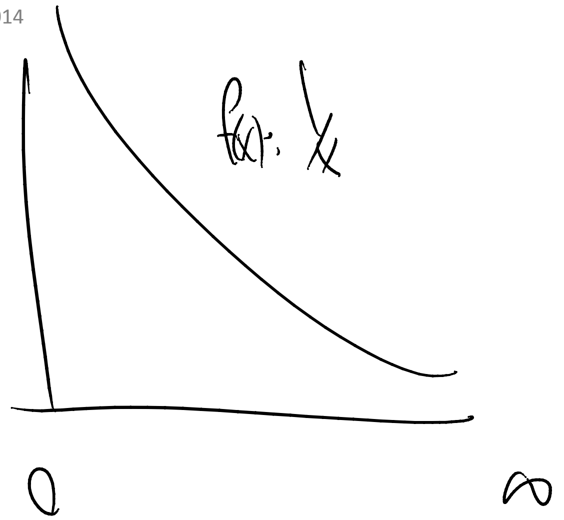
02/20/2014

Thursday, February 20, 2014
11:40 AM



$$f(x) \in C^\infty(\underbrace{(0,\infty)}_{\text{open}})$$

doesn't include 0

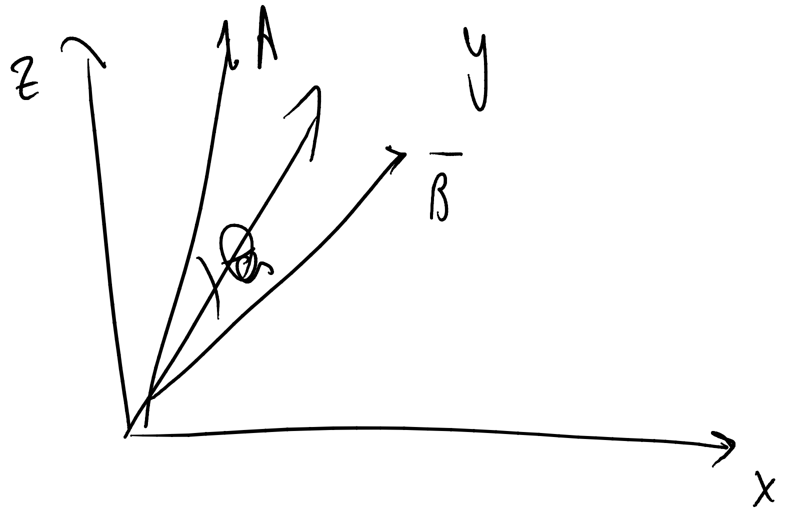
$$f \in C^k(\underbrace{[a,b]}_{\text{closed set}})$$

then it's also bounded

$$C^k_b([a,b])$$

$b \rightarrow$ bounded

Cauchy Schwarz inequality

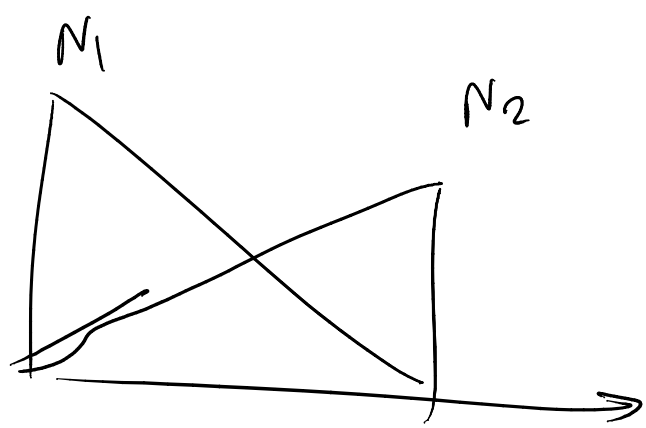


$$|A \cdot B| = |A|\,|B|\,|\cos\theta| \leq |A|\,|B|$$

$$|A \cdot B| \leq \sqrt{(A \cdot A)}\,\sqrt{(B \cdot B)}$$

$$H^1(D) = \left\{ f \;\middle|\; \int_D f^2 dv < \infty,\ \int_D f'^2 dv < \infty \right\}$$

bar



Finite Element

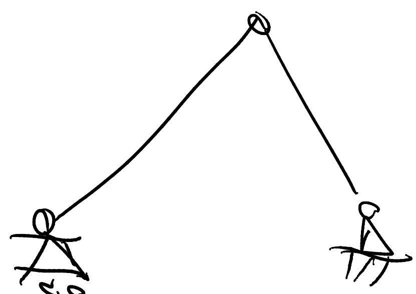
1. bar example

$$K a = F$$

formation of is called assembly .

