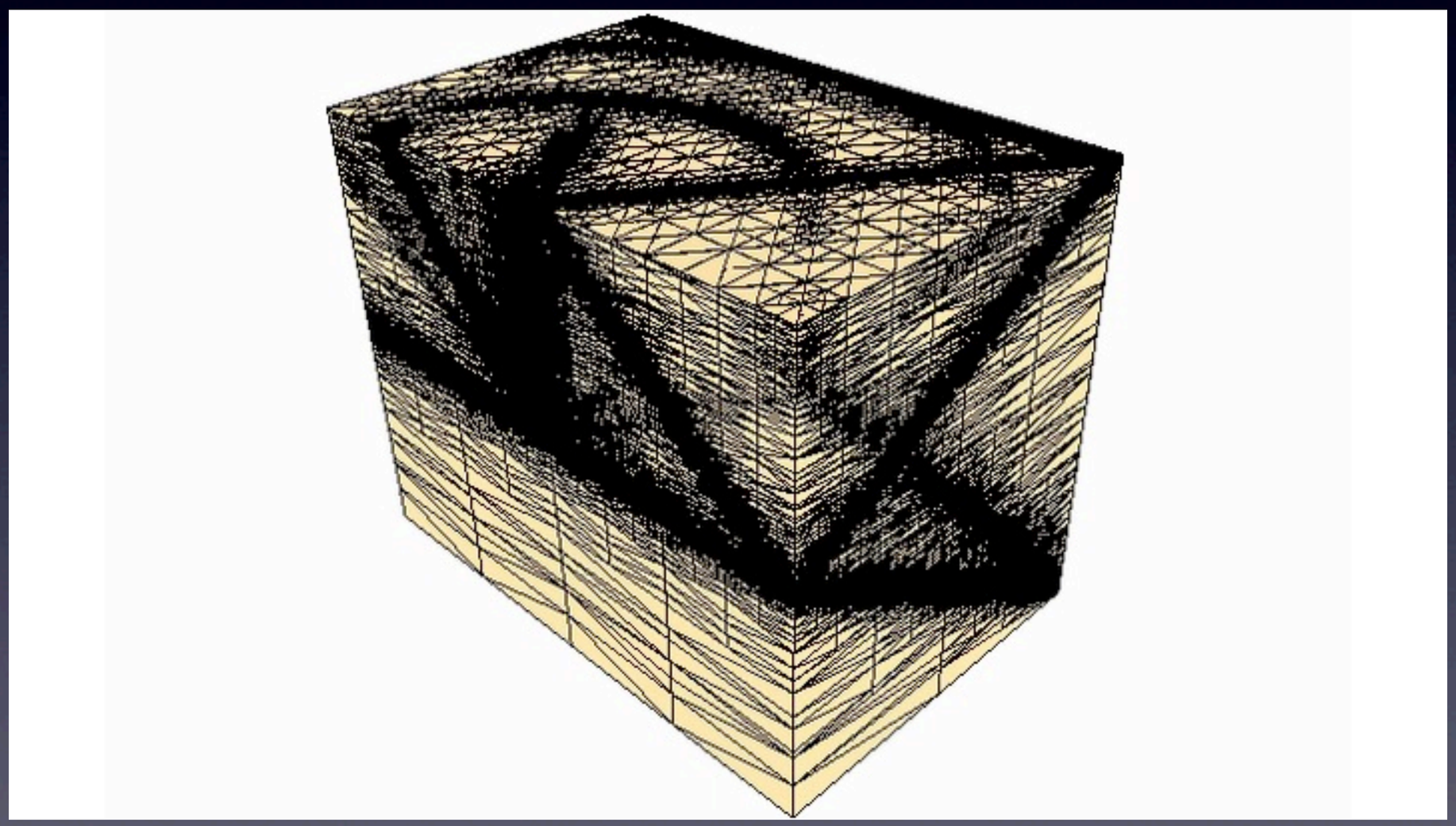
Crack-tip Wave Scattering

click to play movie

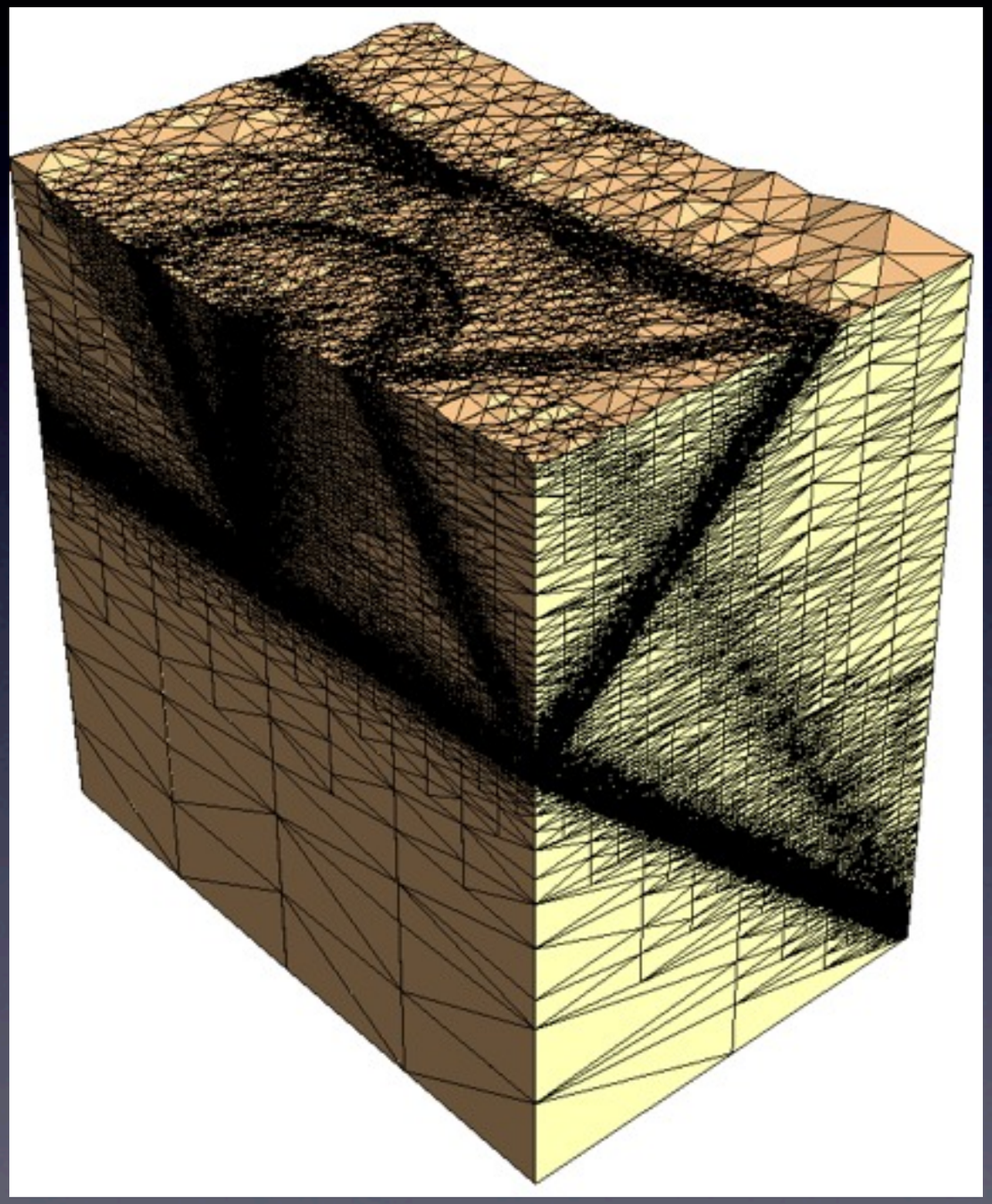


Crack-tip Wave Scattering

click to play movie

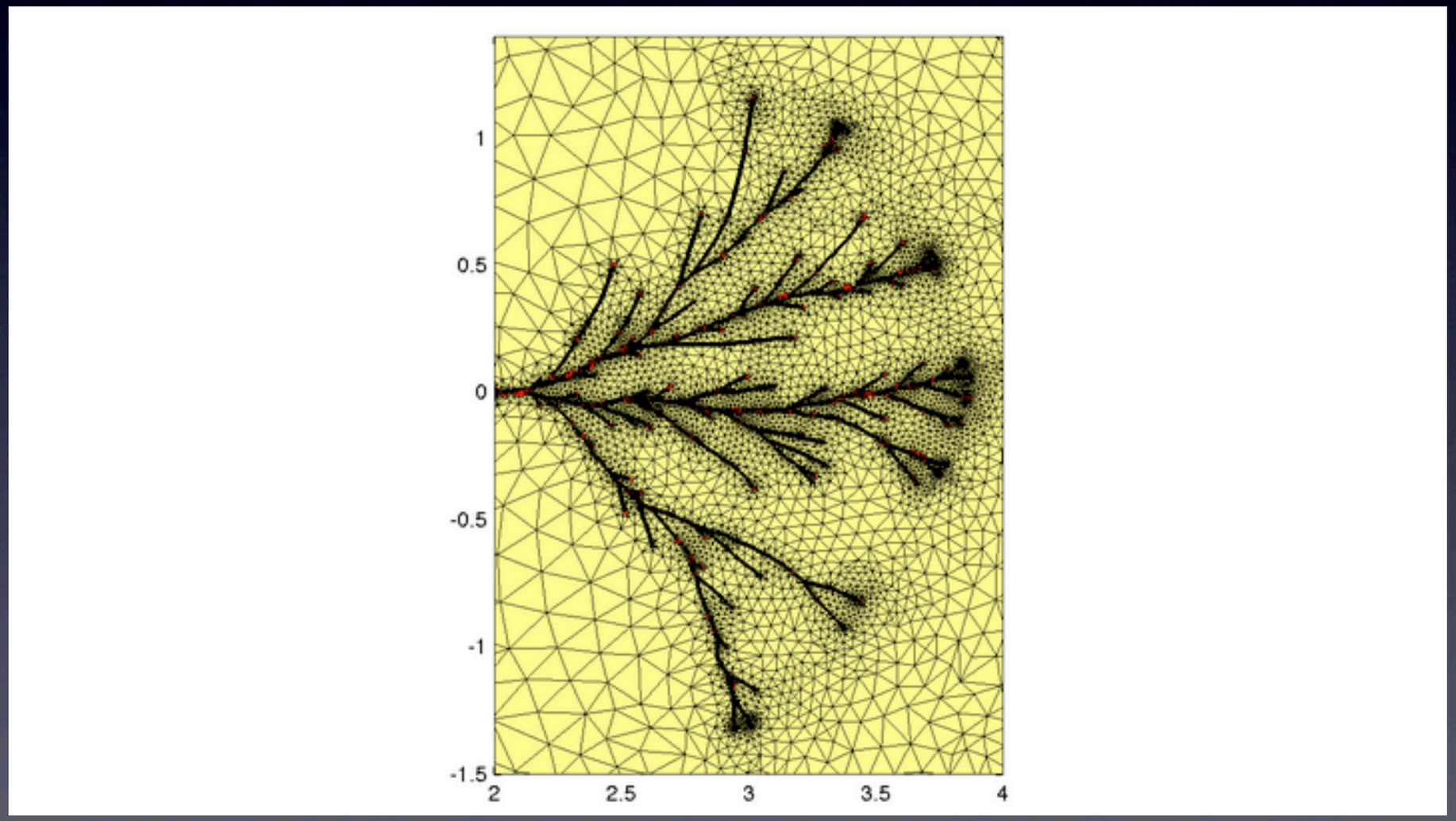


Crack-tip Wave Scattering

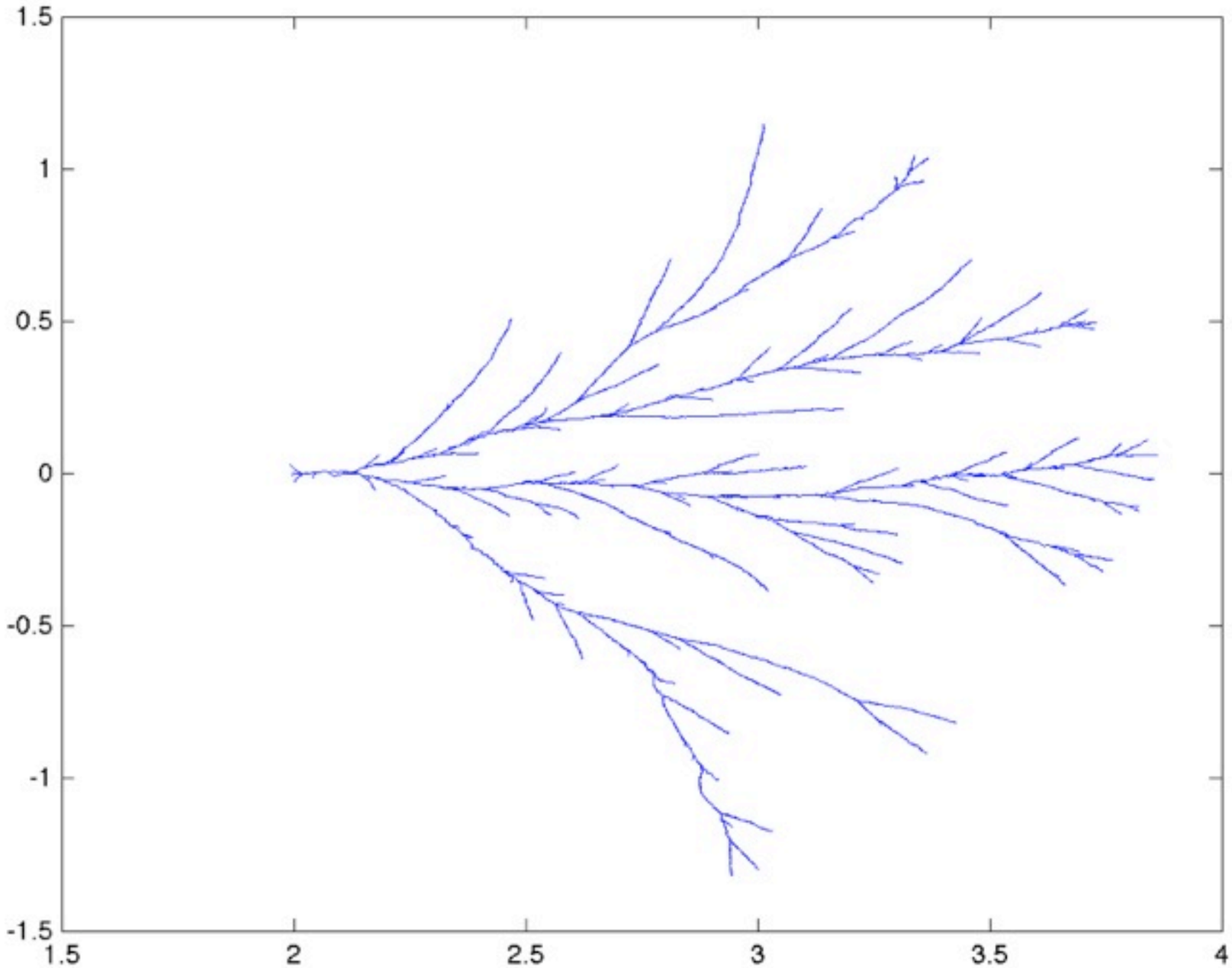


Dynamic fracture with damage-delay *interfacial* failure model

click to play movie

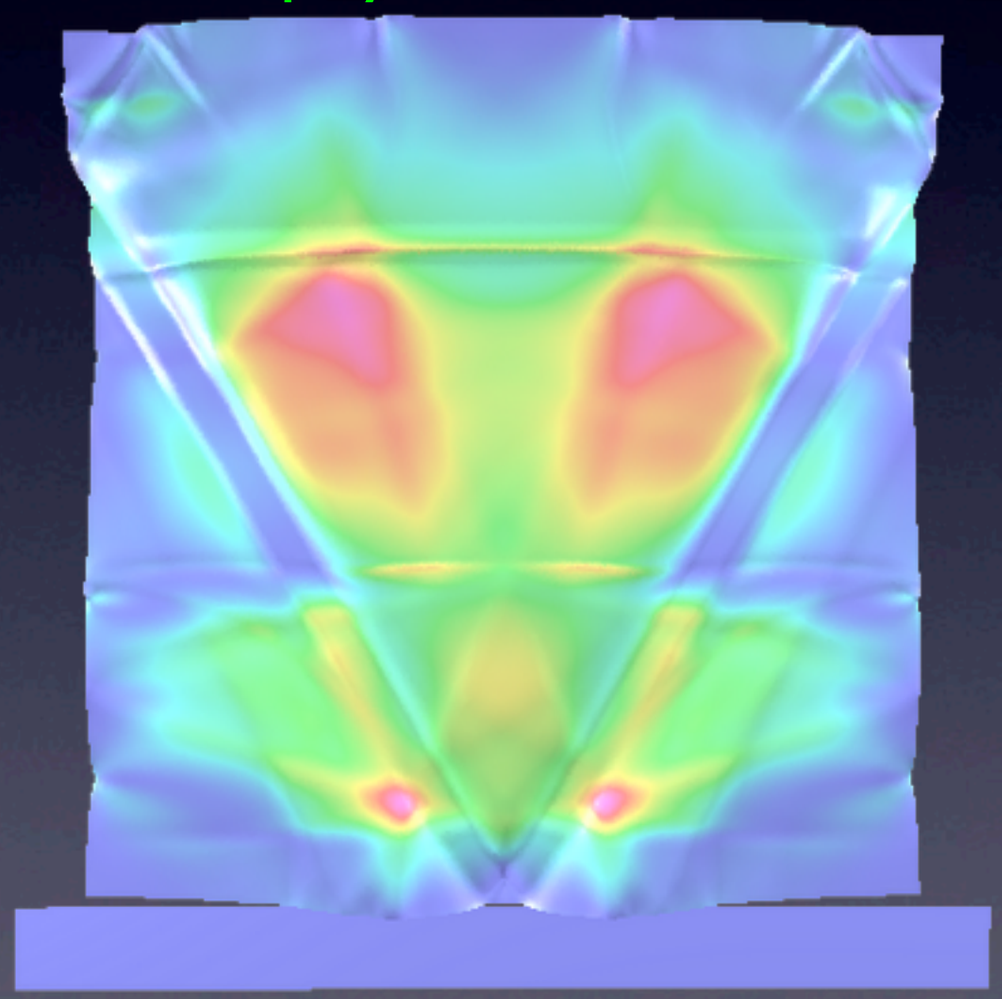


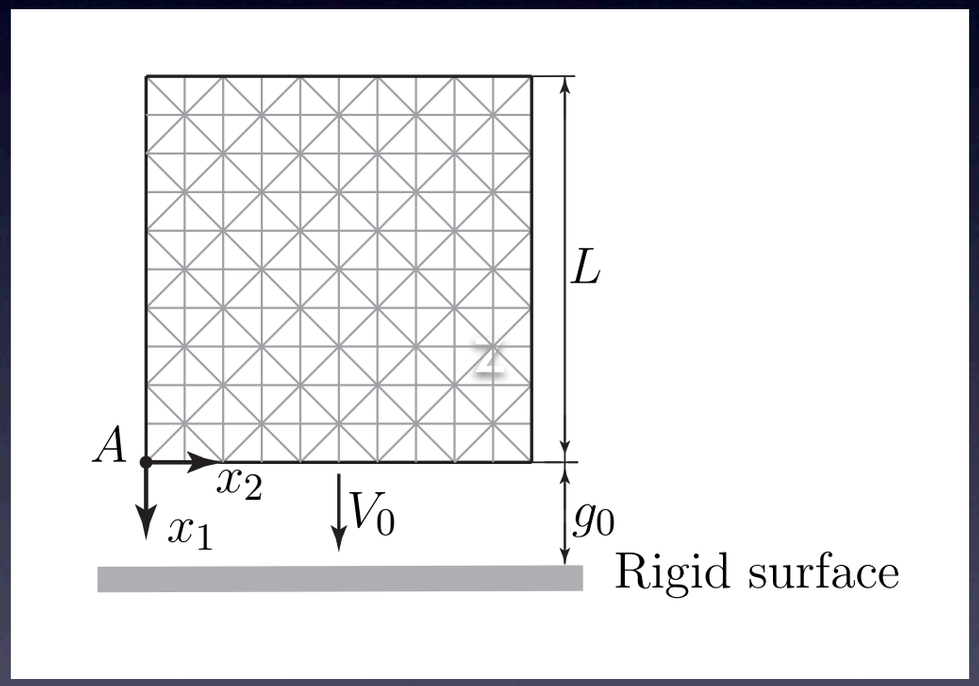
Dynamic fracture with modified damage-delay cohesive model



Square-Plate Impact Example

click to play movie





Spacetime fields

[*0, 1, d, and (d+1)*-forms]

- Displacement (0-form): $\mathbf{u}$
- Strain-velocity (1-form): $\boldsymbol{\varepsilon} := \boldsymbol{E} + \boldsymbol{v}$
  - Linearized strain + velocity
- Spacetime Momentum Flux (d-form): $\boldsymbol{M} := \boldsymbol{p} - \boldsymbol{S}$
  - Linear momentum density - stress
- Body force density ($(d+1)$-form): $\boldsymbol{b}$

3-field SDG formulation

