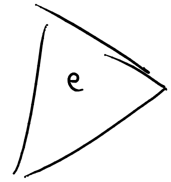
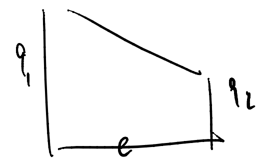
03/11/2014

Tuesday, March 11, 2014

11:40 AM



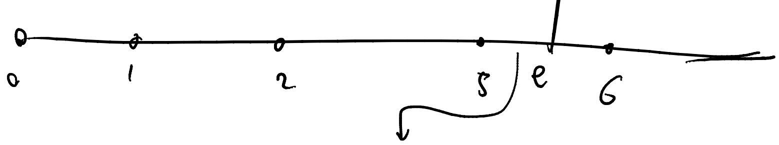
$$k^e = \int_e B^T D B \, dv$$



$$\left\{\begin{array}{l} f_D^e = k^e a^e \\ f_r = \int_e N^T r \, dv = \underbrace{\left(\int_e N^T N \, dv\right)}_{M} \begin{pmatrix} r_1 \\ \vdots \\ r^e_{dof} \end{pmatrix} \\ f_N = \int_{\partial e^N} N^T \bar{f} \, ds \qquad\qquad f_r^e + f_N^e - f_D^e \\ F_n \end{array}\right.$$

$$k^e \qquad \begin{array}{c|c} & \begin{array}{cc} 5 & 6 \end{array} \\ \hline 5 & \\ 6 & \end{array} \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{array}{c} 1 \; 2 \; 3 \; 4 \; 5 \; 6 \; 7 \\ \begin{bmatrix} & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & & & \\ & & & & \frac{AE}{L} & -\frac{AE}{L} & \\ & & & & -\frac{AE}{L} & \frac{AE}{L} & \\ & & & & & & \end{bmatrix} \end{array}$$



$$\begin{array}{c|c} 5 & \\ 6 & \end{array} \; f^e = \begin{bmatrix} .5 \\ .6 \end{bmatrix}$$

$$F \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} \begin{bmatrix} \\ \\ \\ \\ .5 \\ .6 \\ \\ \end{bmatrix}$$

Physical insight

1. Stiffness matrix

2. Assembly

3. Free & prescribed dof.

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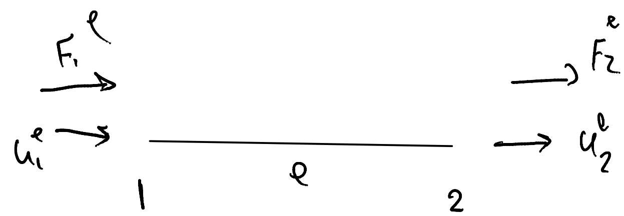
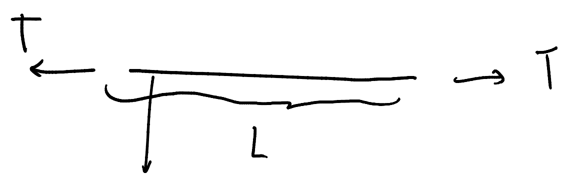
1. Stiffness

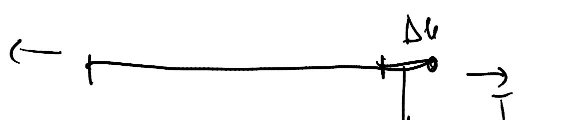


$$\begin{bmatrix} F_1^e \\ F_2^e \end{bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1^e \\ u_2^e \end{bmatrix}$$

$$F_2^e = \frac{AE}{L}\left(u_2^e - u_1^e\right)$$



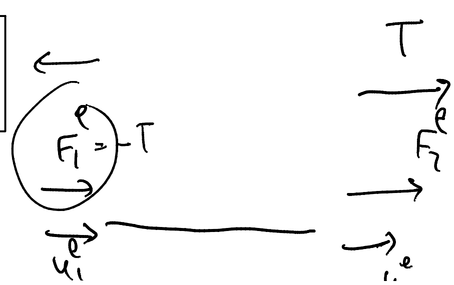
area section A
elastic modulus E



change in length

$$\varepsilon = \frac{\Delta u}{L} \qquad \sigma = \varepsilon E = \frac{\Delta u}{L} E \qquad T = \sigma A = \frac{AE}{L} \Delta u$$

$$\boxed{T = \frac{AE}{L} \Delta u}$$



$$\begin{bmatrix} F_1^e \\ F_2^e \end{bmatrix} = \frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} u_1^e \\ u_2^e \end{bmatrix}$$

$$\underbrace{F_2^e}_{T} = \frac{AE}{L}\,\underbrace{(u_2^e - u_1^e)}_{\Delta u}$$

$$F_1^e = \frac{AE}{L}(u_1^e - u_2^e) \Rightarrow -\underbrace{F_1^e}_{T} = \frac{AE}{L}\Delta u$$

