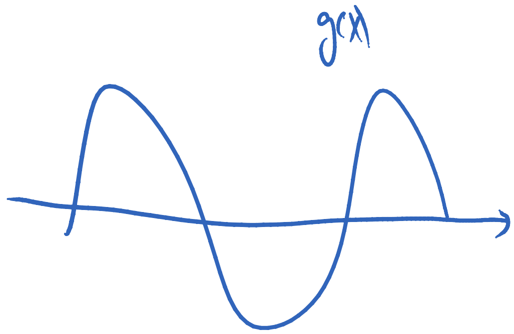
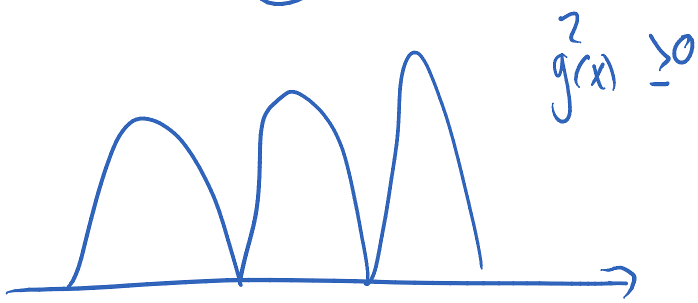
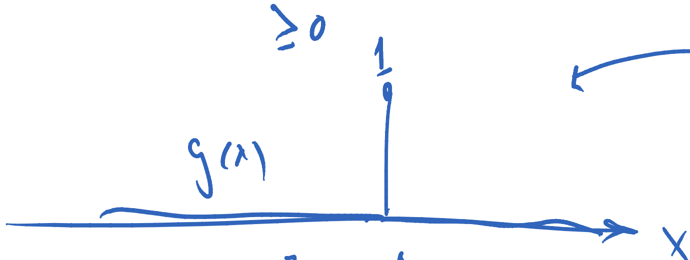
02/04/2014

Tuesday, February 04, 2014
11:58 AM





$$\int \underbrace{g^2(x)}_{\geq 0}\, dx = 0 \implies g(x) = 0 \text{ a.e}$$



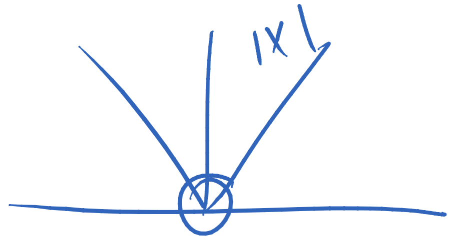
$\int g^2(x)\, dx = 0$

g(x) is continuous

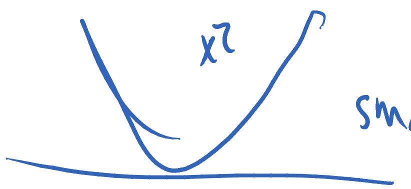
$$\int_D g^2(x)\, dx = 0 \iff \forall x \in D : g(x) = 0$$

Why not this?

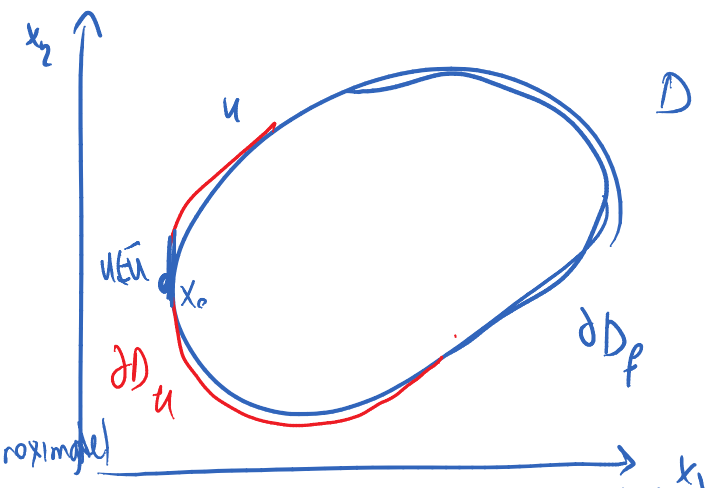
$$\int_{D} \underbrace{|g(x)|}_{\geq 0}\, dx = 0 \iff \forall x \in D \quad g(x) = 0$$