Problem (Weighted residual form). *For each $\mathcal{Q} \in \mathcal{P}$, find $(\mathbf{u}, \boldsymbol{\varepsilon}) \in \mathcal{V}_{\mathbf{u}} \times \mathcal{V}_{\boldsymbol{\varepsilon}}$ such that for every $\mathcal{Q} \in \mathcal{P}$*

$$\int_{\mathcal{Q}} \left[\mathbf{i}\hat{\boldsymbol{\varepsilon}} \wedge (\mathbf{d}\boldsymbol{M} - \rho \boldsymbol{b}) + \mathbf{d}\boldsymbol{\varepsilon} \wedge \mathbf{i}\hat{\boldsymbol{M}} + (\mathbf{d}\mathbf{u} - \boldsymbol{v}) \wedge \hat{\boldsymbol{f}} \right]$$

$$+ \int_{\partial\mathcal{Q}} \left[\mathbf{i}\hat{\boldsymbol{\varepsilon}} \wedge (\boldsymbol{M}^* - \boldsymbol{M}) + (\boldsymbol{\varepsilon}^* - \boldsymbol{\varepsilon}) \wedge \mathbf{i}\hat{\boldsymbol{M}} + (\mathbf{u}^* - \mathbf{u}) \wedge \hat{\boldsymbol{f}} \right] = 0$$

$$\forall\ (\hat{\mathbf{u}}, \hat{\boldsymbol{\varepsilon}}) \in \mathcal{V}_{\mathbf{u}} \times \mathcal{V}_{\boldsymbol{\varepsilon}}$$

in which $\hat{\boldsymbol{f}} = k^{\mathcal{Q}} \mathbf{1}(\hat{\mathbf{u}}) \star \mathrm{dt}$.

