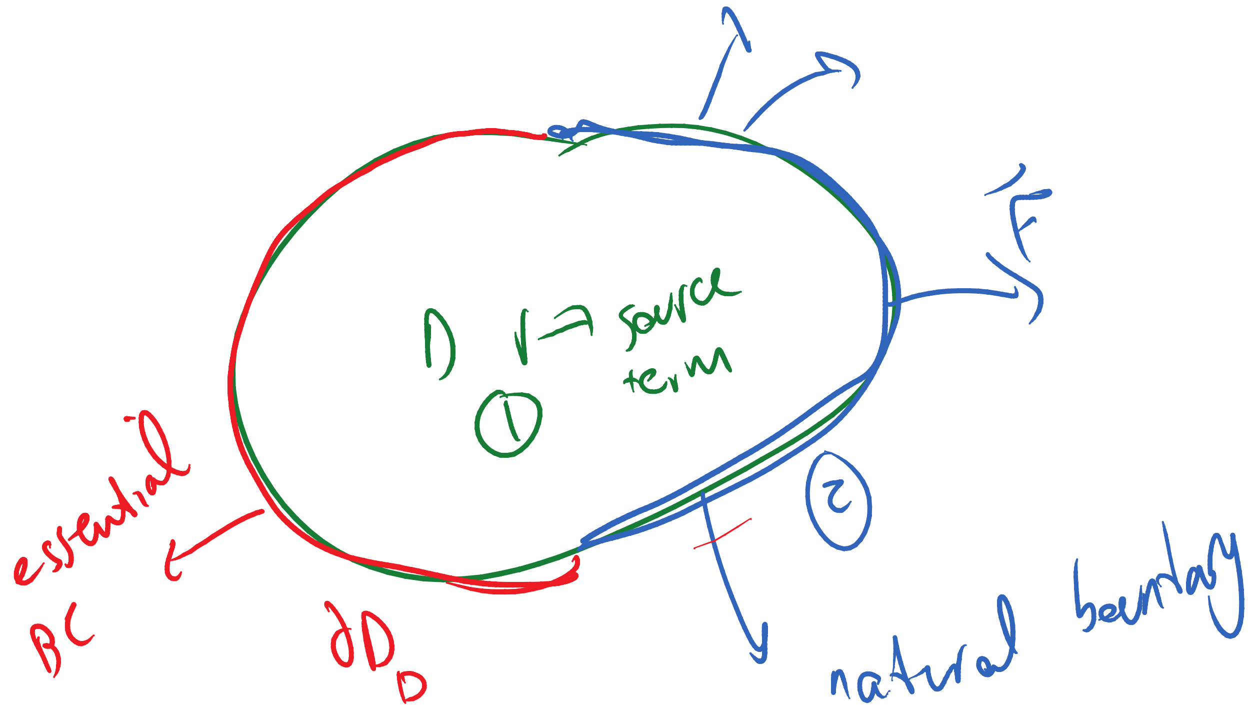
02/25/2014

Tuesday, February 25, 2014
11:40 AM



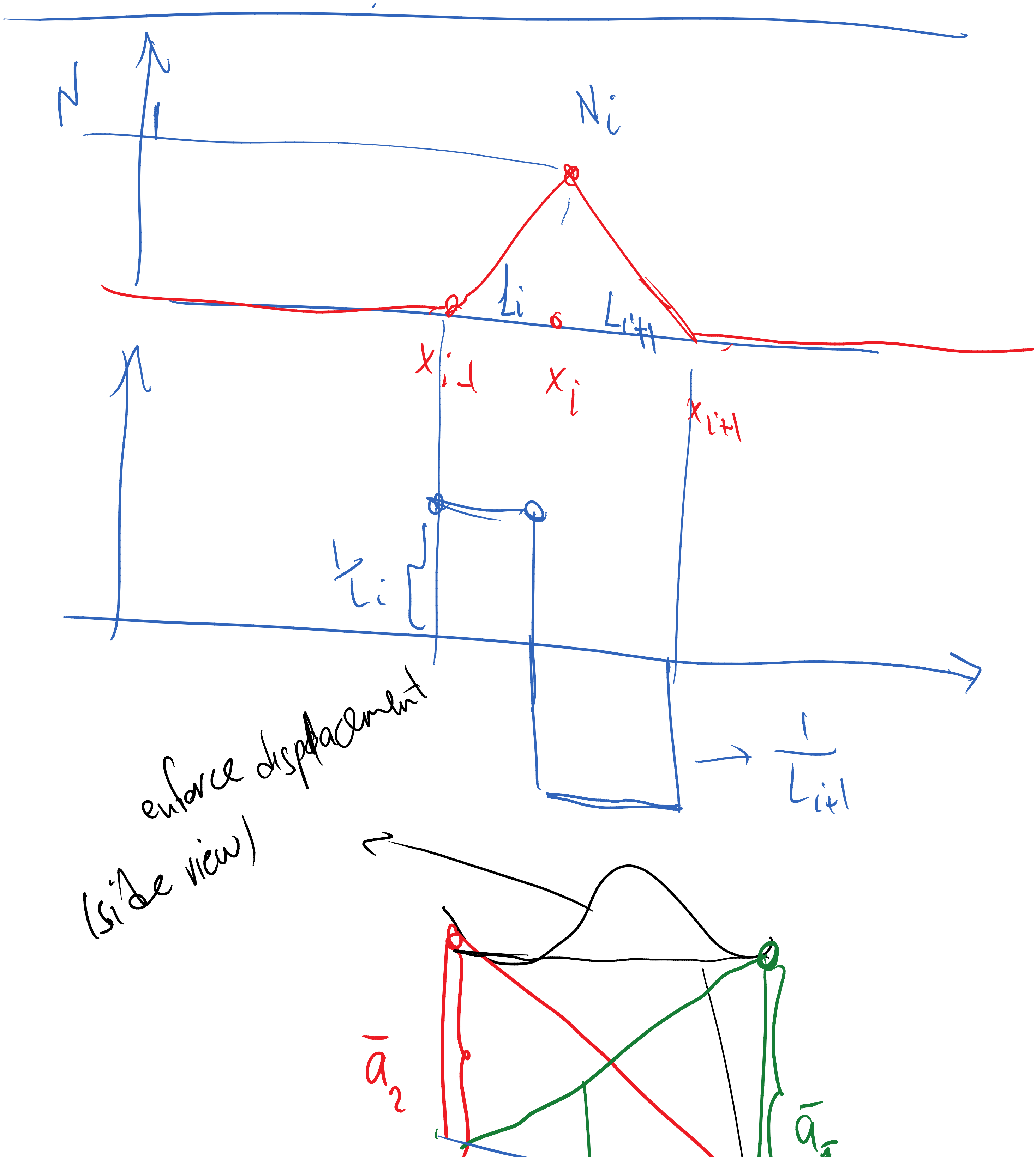
$$F_N = \int_{\partial D_N} w \cdot \vec{F} \, ds$$

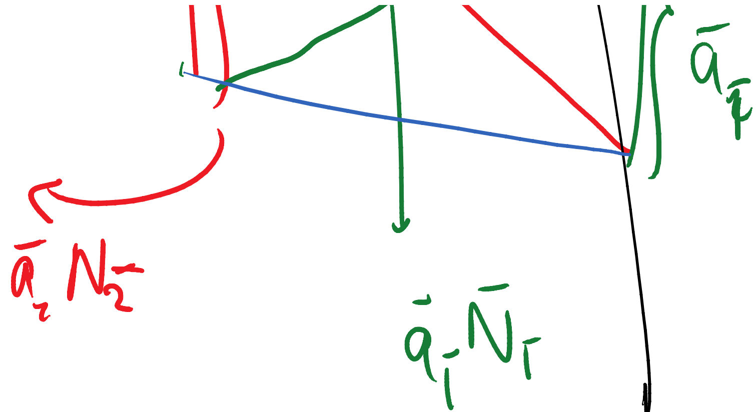
$$K_{IJ} = \int_D \frac{d\phi_I}{dx} EA \frac{d\phi_J}{dx}$$