- Assembly
- Natural & Essential BCs

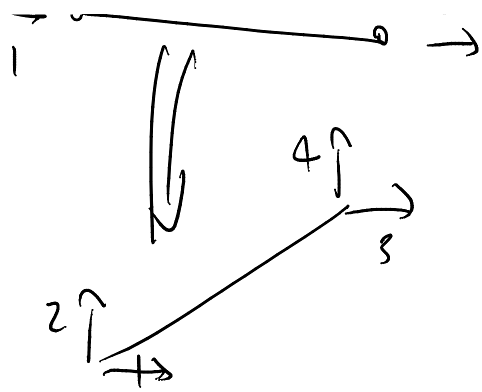
2.



Truss element:

bar element in 2D & 3D

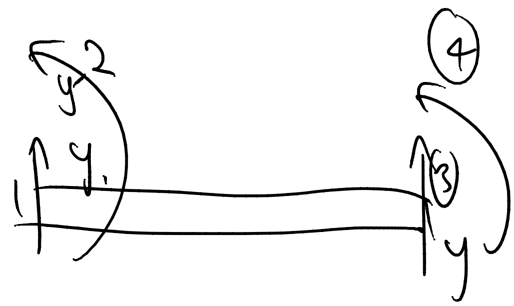
Coordinate transformation



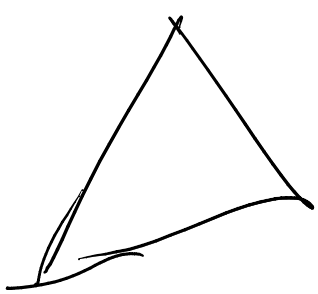
3. beam element

higher continuity at nodes

C^1 rather C^0



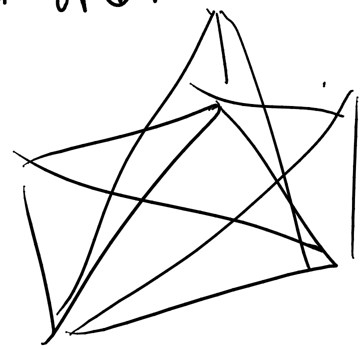
4. 2D & 3D elements



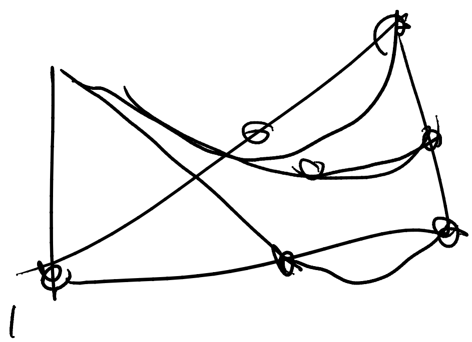
– numerical integration (quadrature)