Problem (Weak form). *Find $(\mathbf{u}, \boldsymbol{\varepsilon}) \in \mathcal{V}_{\mathbf{u}} \times \mathcal{V}_{\boldsymbol{\varepsilon}}$ such that for every $\mathcal{Q} \in \mathcal{P}$ such that*

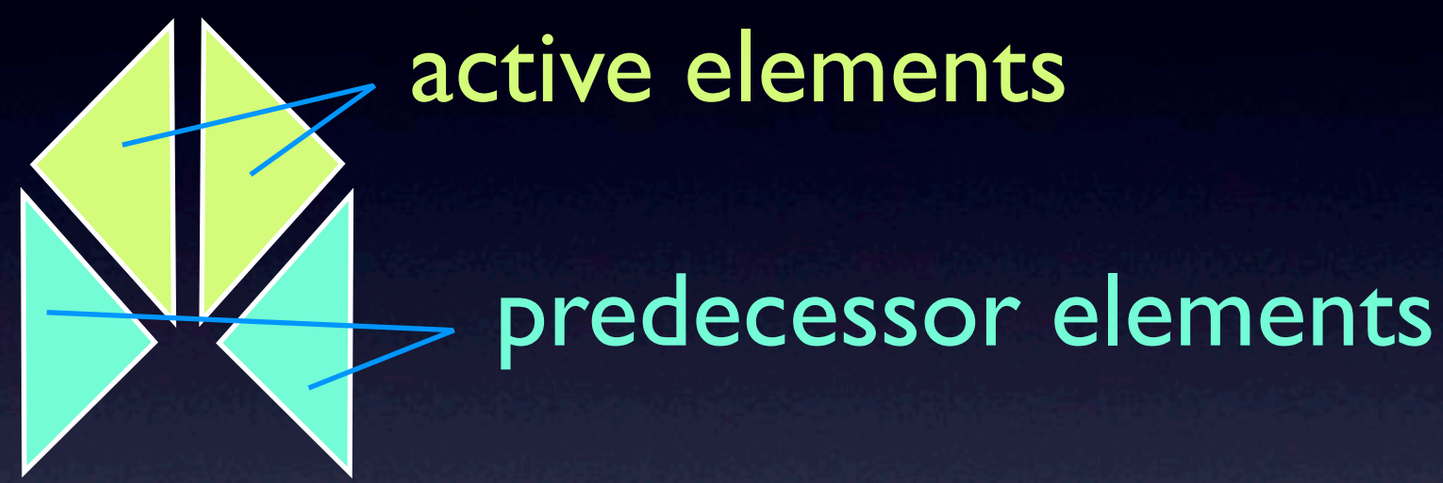
$$- \int_{\mathcal{Q}} \left[\mathbf{d}\mathbf{i}\hat{\boldsymbol{\varepsilon}} \wedge \boldsymbol{M} + \mathbf{i}\hat{\boldsymbol{\varepsilon}} \wedge \rho \boldsymbol{b} - \boldsymbol{\varepsilon} \wedge \mathbf{d}\mathbf{i}\hat{\boldsymbol{M}} + \mathbf{u} \wedge \mathbf{d}\hat{\boldsymbol{f}} + \boldsymbol{v} \wedge \hat{\boldsymbol{f}} \right]$$

$$+ \int_{\partial\mathcal{Q}} \left[\mathbf{i}\hat{\boldsymbol{\varepsilon}} \wedge \boldsymbol{M}^* + \boldsymbol{\varepsilon}^* \wedge \mathbf{i}\hat{\boldsymbol{M}} + \mathbf{u}^* \wedge \hat{\boldsymbol{f}} \right] = 0$$

$$\forall\ (\hat{\mathbf{u}}, \hat{\boldsymbol{\varepsilon}}) \in \mathcal{V}_{\mathbf{u}} \times \mathcal{V}_{\boldsymbol{\varepsilon}}$$

