Theoretical and computational investigation of dynamic contact and fracture

Reza Abedi

Mechanical, Aerospace & Biomedical Engineering
University of Tennessee Knoxville / Space Institute
Part I:

Traction separation relations (TSRs) and Linear Elastic Fracture Mechanics (LEFM)
Cohesive models
Traction Separation Law

- Cohesive models remove stress singularity predicted by Linear Elastic Fracture Mechanics (LEFM)

- Cohesive models are easily integrated into SDG method (discontinuous basis functions)

\[ s \propto \frac{1}{\sqrt{r}} \]

\[ s = \tilde{\sigma} f(\Delta u / \tilde{\delta}) \]

\[ \tilde{\sigma} : \text{Stress scale} \]

\[ \tilde{\delta} : \text{Displacement scale} \]

\[ \Lambda : \text{Length scale} \]
Why does the spike form?
The velocity field is mapped to height field ⇒ high material velocities at the crack tip
Is the material velocity infinite at the crack tip (agreeing with LEFM theory)?
Cohesive Crack Propagation
zoom view

click to play movie

Color: log(strain energy); Height: velocity
Velocity-based length scales
LEFM vs. TSR

- LEFM length scale (singular radius)
  \( r_v \) = size of the region where singular term dominates

- Cohesive length scale
  \( \Lambda_v \) = plateau region size
Singular velocity response?

\[ \frac{\Lambda_{v(k)}}{r_{v(k)}} = \beta_{(k)} \tilde{\zeta}_{(k)} \left( \frac{\pi C_2}{(1 - \nu)c_k} \right)^2 \frac{\tilde{\phi}_{(k)}}{G} \left( \frac{\tilde{\sigma}^k}{\tilde{\sigma}_{(k)}} \right)^2 \left( \frac{A_{(k)}(\hat{v}) \hat{v}}{c_2} \right)^{-2} \]

- Strongly sensitive to crack speed \( \hat{v} \)
  \[- \frac{\Lambda_{v(k)}}{r_{v(k)}} \rightarrow \infty \text{ as } \hat{v} \rightarrow 0^+ \]
  \[- \frac{\Lambda_{v(k)}}{r_{v(k)}} \rightarrow 0 \text{ as } \hat{v} \rightarrow c_R \]

- Weak dependence on cohesive nondimensional parameter \( \tilde{\sigma} / \bar{\sigma} \)

- No evidence of “singular” response when \( r_{v} < \Lambda_{v} \)

- Follows singular form at positions \( r \in [\Lambda_{v}, r_{v}] \) when \( \Lambda_{v} \ll r_{v} \)
Quasi-singular velocity response

\[
\frac{|v(k)|_\infty}{v_{C_k}} = \frac{1}{\sqrt{\gamma/\beta(k)\zeta(k)}} \left(\frac{1 - \nu}{c_k}\right) \left(\frac{G}{\tilde{\phi}(k)}\right)^{\frac{1}{2}} A(k) (\hat{v}) \frac{\hat{v}}{c_2}
\]

- \(|v(k)|_\infty\) bounded for all velocities.

- \(|v(k)|_\infty \to \infty\) as \(\hat{v} \to c_R\)

Low-amplitude loading, \(\sigma' \ll 1\).

High-amplitude loading, \(\sigma' \to 1^-\).
Relation to LEFM

\[ s = \frac{K \Sigma}{\sqrt{2\pi r}} + \text{higher order terms} \]

- LEFM solution is “acceptable” when Small Scale Yielding (SSY) holds:
  
  **SSY:** The size of nonlinear fracture zone is very small compared to relevant length scales in the problem.

- Singular radius is the relevant length scale for dynamic case:
  
  \[ r_k \equiv \text{size of region where singular term dominates for stress} \]

- Fracture zone size: dynamic cohesive process zone is an estimate for “nonlinear fracture zone:

  \[ A_k = \zeta_k \pi \frac{\mu}{1 - \nu} \frac{\tilde{\phi}_k}{(\tilde{\sigma}_k)^2} \frac{1}{A_k(\hat{\upsilon})} \]
Linear Elastic Fracture Mechanics (LEFM) SSY indicator

\[
\frac{A_{(k)}}{r_{(k)}} = \zeta_{(k)} \pi^2 \tilde{\Phi}_{(k)} \left( \frac{\tilde{\sigma}^{(k)}}{\tilde{\sigma}_{(k)}} \right)^2
\]