displacement $\phi_I \longrightarrow$ strain $\frac{d\phi_I}{dx}$

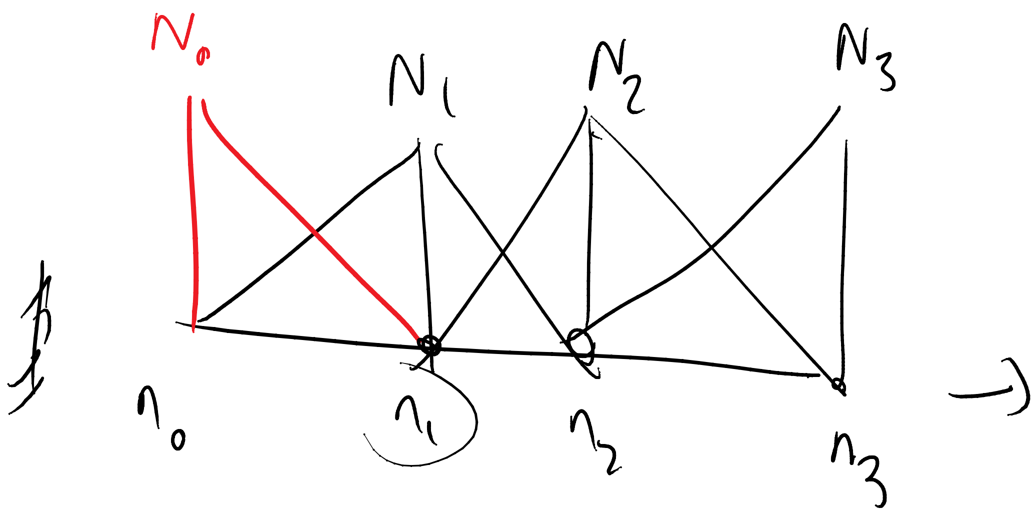
Kinematic relation

$B = ?$





$$\phi_p = \bar a_2 \bar N_{\bar 2}(x) + \bar a_{\bar 1} \bar N_{\bar 1}(x)$$



free d.o.fs

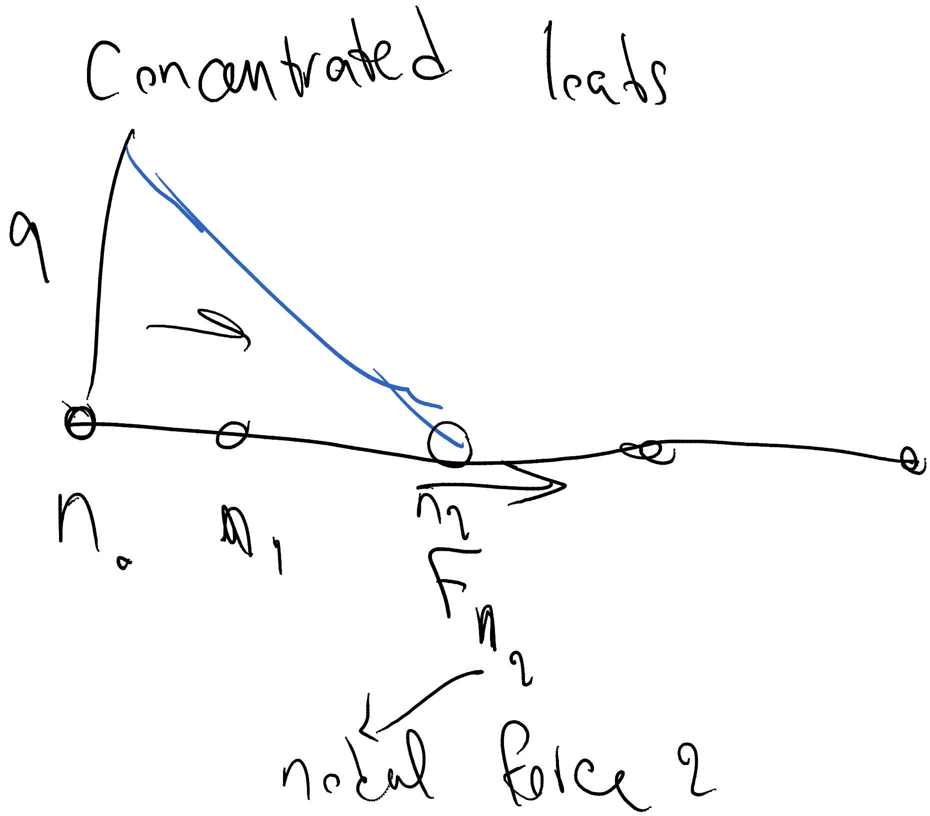
$$N = [N_1(x) \quad N_2(x) \quad N_3(x)]$$