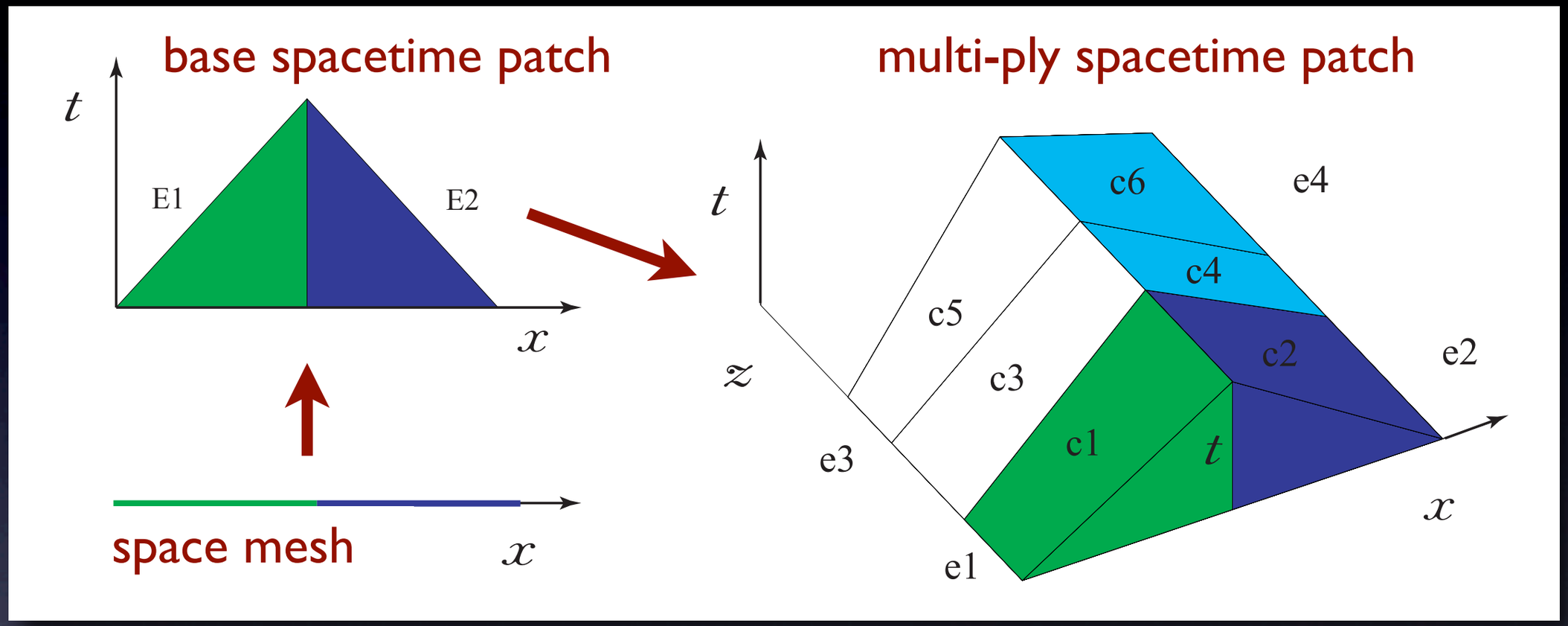
New-Generation SDG FEM framework

- Old model: patches + (elements = integration cells)



- New model: patches + elements + integration cells

Generation of multi-ply spacetime meshes (thick beam example)

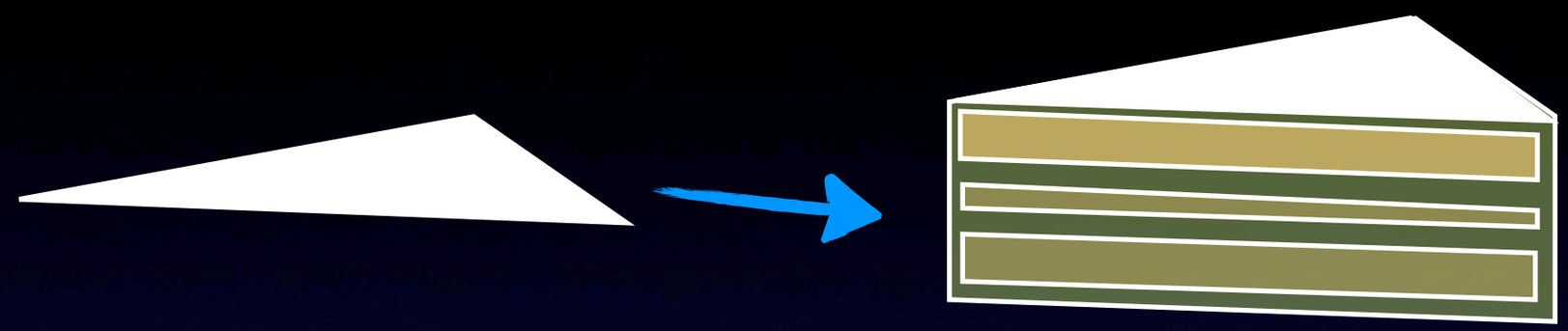


- Generate lower-dimension spacetime mesh
- Extrude in thickness direction to get stack of cylindrical spacetime cells
- Elements may consist of one or more cells

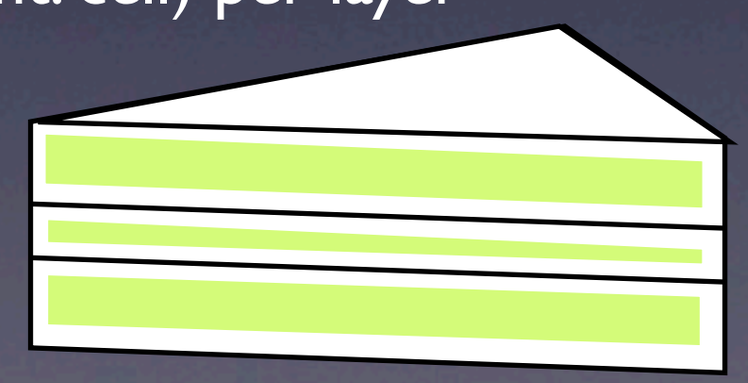
Application to multi-ply plates

(only front mesh shown below)

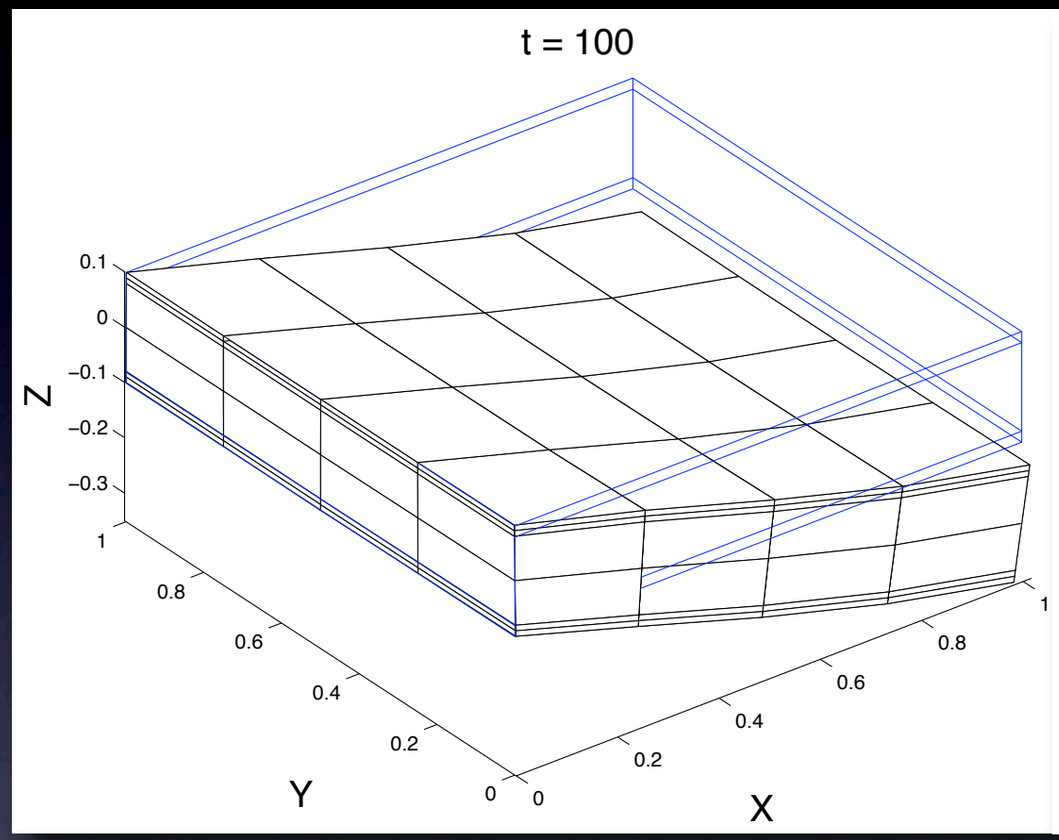
- Extrude *2d* spatial geometry in transverse 'thickness' direction