- not strongly sensitive to crack velocity (as opposed to velocity field for quasi-singular response)
- \( \sigma' = \frac{\tilde{\sigma}}{\tilde{\sigma}} \propto \frac{\tilde{\sigma}}{\tilde{\sigma}} \) is a nondimensional cohesive parameter.
- SSY indicator: General loading and geometry, dynamic loading
  \[\lambda' = \zeta \pi^2 \sigma'^2 \ll 1\]

Low-amplitude loading, \( \sigma' \ll 1 \).
High-amplitude loading, \( \sigma' \to 1^- \).
Traction Separation Laws
Dimensional Analysis

- **Intrinsic cohesive scales**

\[
\tilde{\phi} = \tilde{\sigma} \tilde{\delta}
\]
Energy

\[
\tilde{p} = \rho \tilde{v} = \frac{\tilde{\sigma}}{c_d}
\]
Linear momentum

\[
\tilde{E} = \frac{\tilde{v}}{c_d} = \frac{\tilde{\sigma}}{\rho c_d^2} \propto \frac{\tilde{\sigma}}{\|C\|}
\]
Strain

\[
\tilde{v} = \frac{\tilde{\delta}}{\tilde{\tau}} = \frac{\tilde{\sigma}}{\rho c_d}
\]
Velocity

\[
\tilde{L} = c_d \tilde{\tau} = \frac{\rho c_d^2 \tilde{\delta}}{\tilde{\sigma}} \propto A^0
\]
Length

\[
\tilde{\tau} = \frac{\rho c_d \tilde{\delta}}{\tilde{\sigma}}
\]
Time

\[\tilde{\tau}\] is used for:

- Crack initiation and propagation
- Minimum numerical time step

- **Nondimensional parameters**

  - Spacetime: \( \frac{L}{\tilde{L}} \) and \( \frac{T}{\tilde{\tau}} \)

  - Applied loads:

    \[
    \frac{\tilde{u}}{\tilde{\delta}}, \frac{\tilde{\nu}}{\tilde{\nu}}, c_d \tilde{E}/\tilde{\nu}, \frac{\tilde{p}}{\tilde{\rho}}, \frac{\tilde{\sigma}}{\tilde{\sigma}}
    \]

- \( \frac{L}{\tilde{L}} \) Determines brittleness; related to brittleness indicators of Carpinteri (03), Harder (91)

- \( \frac{\tilde{\sigma}}{\|C\|} \) is not a nondimensional parameter (it is often used as a nondimensional cohesive strength)
Part II:
Dynamically consistent solutions for contact modes
Integrated Fracture/Contact model
Mesoscopic interface subdivisions

Unkowns are:
1. Target fluxes on individual subdivisions
2. Absolute area fractions of subdivisions:

- Bonded & contact–stick (I) : \((1 - D) + D\eta\gamma\)
- Contact–slip (II) : \(D\eta(1 - \gamma)\)
- Separation (III) : \(D(1 - \eta)\)

- \(D = 1\) for pure contact problems; obtained from a fracture evolution law otherwise.
- Relative contact \((\eta)\) and stick \((\gamma)\) area fractions discussed on pages 4 & 5.
Riemann problem set-up

\[ \begin{align*}
\begin{array}{c}
(S^+, \mathbf{v}^+) \\
\hline
\end{array}
\end{align*}\]

---

**Known:**
upstream (interior traces): \( S^\pm, \mathbf{v}^\pm \)

**Goal:**
Riemann solutions on contact interface: \( \mathcal{S}^\pm, \mathbf{v}^\pm \)

**Property:**
\( w^i \pm \) preserved along characteristic directions

- **\( S \):** stress, \( \mathbf{v} \): velocity
- **\( c_d, c_s \):**
dilatational and shear wave speeds
- **\( w^i \pm \):** characteristic values =
  \[ s^i \pm = Z^i \pm \mathbf{v}^i \pm \]
- **\( Z^i \):** Impedance values =
  \[
  \begin{cases} 
  \rho^\pm c_d^i & i = 1 \\
  \rho^\pm c_s^i & i \neq 1 
  \end{cases}
  \]
- **\( s = Sc \):** traction
- **\( \rho \):** material density
### Riemann solutions