$$\bar N = [N_{\bar 1}(x)] = [N_0(x)]$$

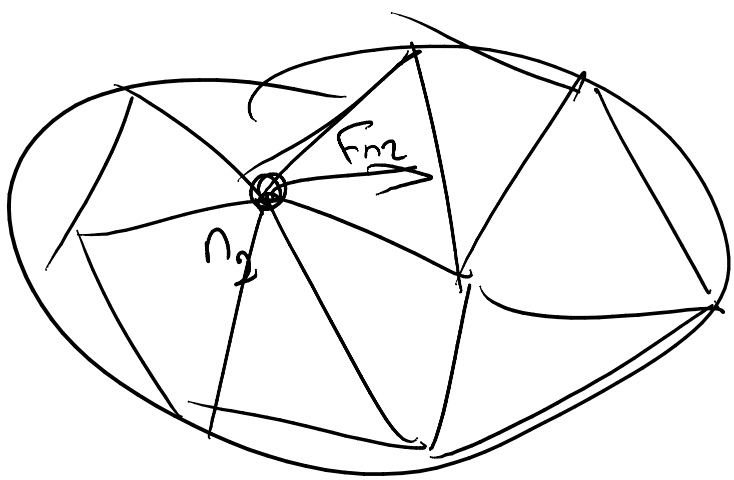
$$N(n_1) = [1 \quad 0 \quad 0]$$

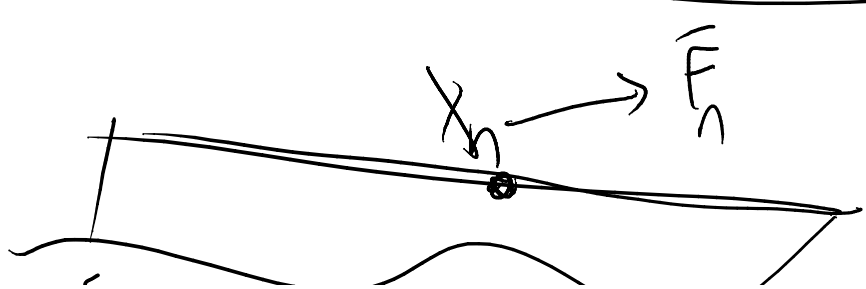
$$\bar N(n_1) = [0]$$

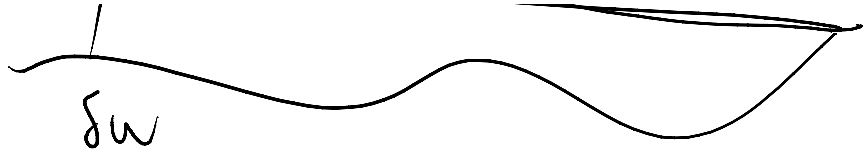
Concentrated loads



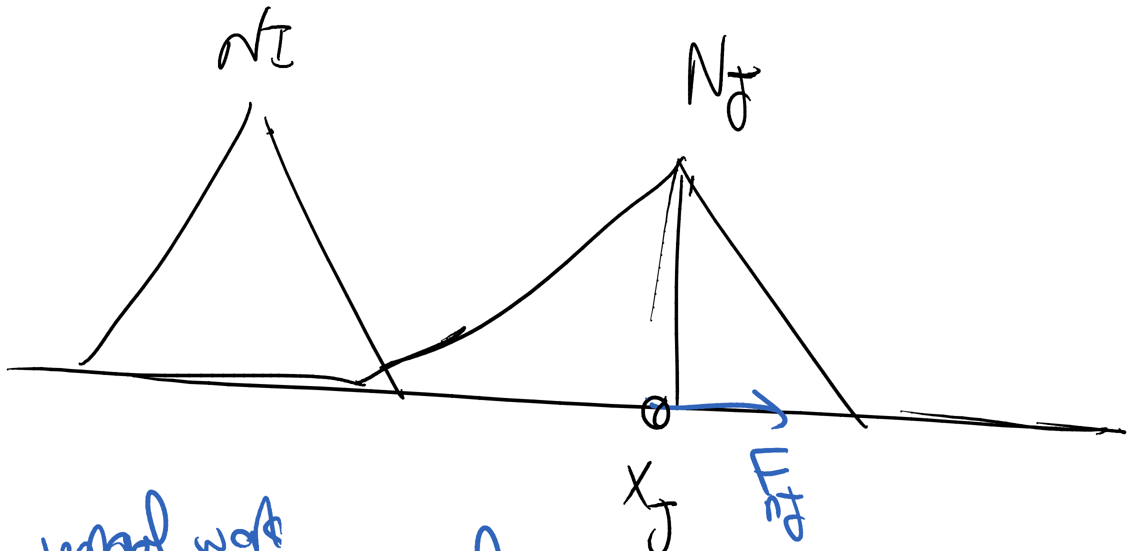
dz^2







external work $\delta w(x_n) \cdot \bar{F}_n$