- Product bases: (in-plane) x (transverse) x (time)
  - ✦ Independent selections for (u,v) and (w) d.o.f.
  - ✦ Elements with multiple int. cells for simple layered plate model
    - First and higher-order shear-deformation-theory models
    - 'Zig-zag' layered plate model; C^0 piecewise-linear basis
  - ✦ Multi-element model: one (element/int. cell) per layer
    - SDG models with per-ply dofs
    - Interface models for delamination
  - ✦ Higher-order and *p*-adaptive models
    - avoid shear correction factors; transitions to full 3d x time models



First-order shear deformation theory (FSDT) plate model



through-thickness polynomial space

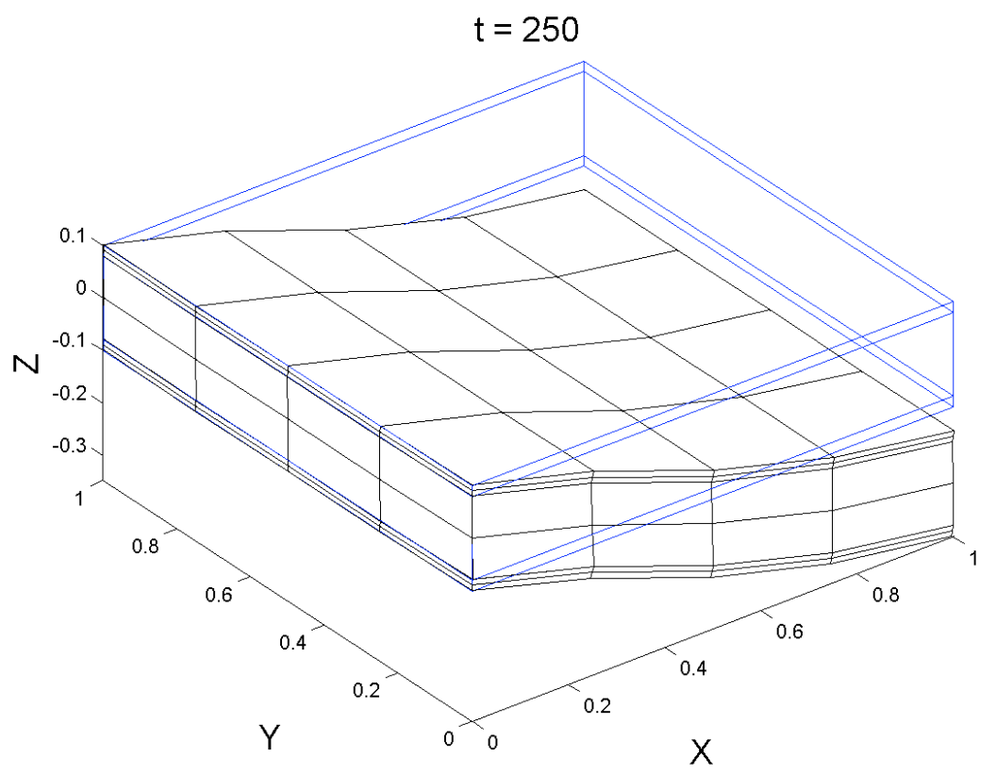
$$\mathcal{P}^{mn} := \mathcal{P}^m(x,y,t) \times \mathcal{P}^n(z)$$

$$u_1, u_2 \in \mathcal{P}^{m1}$$

$$u_3 \in \mathcal{P}^{m0}$$

- Start with general three-dimensional theory; configure elements and restrict basis functions to implement desired plate theory
- FSDT
  - Treat whole stack as one element with multiple integration cells
  - Support of basis functions spans entire multi-cell stack

SDG-zigzag model



per-layer polynomial space

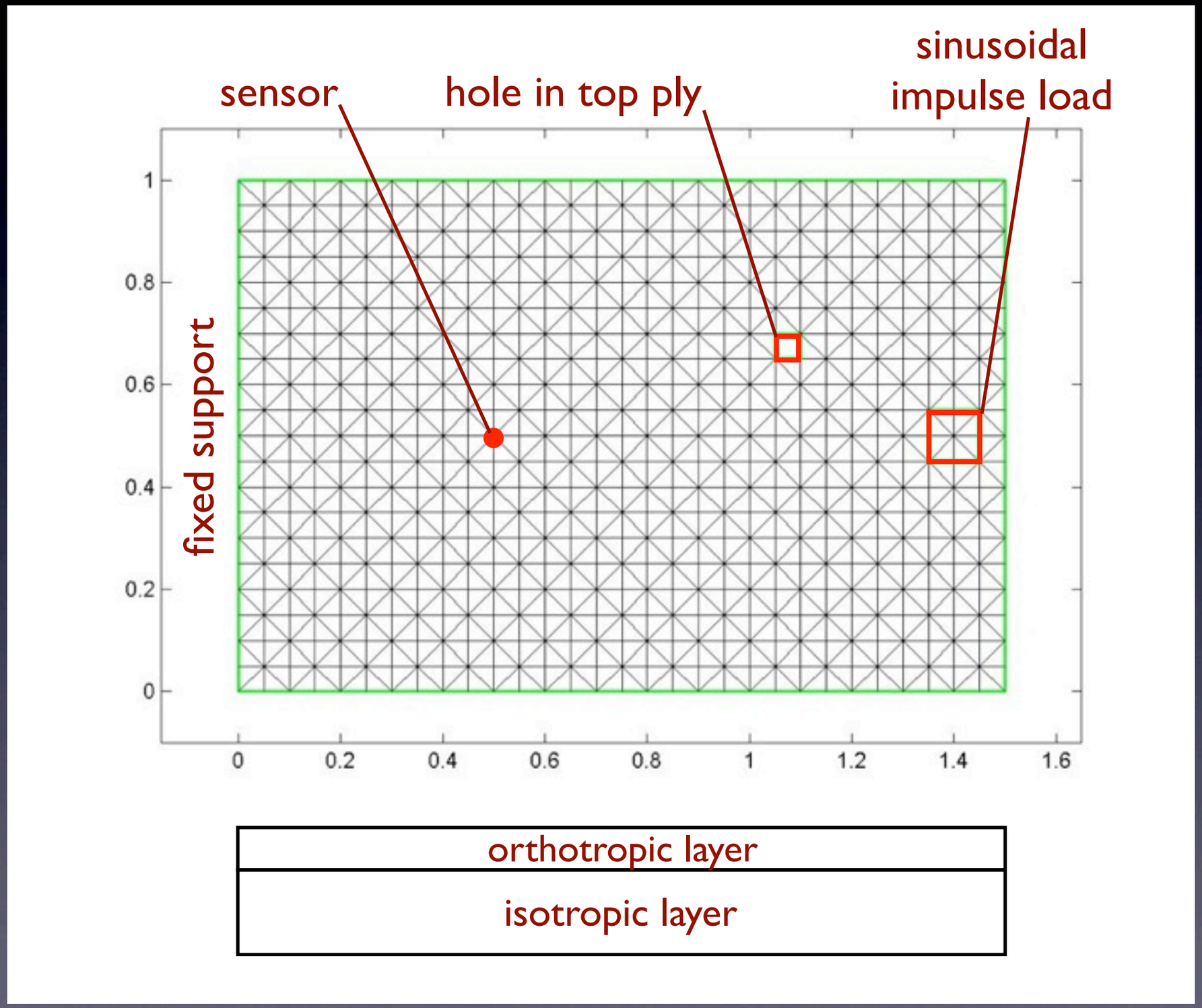
$$\mathcal{P}^{mn} := \mathcal{P}^m(x, y, t) \times \mathcal{P}^n(z)$$

