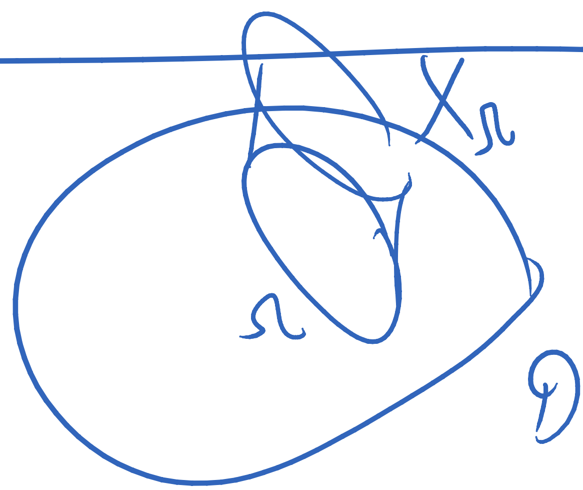
02/06/2014

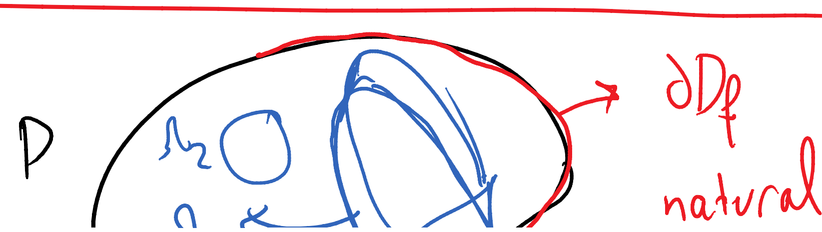
Thursday, February 06, 2014
11:39 AM

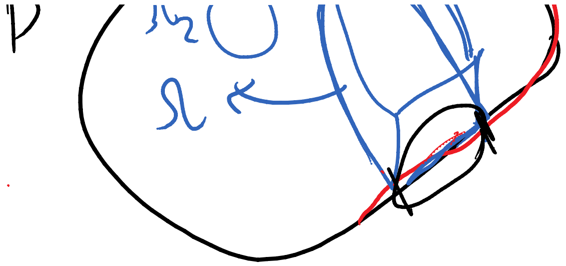
$$\underbrace{\left(\int_D \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \left[L_M|\phi_1|, \dots\, L_M|\phi_n| \right] dv \right)}_{1\times n \quad \text{matrix}} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix}$$

$$\int_D \chi_\Omega\, f(x)\, dv$$



$$= \int_{D\setminus\Omega} 0 \cdot f(x)\, dv + \int_\Omega 1 \cdot f(x)\, dv = \int_\Omega f(x)\, dv$$



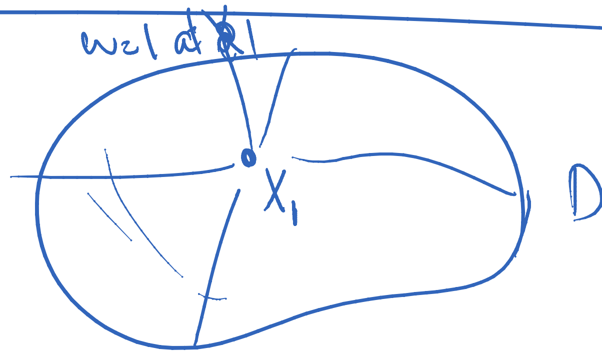


natural boundary condition

$$\int_{\partial D_f} \chi_\Omega \, f(x)\,dv$$

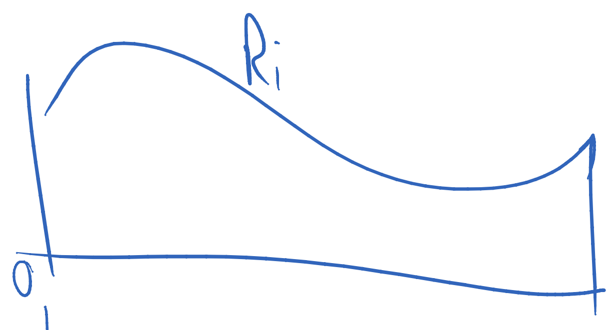
$$(\partial\Omega)_f = \partial\Omega \cap \partial D_f$$

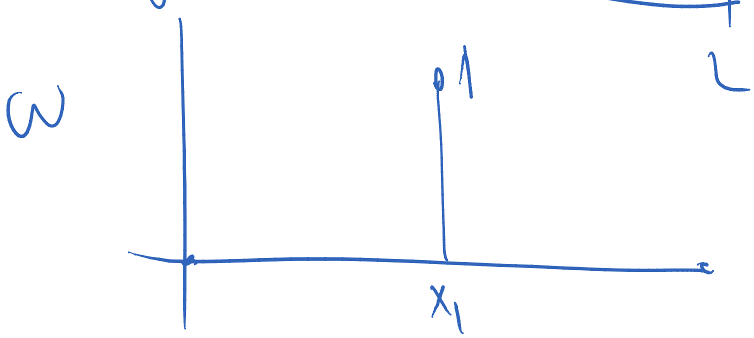
$$= \int_{\partial D_f \cap \partial\Omega}$$



?