non smooth



smooth



discrete (approximate)

$$u^h = \sum_{i=1}^{n} \underset{\text{unknowns}}{a_i} \overbrace{\underbrace{\phi_i(x)}_{\text{trial or test functions}}}^{\text{satisfy homogeneous essential BC's } \phi_i(x_0)=0} + \phi_p(x)$$

satisfies essential boundary condition: $\phi_p(x_0) = \bar{u}(x_0)$

$$u^h(x_0) = \bar{u}$$

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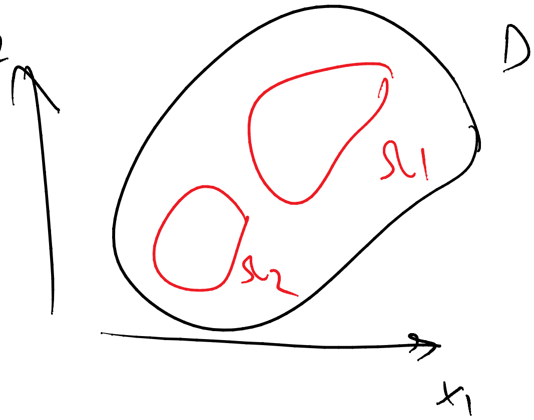
1. We started with balance law:

CONTINUUM



$$\forall \Omega \subseteq D \qquad \int_{\partial\Omega} \underbrace{\vec{F}\cdot d\vec{s}}_{\text{flux}} - \int_{\Omega} \vec{r}\cdot dv = 0$$

DISCRETE

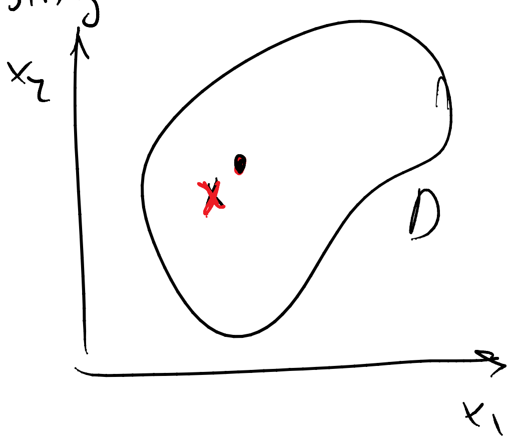
we choose n subdomains Ω_i $i=1,\dots,n$

we choose n Ω_i

for $i=1,\dots,n$

$$\int_{\partial\Omega_i} \vec{F}\cdot d\vec{s} - \int_{\Omega_i} r\, dv = 0$$

2. strong form

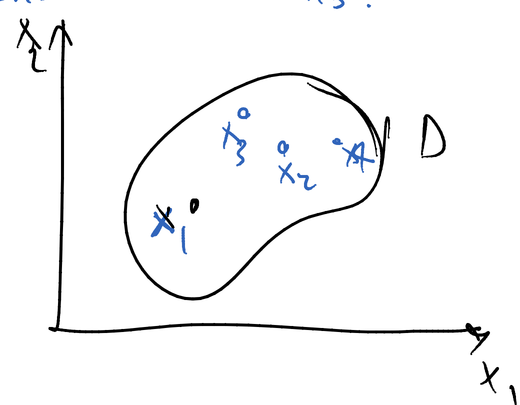


$\forall x \in D$

$$\nabla\cdot F(u) - r = 0$$

$$L_M(u) - r = 0$$

In discrete setting we choose n x_i's:



for $i=1,\dots,n$

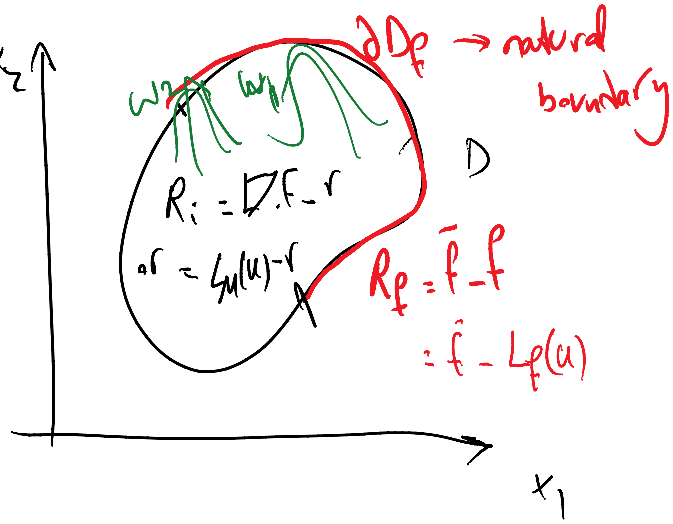
$$\nabla\cdot F(u(x_i)) - r = 0$$

$$L_M(u) - r = 0$$

$$\nabla\cdot F(u(x_i)) - r = 0$$

$$L_M(u)(x_i) - r = 0$$

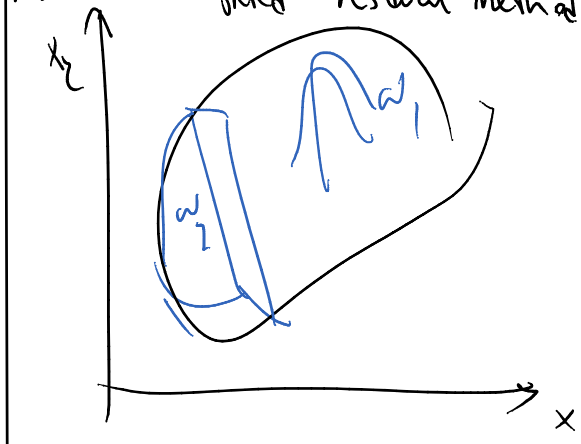
3. Weighted residual



$\forall w \in W$:

$$\int_D w \cdot R_i \, dV + \int_{\partial D_f} w \cdot R_f \, ds = 0$$

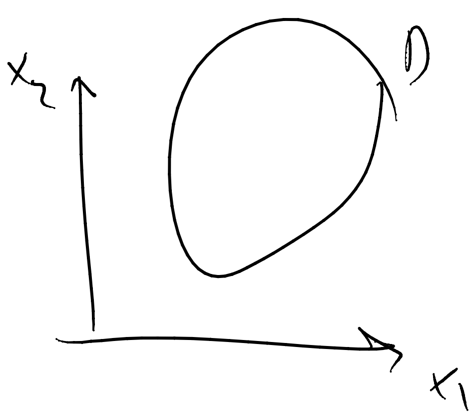
Discrete: Weighted residual method



for $i = 1, \dots, n$

$$\int_D w_i \cdot R_i \, dV + \int_{\partial D_f} w_i \cdot R_f \, ds = 0$$

4. weak statement

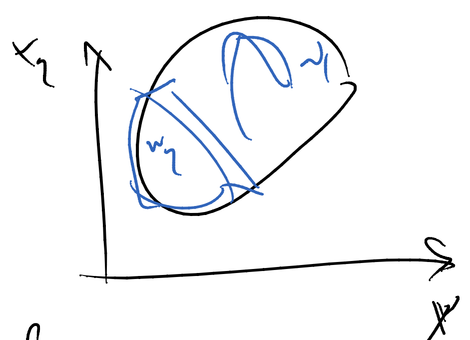


$$\forall w \quad \int L_m^f(w) \, L_m(u) \, dV =$$

discrete

instead of $\forall w$

we choose n w's



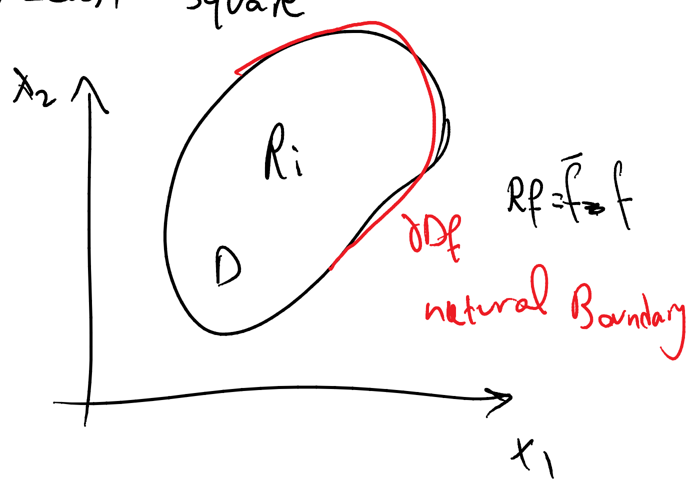
$$\forall \omega \quad \int_D L_m^{+}(\omega)\, L_m(u)\, dv = \int_{\partial D_f} \omega\, \bar{f}\, ds$$

$$\int_D L_m^{f}(a_i)\, L_m(u)\, dv + \int_{\partial D_f} a_i \cdot \bar{f}\, ds = 0$$

$\cancel{x}_1$

(5.) Least Square

not directly following (*)



$R_f = \bar{f} - f$

$$R^2 = \int_D R_i^2\, dv + \int_{\partial D_f} R_f^2\, ds = 0$$

for all $\cancel{u}, \tilde{u} \in \bar{V}$ (satisfies essential BC)

$$\underbrace{R^2(u)}_{=0} \leq R(\tilde{u})$$

$R^2(u^h) =$ known

$R^2\left(\sum a_i\, \phi_i(x) + \phi_p(x)\right)$

unknown

$= R^2(a_1, \ldots, a_n)$

$R^2(u^h) \leq R^2(\tilde{u}^h)$

for all $\tilde{u}^h =$

$\sum \tilde{a}_i\, \phi_i(x) + \phi_p(x)$

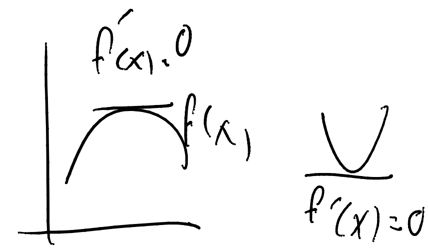
$R^2(a_1, \ldots, a_n)$

$R^2(a_1, \ldots, a_n)$

we want to minimize it

So

gradient of R^2 should be zero

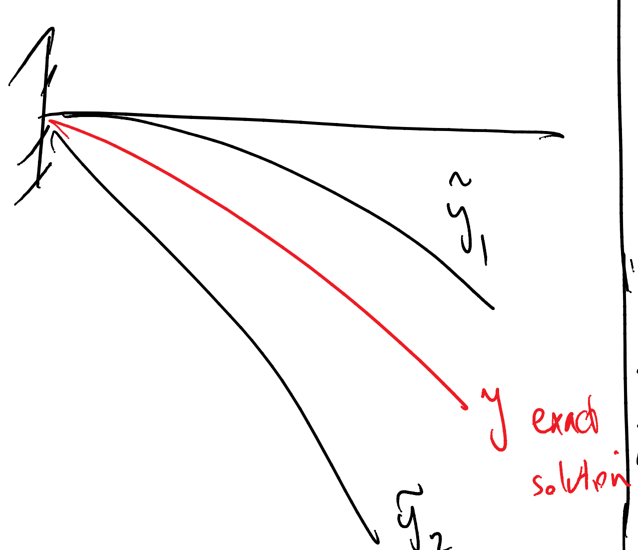


in discrete setting

$$\boxed{\frac{\partial R^2(a_1,\dots,a_n)}{\partial a_i} = 0 \qquad \forall i = 1,\dots,n}$$

6. (not following 1-5, it's an independent statement)