$$u_1, u_2 \in \mathcal{P}^{m1}$$

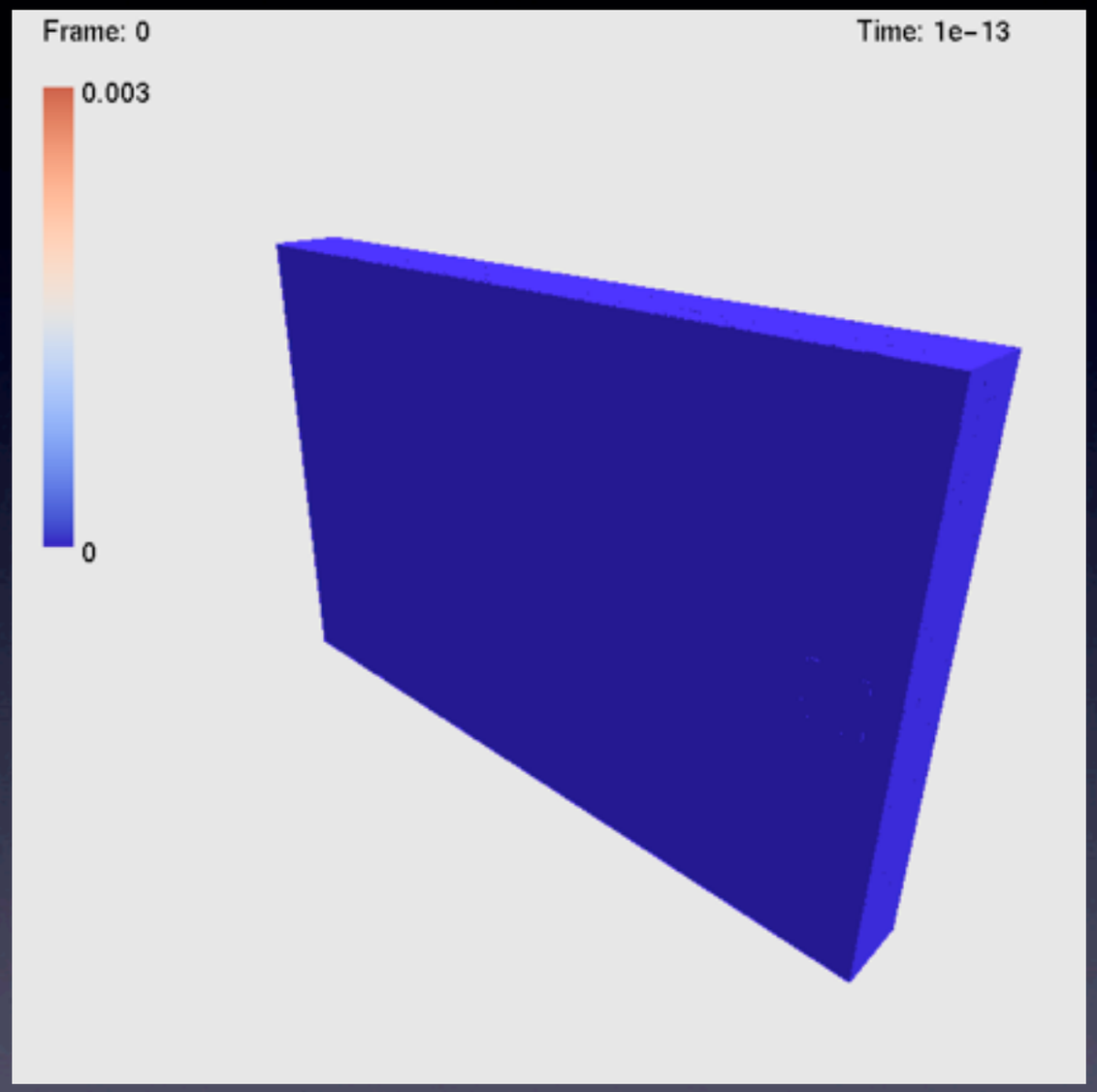
$$u_3 \in \mathcal{P}^{m0}$$

- Single-cell elements at each layer
- Basis functions have layer-wise support (per-layer d.o.f.)
- SDG jump conditions weakly enforce continuity at layer boundaries