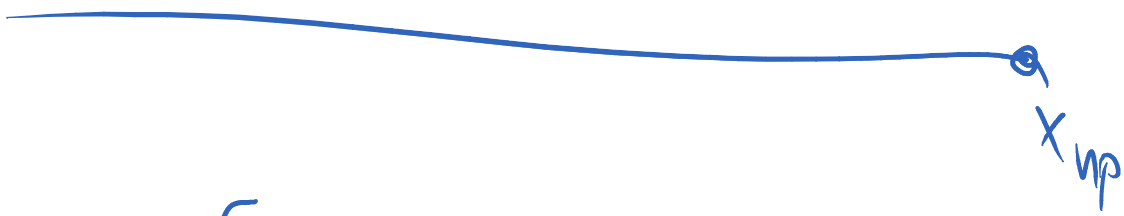


internal work = external work $+ N_I(x_J) F_{nJ}$ δ_{IJ} $= F_{nI}$

$$\int_0^L \frac{dN_I}{dx} EA \frac{dN_J}{dx} = \int_0^L N_I q \, dx + N_I(x_p)\bar{F} + F_{nI}$$

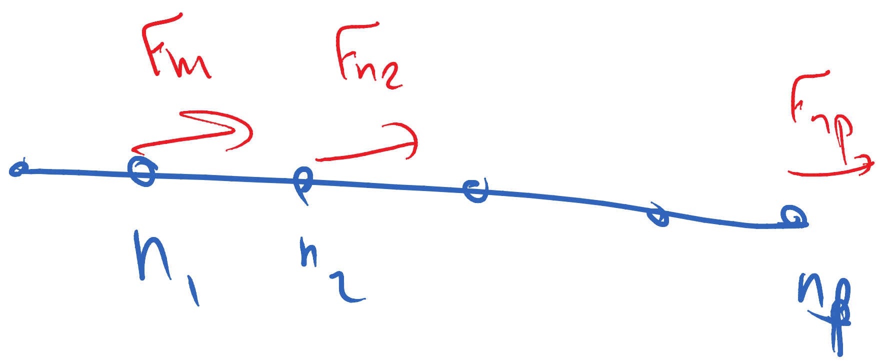
$$\int_0^L \frac{dN_I}{dx} EA \frac{dN_J}{dx} = \int_0^L N_I q \, dx + N_I(x_{np}) \bar{F} + \bigcirc$$