$$\int_D \omega \cdot R_i(x)\,dv = R_i(X_1) = 0$$



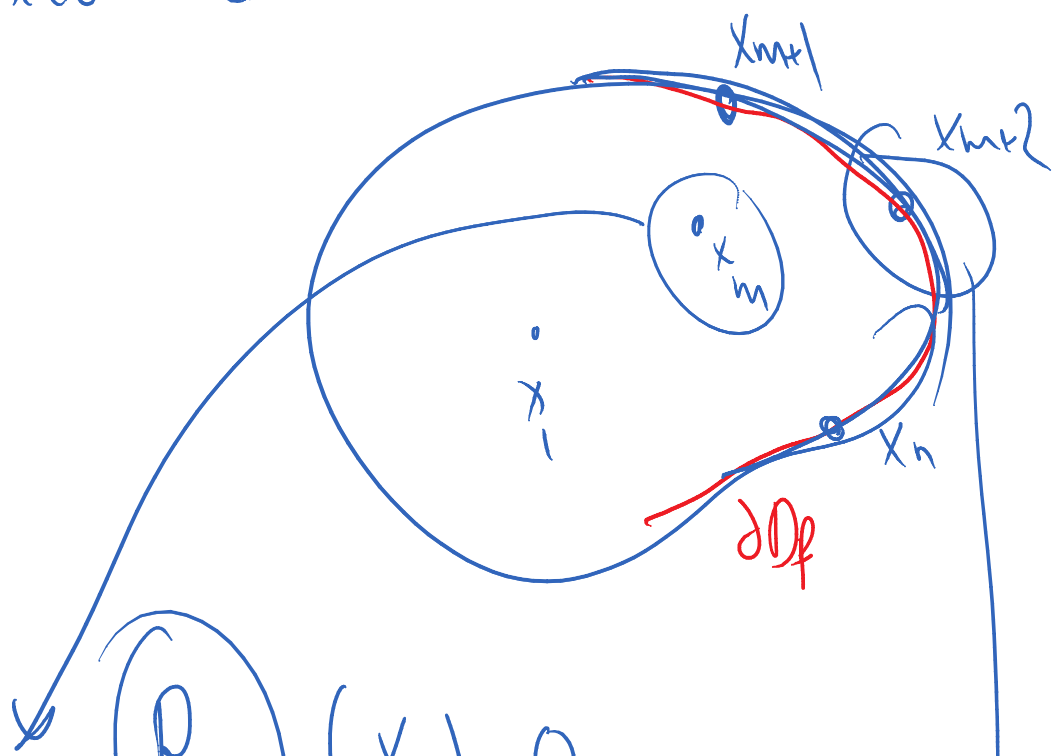


$$\int_0^L \omega R_i = \int_{\{x_i\}} R_i(x_i)\,dx + 0$$

$$\omega(x) = \delta(x - x_i)$$

$$\int_D w(x)\, R_i(x) = \int_D \delta(x - x_i)\, R_i(x)\, dV = R_i(x_i)$$

Mixed Collocation Method



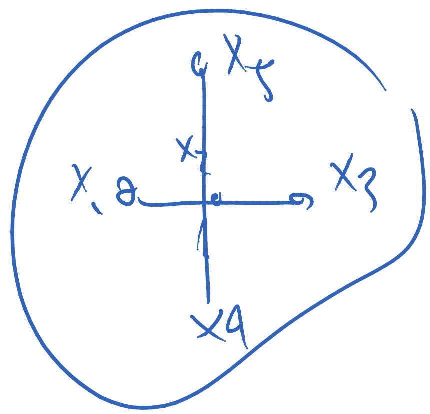
$$R_i(x_m) = 0$$

$$R_i(X_m) = 0$$

$$R_f(X_{m+1}) = 0$$

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$$

$$\frac{\partial^2 \phi_j}{\partial x_1^2}(2) + \frac{\partial^2 \phi_j}{\partial x_2^2}(2) = 0$$



$$\phi = x^2 + y^2$$

$$\Delta(x_2) \simeq \frac{f(x_1) + f(x_3) + f(x_4) + f(x_5) - 4f(x_2)}{h^2}$$

Least square:

$$R^2 = \int R_i^2(a_1, \dots, a_n)\, dV$$

$$R = \int_D R_i(a_1,\dots,a_n)\, dV$$

$$\frac{\partial R^2}{\partial a_i} = 0$$

$$\frac{\partial R^2}{\partial a_i} = \int_V \frac{\partial \left(R_i^2(a_1,\dots,a_n)\right)}{\partial a_i}\, dV$$

$$\boxed{\frac{\partial R}{\partial a_i} = \int 2 R_i(a_1,\dots,a_n) \frac{\partial R_i(a_1,\dots,a_n)}{\partial a_i}\, dV = 0} \quad (1)$$

differential operator

$$R_i(a_1,\dots,a_n) = L_u(\tilde{u}) - r$$

$\rightarrow$ source term

$$\tilde{u} = \sum_{j=1}^{n} a_j \phi_j(x) + \phi_p(x)$$

$$R_i(a_1,\dots,a_n) = \underbrace{L_u\left(a_j \phi_j(x) + \phi_p(x)\right)}_{\text{linear}} - r(x)$$

linear

$$\boxed{R_I = a_j L_M(\phi_j(x)) + L_M(\phi_p) - r(x)}$$

$$\boxed{\frac{\partial R_I}{\partial a_i} = L_M(\phi_i(x))} \quad (2)$$

(1) & (2) $\Longrightarrow$

$$\boxed{\int_D L_M(\phi_i(x))\, R_i(a_1, \ldots, a_n)\, dv = 0}$$

$$\int \omega \cdot R_i(a_1, \ldots, a_n)\, dv = 0$$

$$\boxed{\omega_i = L_M(\phi_i(x))}$$ for least square method

Galerkin method

$$\boxed{\omega_i = \phi_i}$$

Galerkin method

$$\boxed{\omega_i = \delta(x - x_i)}$$

Collocation Method

$$\boxed{\omega_i = \chi_{\Omega_i}}$$

Subdomain