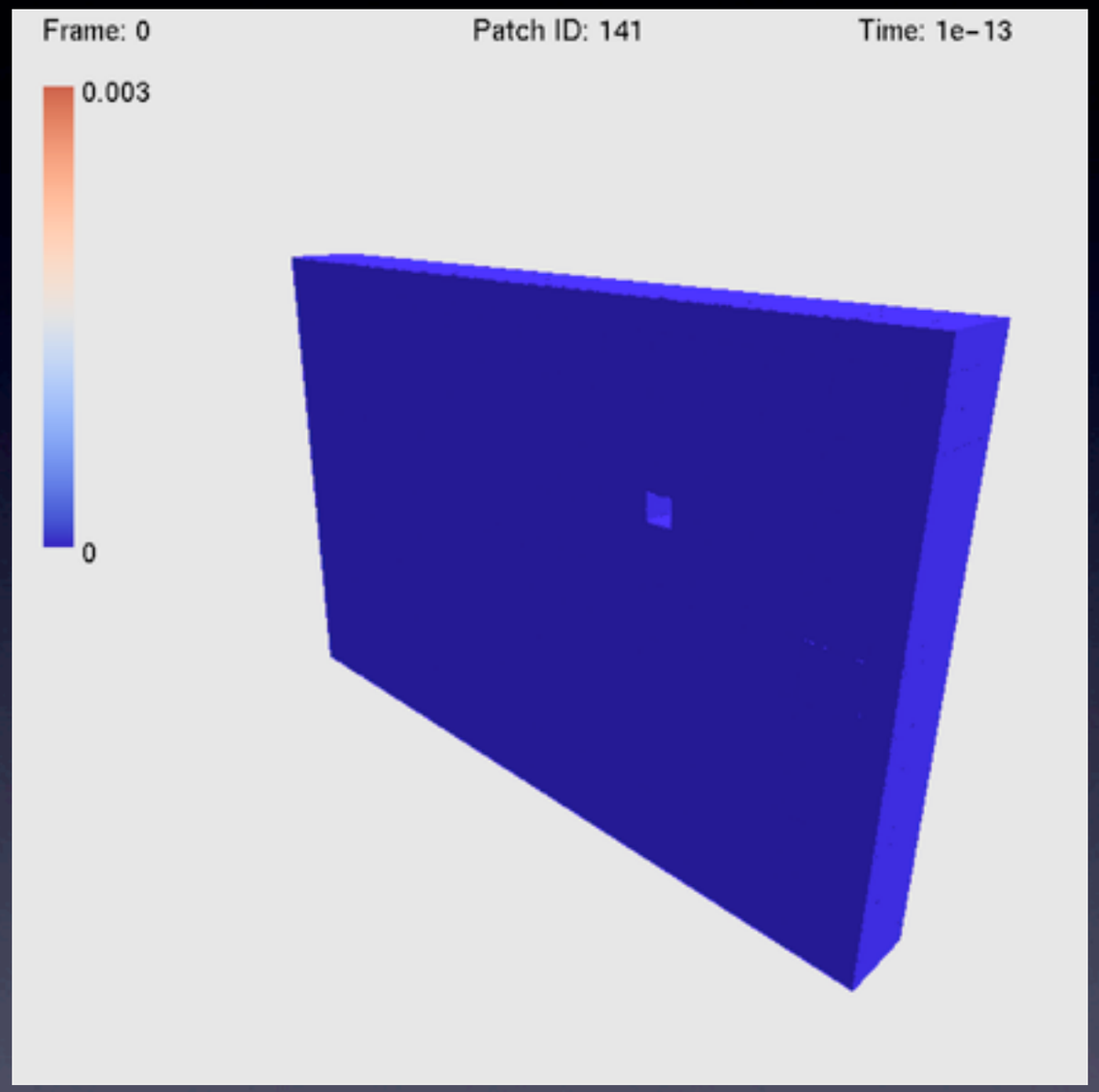
2-ply plate with flaw



2-ply plate with flaw



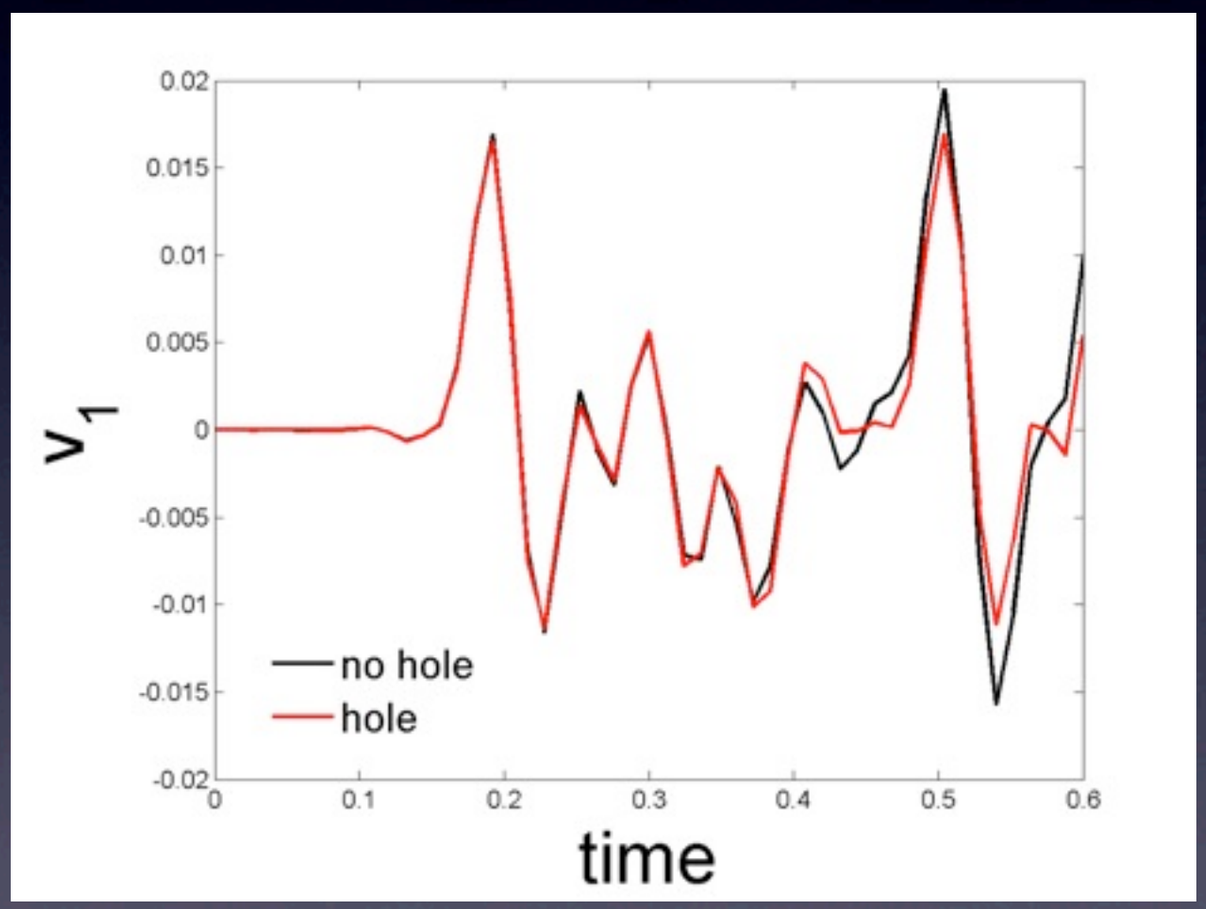
No hole

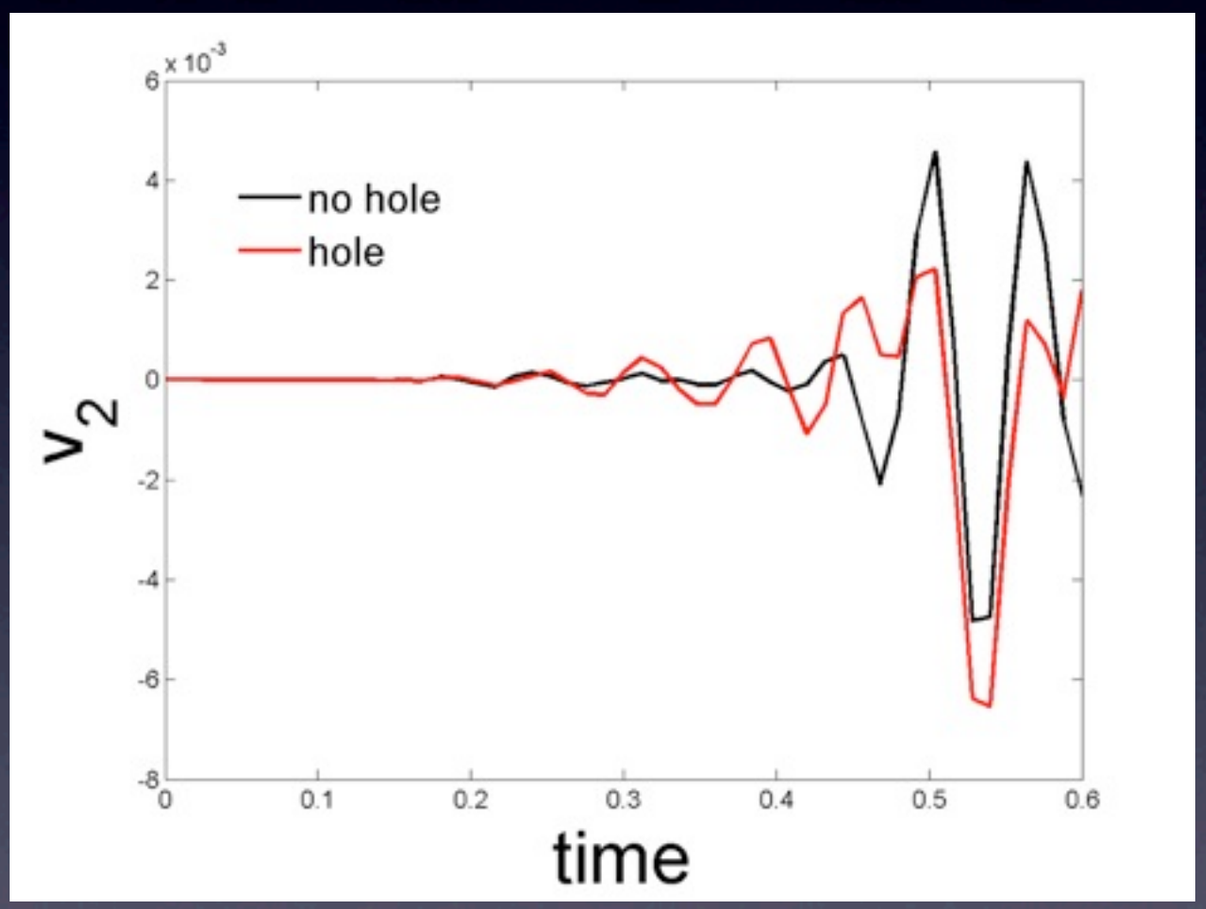


With hole

2-ply plate with flaw

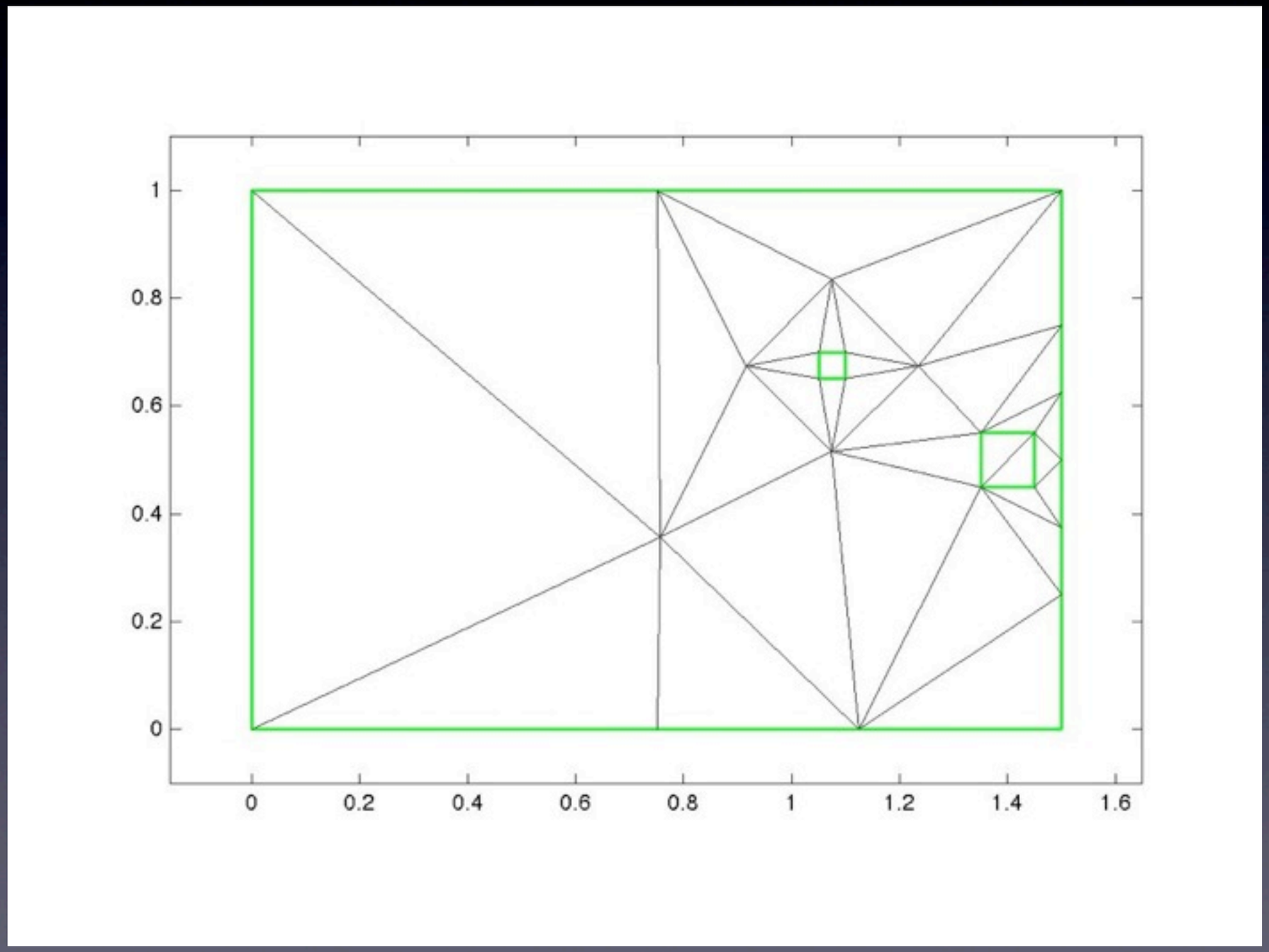
Effect of flaw on velocity at sensor position