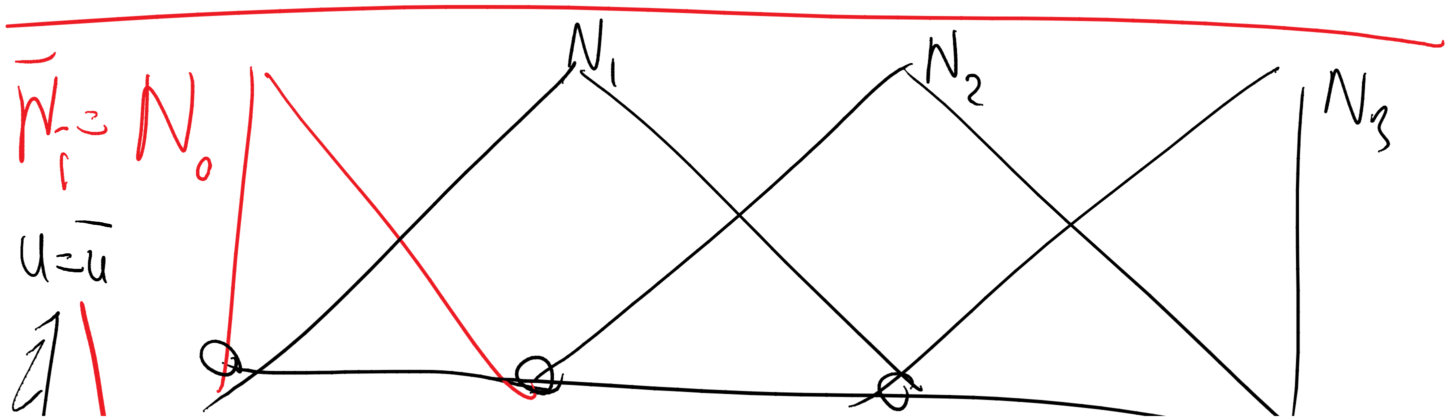
new term

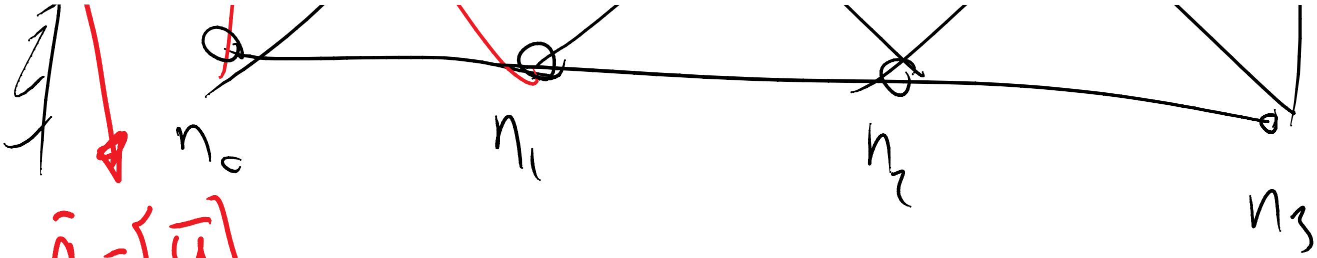


$$F_n = \begin{bmatrix} F_{n_1} \\ \vdots \\ F_{n_p} \end{bmatrix}$$

nodal force







$\bar{a} = [\bar{u}]$

$N = [\underbrace{N_1 \quad N_2 \quad N_3}]$ 1x3 matrix

→ # free d.o.f n_f

$\bar{N} = [\bar{N}_i] = [N_0]$ 1x1

→ # prescribed d.o.fs

$$B = \left[\frac{dN_1}{dx} \quad \frac{dN_2}{dx} \quad \frac{dN_3}{dx}\right] \qquad 1\times 3$$

$$\bar{B} = \left[\frac{d\bar{N}}{dx}\right] \qquad 1\times 1$$

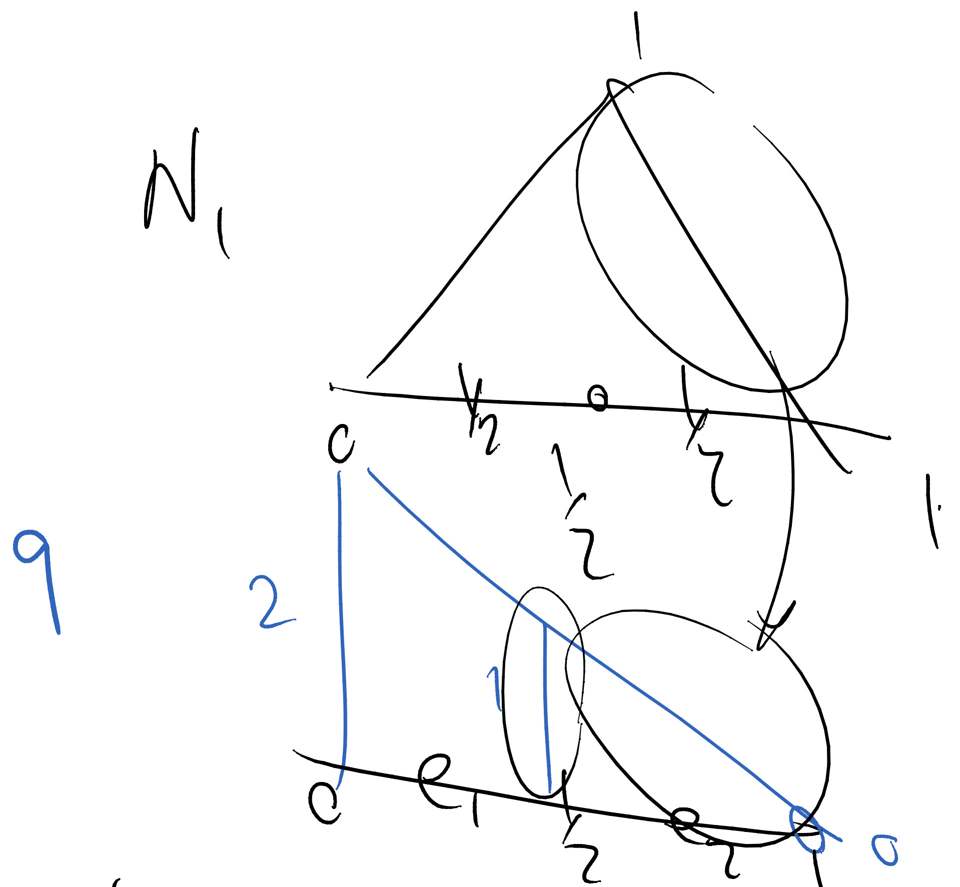
$$F_D = \left(\int_0^L B^T EA \bar{B}\, dx\right) \bar{a}$$