Energy method



for $y, \tilde{y} \in V$ { satisfies essential

$$\tilde{u}^h(x) = \sum \tilde{a}_i \phi_i(x) + \phi_p(x)$$

unknowns

for discrete solution

$$u^h(x) = \sum a_i \phi_i(x) + \phi_p(x)$$

example

$$\Pi(u) = \int_0^L EA\,(u')^2\,dx - \int_0^L q\,u(x)$$

$$\Pi(a_1,\dots,a_n)$$

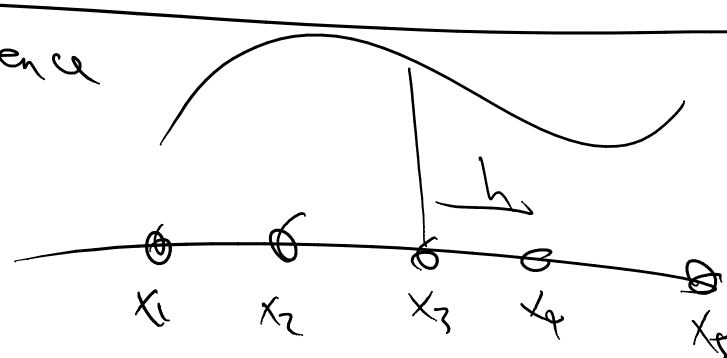
for $\tilde{y}, y \in V$ { ... essential BC }

$\Pi(y) \leq \Pi(\tilde{y})$

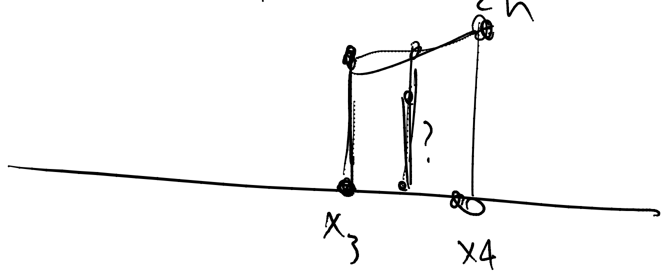
$\Pi(a_1, \dots, a_n)$

$\frac{\partial \Pi}{\partial a_i} = 0$ for $i = 1, \dots, n$

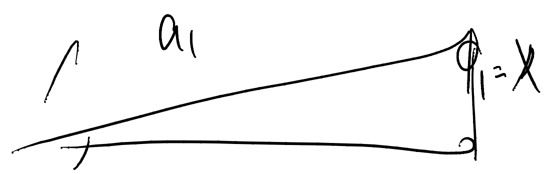
Finite difference

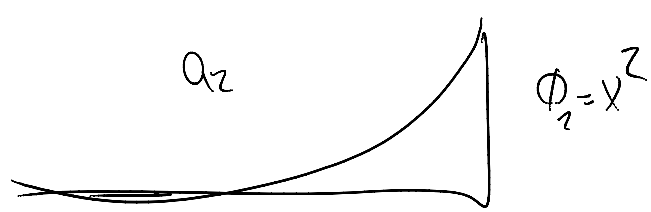


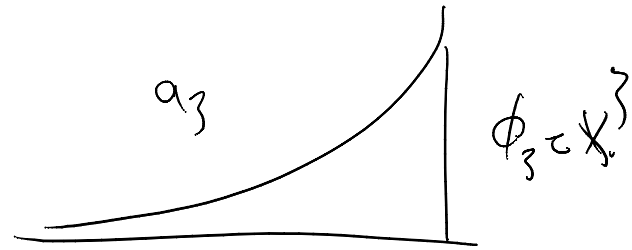
$$u'(x_3) = \frac{u(x_4) - u(x_2)}{2h}$$



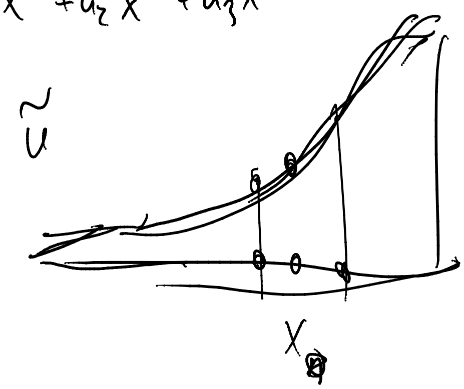
3:







$$u^h = a_1\phi_1(x) + a_2\phi_2(x) + a_3\phi_3(x) =$$
$$a_1 x + a_2 x^2 + a_3 x^3$$



$$u^h(x_0) = a_1\phi(x_0) + a_2\phi_2(x_0) + a_3\phi_3(x_0)$$

as soon as we have a_i's
we can "directly" evaluate the
function everywhere