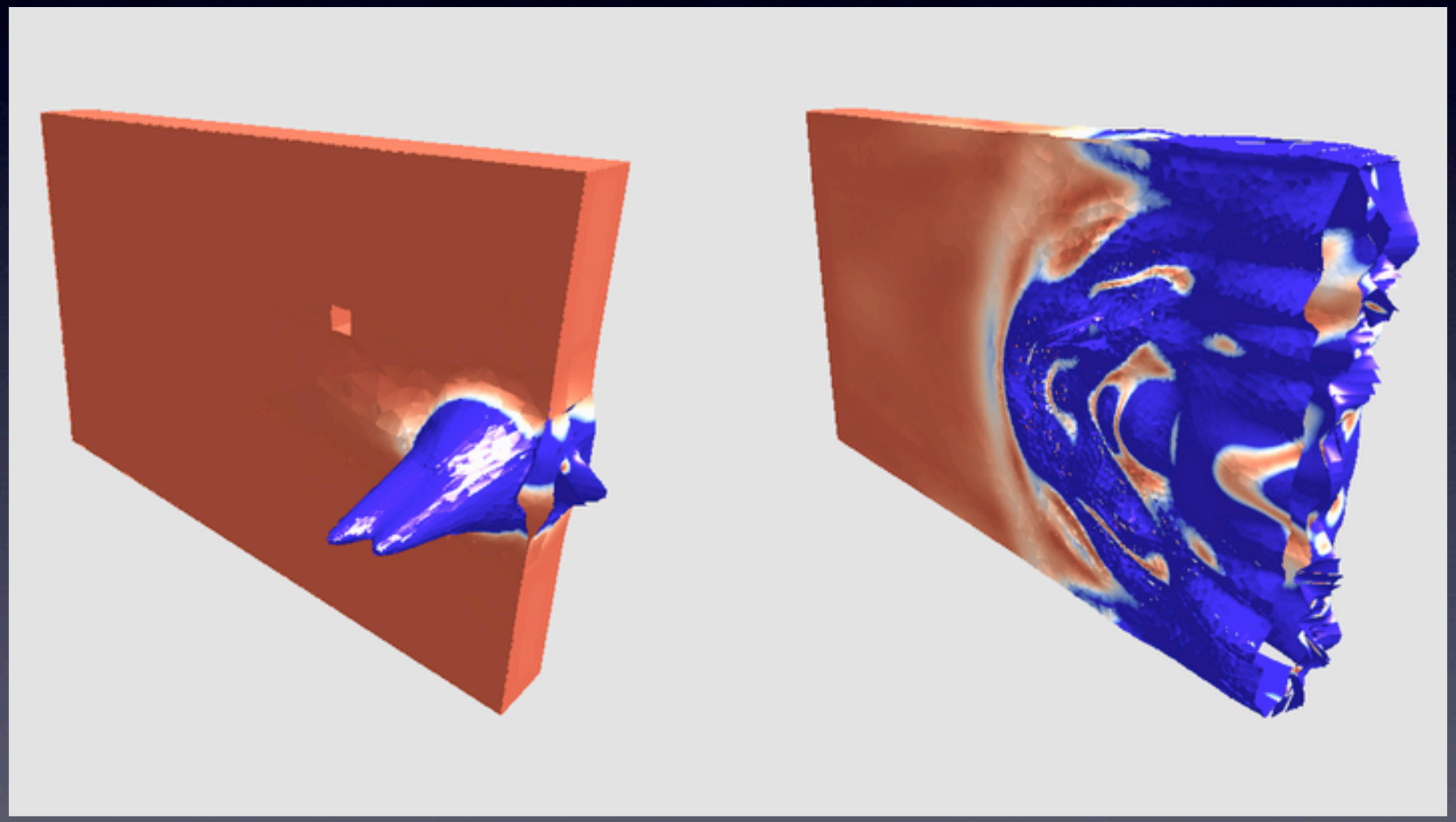
2-ply plate with flaw

initial mesh for *h*-adaptive solutions



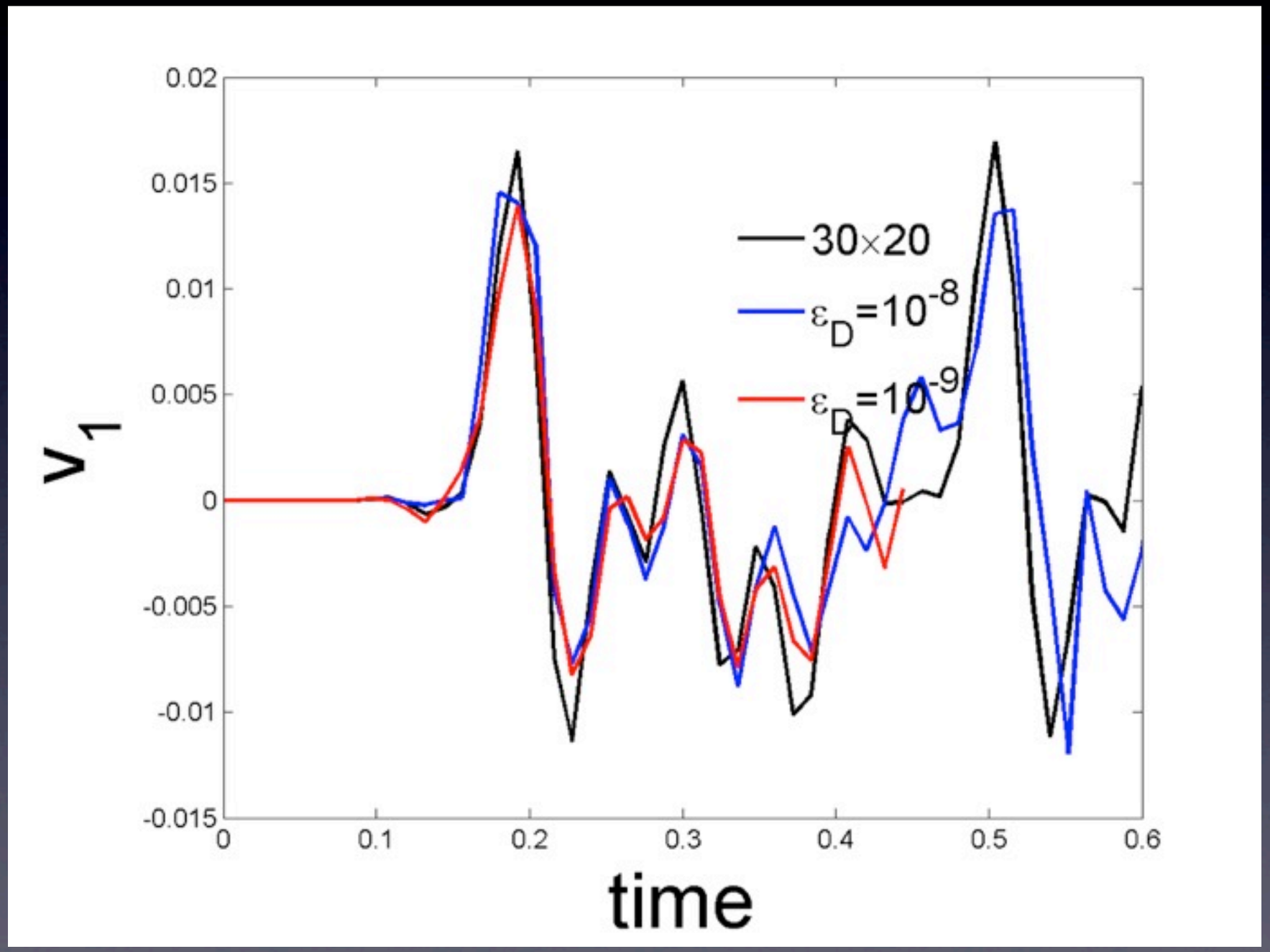
2-ply plate with flaw
h-adaptive solution: $\varepsilon_D = 10^{-9.5}$

click to play movie



2-ply plate with flaw

Adaptive vs. non-adaptive response



Conclusions

- Still higher resolution is needed
- More horsepower is on the way
  - *hp*-adaptivity (already beats *h*-adaptive)
  - Code optimization and vectorization
  - Parallel meshing and patch solution
    - asynchronous
    - hierarchical
    - heterogeneous
    - dynamic load balancing at same granularity of meshing/solution
- Plate models vs. full 3d?
  - Initial response to surface loading and scattering by flaws require 3d
  - Let *hp*-adaptive solver decide what is needed