– higher order elements

p=1

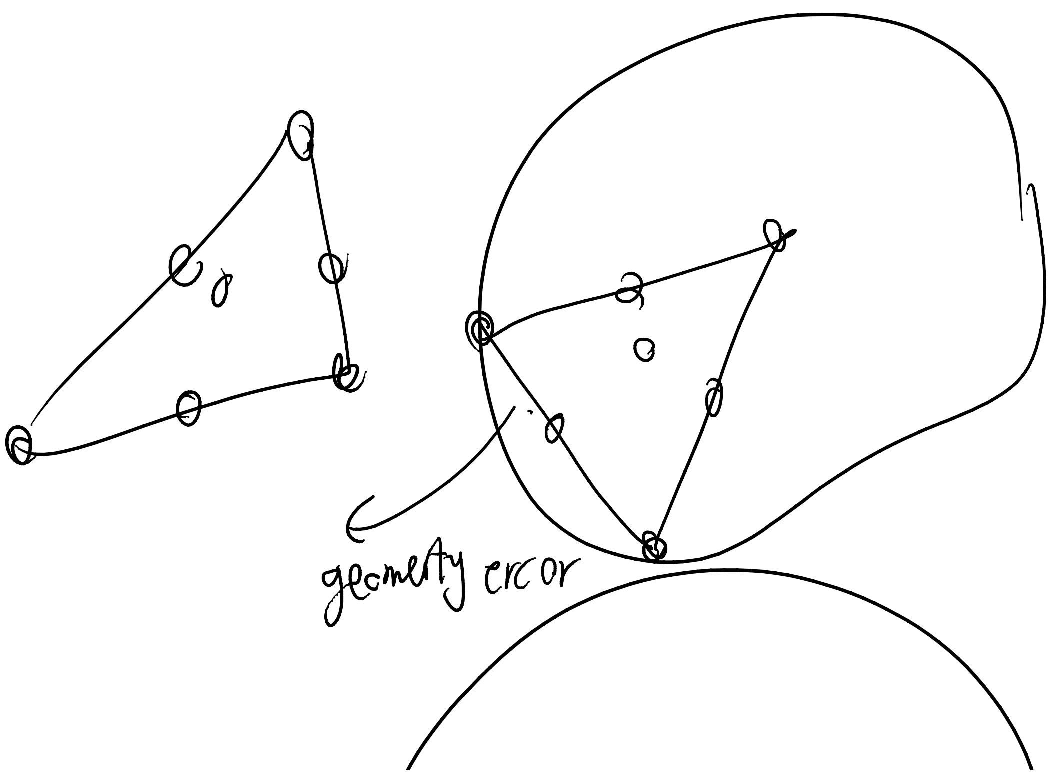


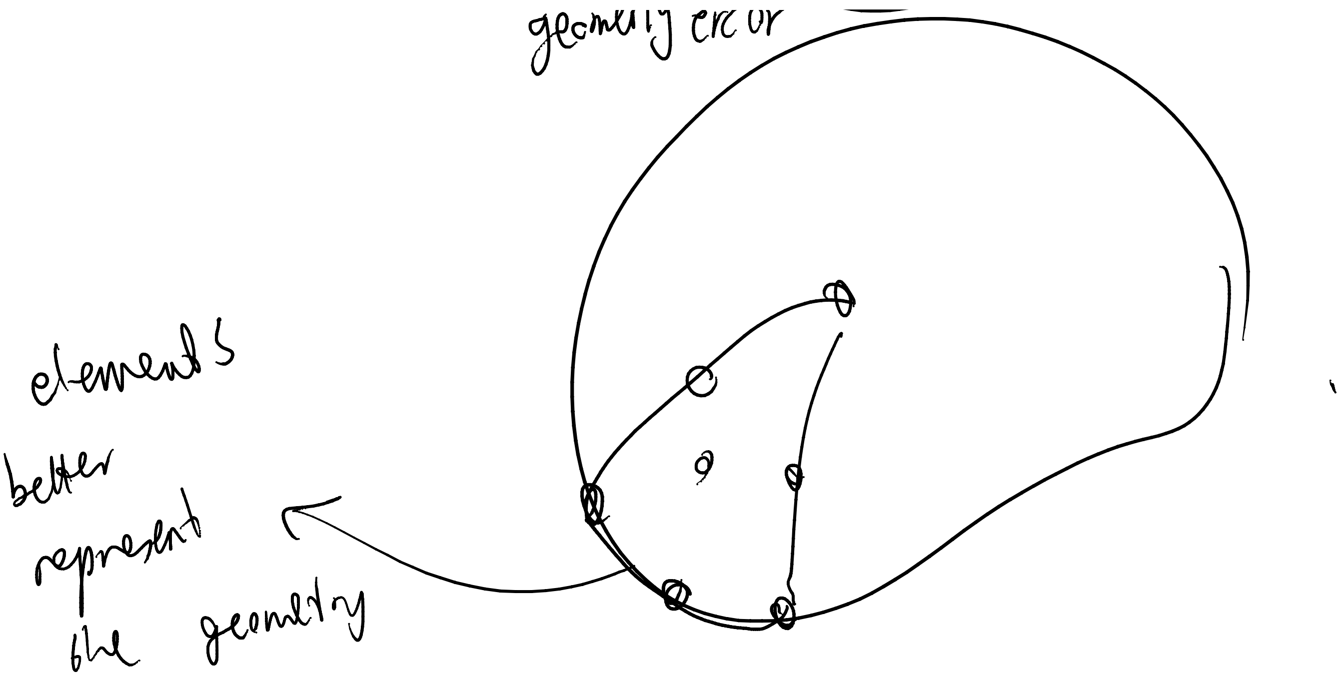
p=2

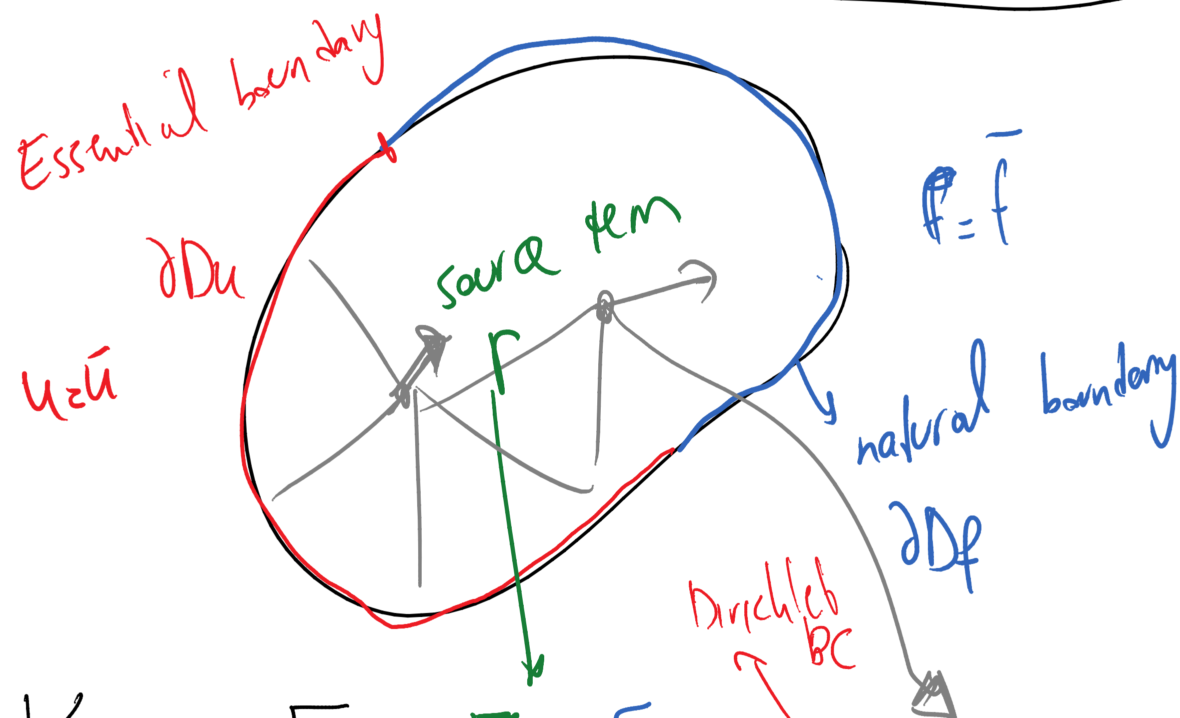


– isoparametric elements
super " elements
sub " elements

p = 2







$$Ka = F = F_r + F_N - F_D + F_n$$

for natural

$$\int^{L} \frac{dw}{dx} EA \frac{du}{dx} dx = \int^{L} w q \, dx + w(L) \bar{F}$$

$$\underbrace{\int_0^L \frac{dw}{dx} EA \frac{du}{dx} dx}_{\mathcal{A}(w,u)} = \underbrace{\int_0^L w q \, dx}_{(w,q)_\Gamma} + \underbrace{(w \bar{F})\Big|_{x=L}}_{(w,\bar{F})_N}$$

$$u = a_J \phi_J + \underbrace{\phi_p}_{\text{particular solution}}$$

$$w = \phi_I$$

$$\forall I \quad \underbrace{\mathcal{A}(\underbrace{\phi_I}_{w_I}, a_J \phi_J + \phi_p)} = (\phi_I, q)_\Gamma + (\phi_I, \bar{F})_N$$

$$\mathcal{A}(\phi_I, \phi_J) a_J + \mathcal{A}(\phi_I, \phi_p) = (\phi_I, q)_\Gamma + (\phi_I, \bar{F})_N$$

I & $J = 1, \ldots, n_f$

number of unknowns = number of free degrees of freedom

$$\left\{ \mathcal{A}(\phi_I, \phi_J) \right\} a_J = (\phi, q)_\Gamma + (\phi, \bar{F})_N - \mathcal{A}(\phi_I, \phi_p)$$