$$= \left(\int \underbrace{\begin{bmatrix} N_1' \\ N_2' \\ N_3' \end{bmatrix}}_{B^T} EA \underbrace{\left[\frac{dN_0}{dx}\right]}_{\bar{B}} dx\right) [\bar{u}]$$

$\underbrace{B^T \quad \text{is}}_{3 \times 1 \text{ matrix}}$

K_{fp}

$$\boxed{F_D = K_{fp}\,\bar{a}}$$



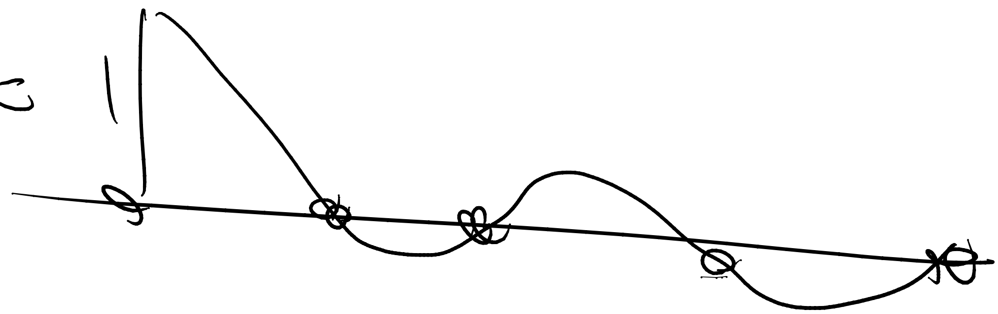
$\int Nq = \int_{e1} Nq + \int_{e2} Nq =$

$$\int N q = \int_{e_1} N q + \int_{e_2} N q =$$

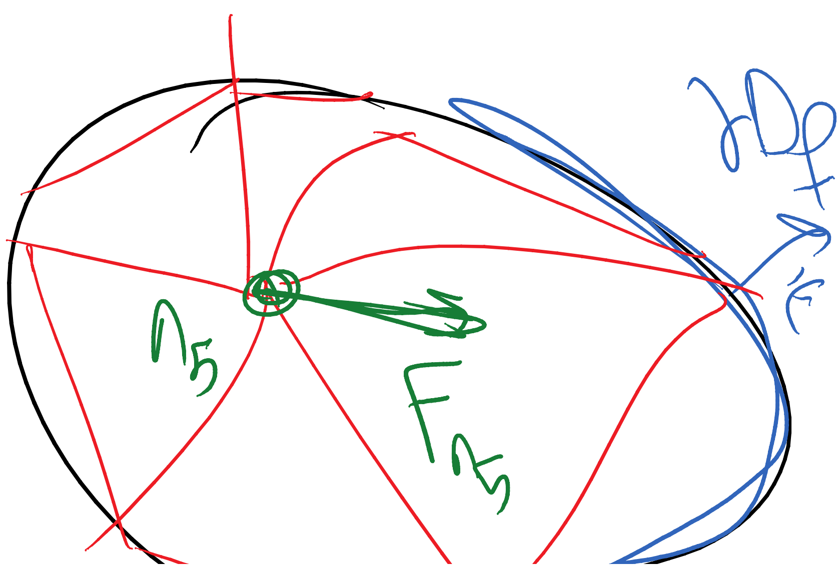
$$\frac{1}{6}\left(\frac{1}{2}\right)\left(2\times 0\times 2 + 1\times 1\times 2 + 1\times 0\times 0 + 1\times 2\right)$$

$$+ \frac{1}{6}\left(\frac{1}{2}\right)\left(2\right)$$

$$= 1$$



Difference of nodal force & natural BC force



$$F_N = \int_{\partial D_N} q \vec{F} \, ds$$