<table>
<thead>
<tr>
<th>contact mode</th>
<th>interface conditions</th>
<th>Riemann solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>bonded &amp; contact–stick</td>
<td>( \mathbf{s}^+ = \mathbf{s}^- := \mathbf{s} ) ( \mathbf{v}^+ = \mathbf{v}^- := \mathbf{v} )</td>
<td>( \mathbf{s}^i = \frac{w^{i+} Z^{i-} + w^{i-} Z^{i+}}{Z^{i-} + Z^{i+}} ) ( \mathbf{v}_i = \frac{[w^i]}{Z^{i-} + Z^{i+}} = \frac{[s^i]}{Z^{i-} + Z^{i+}} + \frac{v_i^+ Z^{i+} + v_i^- Z^{i-}}{Z^{i-} + Z^{i+}} )</td>
</tr>
<tr>
<td>contact–slip</td>
<td>( \mathbf{s}^{1+} = \mathbf{s}^{1-} := \mathbf{s}^1 ) ( \mathbf{v}_1^+ = \mathbf{v}_1^- := \mathbf{v}<em>1 ) ( \mathbf{\tau}^+ = \mathbf{\tau}^- := \mathbf{\tau} = k\langle -\mathbf{s}^1 \rangle + \mathbf{e}</em>{\mathbf{\gamma}} )</td>
<td>( \mathbf{s}^{1,} \mathbf{v}<em>1 = \text{same as contact–stick solution} ) ( \mathbf{s}^i = k\langle -\mathbf{s}^1 \rangle + \mathbf{e}</em>{\mathbf{\gamma}} ) ( \mathbf{v}_i^\pm = \pm \frac{\mathbf{s}^i - w_i^\pm}{Z_i^\pm} = v_i^\pm + \frac{\mathbf{s}^i - s_i^\pm}{Z_i^\pm} )</td>
</tr>
<tr>
<td>separation</td>
<td>( \mathbf{s}^+ = \mathbf{s}^- := \mathbf{s} = 0 )</td>
<td>( \mathbf{s} = 0 ) ( \mathbf{v}_i^\pm = \pm \frac{w_i^\pm}{Z_i^\pm} = v_i^\pm - \frac{s_i^\pm}{Z_i^\pm} )</td>
</tr>
</tbody>
</table>

\( \mathbf{\tau} := s^2 e_2 + s^3 e_3 \): tangential traction at contact interface

\( \mathbf{e}_{\mathbf{\gamma}} \): unit vector along the velocity jump from contact–slip solution

\( \mathbf{e}_{\mathbf{\tau}} \): unit vector along the tangential traction from contact–stick solution
Contact Transitions: Contact-Separation (C-S)

Provable Max. Penetration
No stiff systems

$\sigma$: normal contact–stick
Riemann traction

$\eta = 0$

$\delta > 0$

$\delta < 0$

Contact locus ($\eta = 1$)

Infeasible region (no penetration)

$t', \delta'$: normalized time and penetration
$\eta = g(t') \Rightarrow \delta' = h(t') \Rightarrow \eta = f(\delta')$, where $f = g \circ h^{-1}$

$g(t')$: ideal to design well-behaved & $C^1$ regularization

$f(\delta')$: computationally favorable; $\delta'$ readily available

Example for regularization design ($g \rightarrow f$):

$g(t') = 1 - \cos(\frac{\pi}{2} t') \Rightarrow f(\delta') = 1 - \sqrt{1 - \delta'^2}$

Only $S \rightarrow C$:
- occurs at constant $\sigma$  $\Rightarrow$
- introduces shocks  $\Rightarrow$
- requires regularization

$\nabla$

$S \rightarrow C$

Regularized

$\eta = 0$

$\delta$

$\sigma$

$\eta = 0$

$\delta$

$\nabla$

$S \rightarrow C$
Contact Transitions: Stick-Slip

Coulomb friction law:

- In slip mode: tangential traction ($\tau$) is aligned with velocity jump ($[\nu]$):
  $$\tau = k(-\dot{s})e_\nu$$

  But $[\nu] = 0$ at the transition $\Rightarrow$
  - $e_\nu$ is not defined at the transition
  - Transition is apparently discontinuous
    - Many “numerical treatments” are presented to address the problem

