01/28/2014

Tuesday, January 28, 2014
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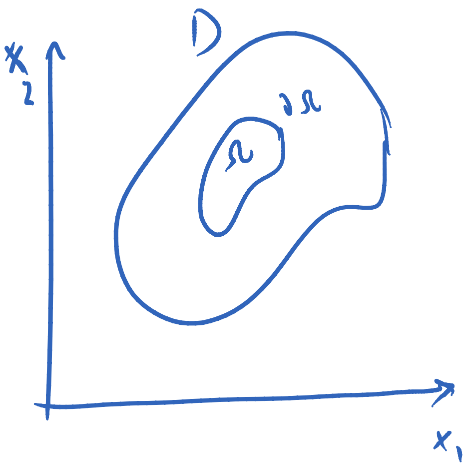
Summary of balance law approach:

①



$\forall \Omega \subseteq D \quad \int_{\partial\Omega} F \cdot \vec{ds} - \int_{\Omega} r \, dV = 0$

② strong form

to get to strong form we used divergence theorem

$\forall \Omega \subseteq D \quad \int_{\Omega} (\nabla \cdot F - r) \, d\bar{V} = 0$

$\Longrightarrow \quad \boxed{\nabla \cdot F - r = 0 \quad \text{strong form}}$

③ weighted residual statem (continuum)

First needed to close the system:

a) adding constitutive & other <u>bulk</u> equations to balance number of unknowns & differential equations.

$\sigma = E\varepsilon \qquad \varepsilon = u_{,x}$

b) Adding boundary conditions

b1. Essential BC — lower half derivatives

b2. Natural BC — upper half "

M is differential operator order $(\nabla . F)$

Find $u \in V \{ C^M(D) \mid \underbrace{u(\Gamma_u) = \bar{u}(\Gamma_u)}_{\text{Essential BC}} \}$

$\forall w \in \{ C(D) \mid w(\Gamma_u) = 0 \}$

such that

$$\int_D w . \underbrace{(\nabla F - r)}_{R_i} + \int_{\partial D_f} w(\underbrace{\bar{t} - t}_{R_f}) = 0$$

∂D_f → natural Boundary

⊕ Weak statements

Take derivatives to the weight function

Sample for solid mechanics

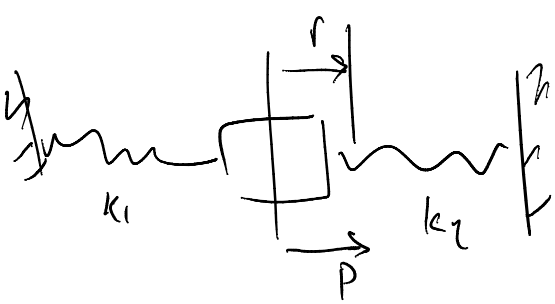
$$\int_D \varepsilon(w)\,\sigma(u)\,dV - \int_{\partial D_f} w\cdot\bar{t}\,ds = 0$$

$$u \in \mathcal{V} = \{ C^1(D) \mid u(\Gamma_u) = \bar{u}(\Gamma_u) \}$$

$$\forall w \in \mathcal{W} = \{ C^1(D) \mid w(\Gamma_u) = 0 \}$$

weak statement balances differentiability requirement.

Why having minus sign when forming the energy of a system?



$$P = -\frac{\partial \Pi_p}{\partial r} \Rightarrow$$

$$\Pi = \frac{1}{2}k_1 r^2 + \frac{1}{2}k_2 r^2 - Pr$$

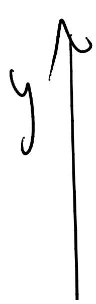
?

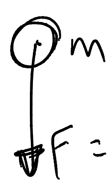
$$\Pi_p = -Pr$$

$$F = -\frac{\partial \Pi_p}{\partial r} = -(-P) = P$$

Potential function $\Pi(r)$

$$\text{force} = -\nabla\Pi(r)$$





$F = -mg$

$$\Pi = mgy$$

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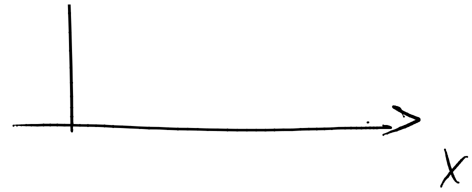


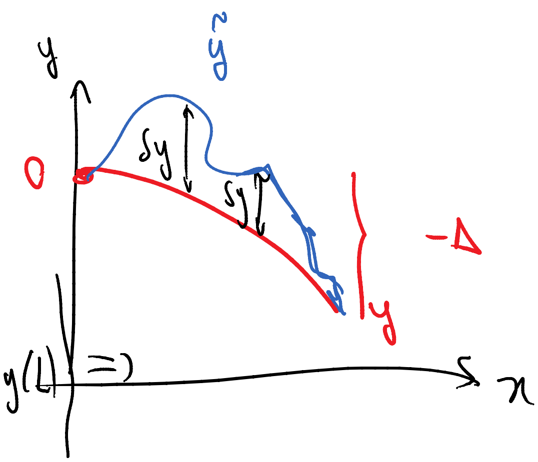
$$\boxed{F = -\nabla \Pi} = \begin{bmatrix} -\dfrac{\partial mgy}{\partial x} \\ -\dfrac{\partial mgy}{\partial y} \end{bmatrix} = \begin{bmatrix} 0 \\ -mg \end{bmatrix}$$

Finding the space of the increments

$$\Pi(y) \leq \Pi(\tilde{y})$$

$$y(L) = -\Delta$$

$$\tilde{y}(L) = -\Delta = y(L) + \delta y(L) \Rightarrow$$



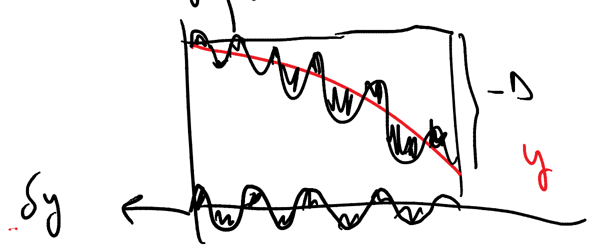
$$\delta y(L) = -\Delta - (-\Delta) = 0$$

First increment for a functional

$$\Delta \Pi = \Pi(\tilde{y}) - \Pi(y)$$

$$= \Pi(y + \delta y) - \Pi(y)$$

$$\tilde{y} = y + \delta y$$



Function of x & y, what is first variation?

$$\delta f(x,y) \simeq \frac{\partial f(x,y)}{\partial x}\Delta x + \frac{\partial f(x,y)}{\partial y}\Delta y$$