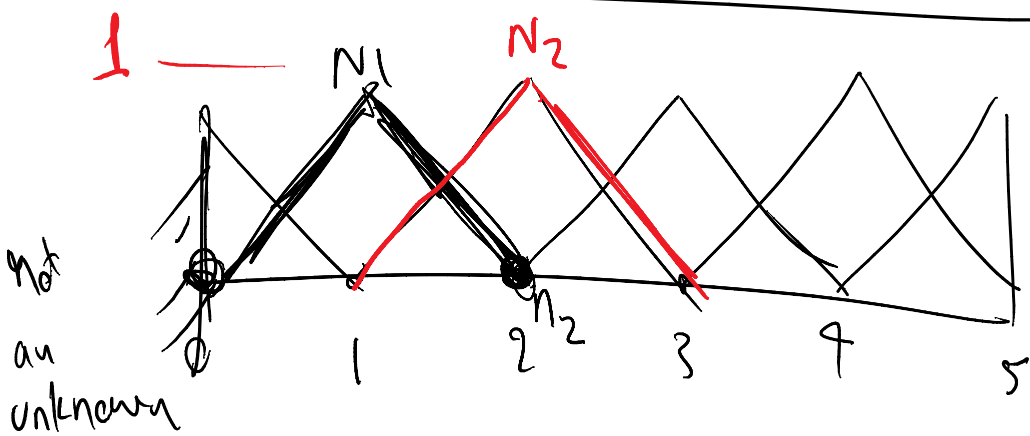
$$\forall I, J \in \{1, \dots, n_f\}$$

$$K = \begin{bmatrix} \mathcal{A}(\phi_1, \phi_1) & \mathcal{A}(\phi_1, \phi_2) & \cdots \\ \mathcal{A}(\phi_2, \phi_1) & \mathcal{A}(\phi_2, \phi_2) & \cdots \\ \vdots & \vdots & \\ \mathcal{A}(\phi_n, \phi_1) & & \mathcal{A}(\phi_{n_f}, \phi_{n_f}) \end{bmatrix}$$

$$F_\Gamma = \begin{bmatrix} (\phi_1, q)_\Gamma \\ (\phi_2, q)_\Gamma \\ \vdots \\ (\phi_{n_f}, q)_\Gamma \end{bmatrix} \quad F_N = \begin{bmatrix} (\phi_1, \bar{F})_N \\ (\phi_2, \bar{F})_N \\ \vdots \\ (\phi_{n_f}, \bar{F})_F \end{bmatrix}$$

$$F_D = \begin{bmatrix} \mathcal{A}(\phi_1, \phi_p) \\ \vdots \\ \mathcal{A}(\phi_{n_f}, \phi_p) \end{bmatrix} \quad F = F_\Gamma + F_N - F_D$$

$$\left\lfloor \mathcal{A}(\phi_{nf}, \phi_p) \right\rfloor$$



a_1 to a_5 are your unknowns

$$K_{5\times 5} \begin{bmatrix} a_1 \\ \vdots \\ a_5 \end{bmatrix} = \begin{bmatrix} F_1 \\ \vdots \\ F_5 \end{bmatrix}$$

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_5 \end{bmatrix}$$

shape function

$$u^h = a_1 N_1 + a_2 N_2 + \dots + a_5 N_5 + \phi$$

$$u^h(\tilde{n_2}) = a_1 N_1(n_2) + a_2 N_2(n_2) + \dots + a_5 N_5(n_2) + \phi_p(n_2)$$

$$= a_1\, 0 + a_2 1 + \dots + \underbrace{\phi_p(n_2)}$$

will show that this is zero

$$\Longrightarrow \boxed{u^h(n_2) = a_2}$$

Kinematic relationship

$$u^h - \phi_p = a_1 N_1(x) + a_2 N_2(x) + \dots + a_{n_f} N_{n_f}(x)$$

$$= \underbrace{\begin{bmatrix} N_1(x) & \dots & N_{n_f}(x) \end{bmatrix}}_{1 \times n_f \text{ matrix}} \underbrace{\begin{bmatrix} a_1 \\ \vdots \\ a_{n_f} \end{bmatrix}}_{n_f \times 1 \text{ matrix}}$$

$$u^h - \phi_p = \overset{\text{matrix}}{\mathbf{N}(x)}\, \mathbf{a}$$

$$\frac{d}{dx}(u^h - \phi_p) = \underbrace{\begin{bmatrix} \frac{dN_1}{dx}(x) & \dots & \frac{dN_{n_f}(x)}{dx} \end{bmatrix}}_{B} \begin{bmatrix} a_1 \\ \vdots \\ a_{n_f} \end{bmatrix}$$

$B(x)$ = Kinematic / displacement strain matrix or transformati

$$\boxed{\begin{gathered}\frac{d(u^h-\phi_p)}{dx} = Ba \\ \frac{dN_I}{dx} = B_I(x)\end{gathered}}$$