Riemann solutions for Coulomb friction law $\Rightarrow$

$$[\nu] = \frac{Z^2_2 + Z^2_+}{Z^2_2 + Z^2_+} (|\dot{\tau}| - |\tau|) e_\tau \Rightarrow (\dot{\tau}: \text{contact-stick tangential stress})$$

- $e_\nu = e_\tau$

- At the transition $|\dot{\tau}| = k(-\dot{s})_+ > 0 \Rightarrow |\tau| > 0$. That is:
  - $e_\tau$ is continuous at the transition
  - The representation $\dot{\tau} = k(-\dot{s})_+ e_\tau$:
    - Eliminates the need for regularization
    - $\gamma$ only takes the binary values of 0 and 1
Numerical verification: Identical bars

Benchmark problem [Hughes (76); Laursen, Chawla (97); Czekanski, Meguid (01); Cirak, West (05); etc.]

Riemann solutions incorporated in Spacetime Discontinuous Galerkin finite element method

Unlike other solutions, SDG results are not overly damped and are free of numerical oscillations and overshoot / undershoot
Contact Transitions: Stick-Slip

Coulomb friction law:

- In slip mode: tangential traction ($\tau$) is aligned with velocity jump ($[\dot{v}]$):
  \[
  \tau = k(\dot{s}^1)_+ e_{\dot{v}}
  \]
  But $[\dot{v}] = 0$ at the transition $\Rightarrow$
  - $e_{\dot{v}}$ is not defined at the transition
  - Transition is apparently discontinuous
    - Many “numerical treatments” are presented to address the problem

Riemann solutions for Coulomb friction law $\Rightarrow$

\[
[\dot{v}] = \frac{Z^2_++Z^2_-}{Z^2_++Z^2_-} (|\tau| - |\tau|) e_{\tau} \Rightarrow (\tau : contact-stick tangential stress)
\]
- $e_{\dot{v}} = e_{\tau}$
- At the transition $|\tau| = k(\dot{s}^1)_+ > 0 \Rightarrow |\tau| > 0$. That is:
  - $e_{\tau}$ is continuous at the transition
  - The representation $\tilde{\tau} = k(\dot{s}^1)_+ e_{\tau}$:
    - Eliminates the need for regularization
    - $\gamma$ only takes the binary values of 0 and 1
Brake simulation

Contact mode transitions (high slip velocity)

Depending on sliding velocity there are different mode transitions:

- $V_0$:
  - Low: stick-slip
  - Medium: stick-slip-separation
  - High: slip-separation

Histories presented at point $A$
Brake simulation
Contact mode transitions (high slip velocity)

click to play movie

Color: strain energy; Height: velocity
Key points of the contact model

- Opposed to contact models that prevent node penetration at discrete level, the formulation is continuum-based.
- Solutions are for dynamic setting.
- Riemann solutions preserve the characteristics of the elastodynamics problem.
- Only separation to contact requires regularization:
  - There is a maximum tunable penetration.
  - The regularization is rather uniform and can be $C^1$ continuous.
  - It does not make the problem stiff (unlike penalty methods for example).
- Numerical results do not exhibit nonphysical oscillations, excessive dissipation, overshoot, and undershoot.
- Fracture (damage evolution) is easily incorporated.
Part III:
An interfacial rate-dependent contact/fracture model
Interfacial rate-dependent damage model

Motivated by mesoscale features:

Conical (parabolic) marking on the crack surface

Fineberg & Marder 1999

Ravi-Chandar, Knauss 1984
Damage evolution law

\[ \dot{D} = \begin{cases} \frac{1}{\tau}[1 - H((f(y) - D)_+)] & D < 1 \\ 0 & D = 1 \end{cases} \]

\[ H(x) = \exp(-a \cdot x) \]

Properties of the evolution law

- a maximum damage rate, exists.

- Target damage value, \( f(y) \), is a function of stress.

- The rate is a function of the difference between damage and its target value.
Key points of the damage model

- Based on a characteristic length $\bar{s}$ and characteristic time $\bar{\tau}$.
- Damage is stress induced (Ravi-Chandar, Yang 1997).
- Rate-dependent damage model:
  - Hardening effects at high-strain rates (Fineberg & Marder 1999)
  - There is a maximum damage rate $\bar{\tau}$.
  - Introduces a length (displacement) scale.
  - Eliminated the mesh dependency of static damage models (strain softening behavior, Bazant & Belytschko 1984)
- No artificial bulk compliance: Perfectly bonded initial response with no effect on bulk material response.
- Differentiable response (facilitates numerics).
- Dynamically consistent fluxes.
- Integrated contact modes.
Deeper understanding of fracture models
Comparison of cohesive and damage models

While cohesive and interfacial damage models have the intrinsic scales, they behave very differently even when same nondimensional parameters are used.

They have fundamentally different crack acceleration pattern ⇒
Very different dynamic crack propagation patterns
Contact/damage examples
Crack closure: cyclic, dynamic loading

click to play movie

Color: log(strain energy); Height: velocity
Contact/damage examples
cyclic loading for a stiff circular inclusion

click to play movie

Color: log(strain energy);
Height: velocity
Part IV:
A probabilistic model for dynamic fracture
Solution-dependent crack path

Arbitrary crack extension:

- Element boundaries are aligned with arbitrary propagation direction by spacetime meshing operations.
- Unlike XFEM, no need to introduce discontinuous features within elements.
- Maximum effective stress governs direction.
Solution-dependent crack path

Probabilistic crack nucleation:

- Fracture process are inherently stochastic
- Deterministic & homogeneous treatments fail at continuum level!
- Cracks nucleate from defects with random distribution of strength

Instabilities: no additional criterion required to capture:

- Branching
- Microcracking
- Surface roughening
Dynamic fracture with damage-delay cohesive model, random defects and nucleation

PMMA:
- $E = 3.24 GPa$
- $\rho = 1190 \frac{Kg}{m^2}$
- $w = 0.020 MPa$
- $\tau = 10^{-2} \mu s$

based on Sharon and Fineberg experiment (1996)
Dynamic fracture
Spacetime mesh front

click to play movie
Dynamic fracture
Displaced shape

click to play movie

![Diagram showing dynamic fracture with displaced shapes.](image)
Refinement Details: 1 x zoom
Refinement Details: 25 x zoom

Crack-branching automatically follows from probabilistic nucleation model.
Refinement Details: 100 x zoom
Crack path oscillation for higher crack speeds agrees with theoretical and experimental findings.
Refinement Details: 2000 x zoom
Part V:
Structural Health Monitoring for composite laminates
Forward analysis
Structural Health Monitoring in Layered Plate Systems

- Flaw detection and identification
  - Holes, through-cracks, delamination, bulk damage, etc.
  - Many layers with strong anisotropic response
- Detect and simulate changes in wave propagation
  - Precise physical excitation and sensing
  - Inverse problem to identify flaws (type, location, size, shape): requires many forward analyses

- Many forward analyses → efficiency
- to not only resolve the minute differences causes by defects, but also differentiate them from numerical artifacts → high accuracy and stability

SDG method is an ideal solver for SHM applications
Forward analysis
Plate with multiscale features (planar modes)

(click to play movie)

Height: velocity

Detection of arrays of defects from wave scattering:
Large defects at lower right corner & very small ones at upper left corner (cannot be seen)
Hybrid 2D/3D plate formulation

- Low order kinematic assumptions for thickness direction can introduce relative errors larger than 10 (e.g. multi-layer composite laminates).
- Difficulties in the derivation of 2D formulations for high order in-thickness kinematics.
- Full 3D analysis may be expensive and not suitable for specific thickness direction kinematic assumptions (e.g. zigzag interpolations)
Application to multi-ply plates
(only front mesh shown below)

- Extrude 2D spatial geometry in transverse ‘thickness’ direction

- Product bases: (in-plane) x (transverse) x (time)
  - Independent selections for (u,v) and (w) d.o.f.
  - Elements with multiple int. cells for simple layered plate model
    - First and higher-order shear-deformation-theory models
    - ‘Zig-zag’ layered plate model; C0 piecewise-linear basis
  - Multi-element model: one (element/int. cell) per layer
    - SDG models with per-ply dofs
    - Interface models for delamination
  - Higher-order and p-adaptive models
    - avoid shear correction factors; transitions to full 3D x time models
2-ply plate with flaw

- Sensor
- Hole in top ply
- Sinusoidal impulse load

Orthotropic layer
- Isotropic layer
2-ply plate with flaw
Effect of flaw on velocity at sensor position
Forward analysis
Plate with multiscale features (planar modes)

**click to play movie**

Color: strain energy (blue: high); Height: velocity
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