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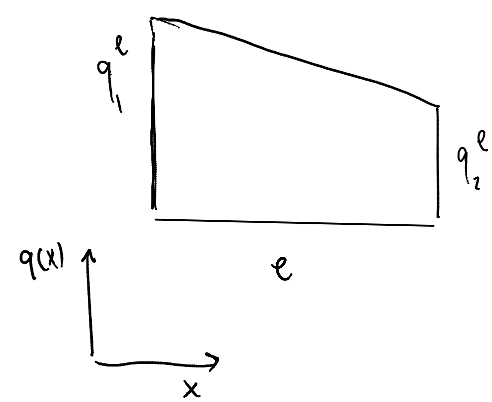
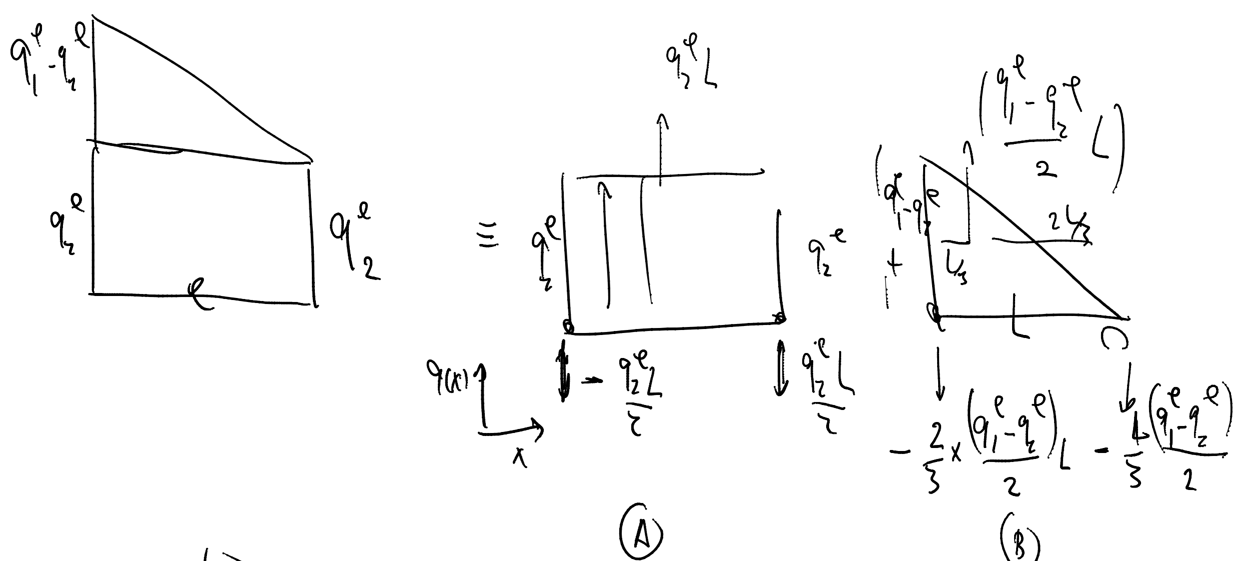
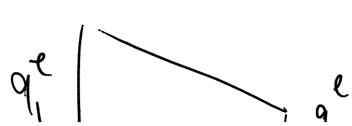
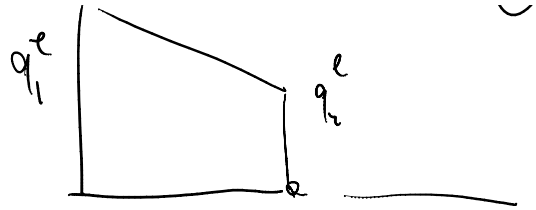
2. Assembly



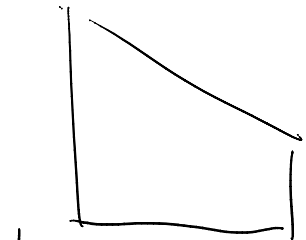
$$F_r^e = \int_e N^T q\,dx = \left(\int_e N^T N\,dx\right)\begin{bmatrix} q_1^e \\ q_2^e \end{bmatrix} = \frac{L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} q_1^e \\ q_2^e \end{bmatrix}$$





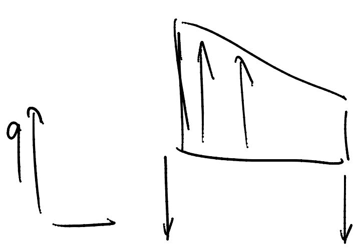


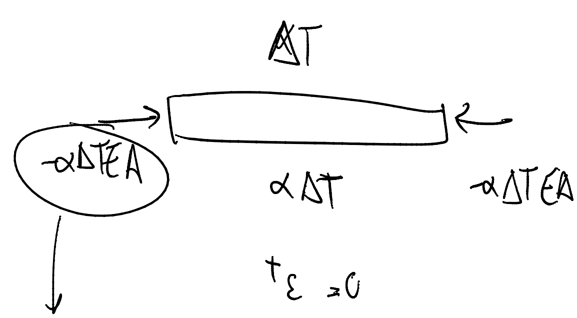
$$\downarrow \; -\left(\frac{q_2^e L}{2} + \frac{2}{3}\,\frac{(q_1^e - q_2^e)}{2}\,L\right) \qquad \downarrow \; -\left(\frac{q_2^e L}{2} + \frac{L}{6}\,(q_1^e - q_2^e)\right)$$



$$\downarrow \; -\left(\frac{2q_1^e + q_2^e}{6}\right)L \qquad -\left(\frac{q_1^e + 2q_2^e}{6}\right)L \; \downarrow$$

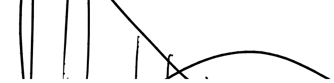
- we derive d equivalent nodal forces without resorting to shape functions
- — sign is because we are deriving <u>equivalent nodal force</u>. That is, nodal forces that balance the distributed load



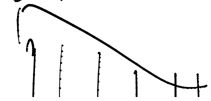


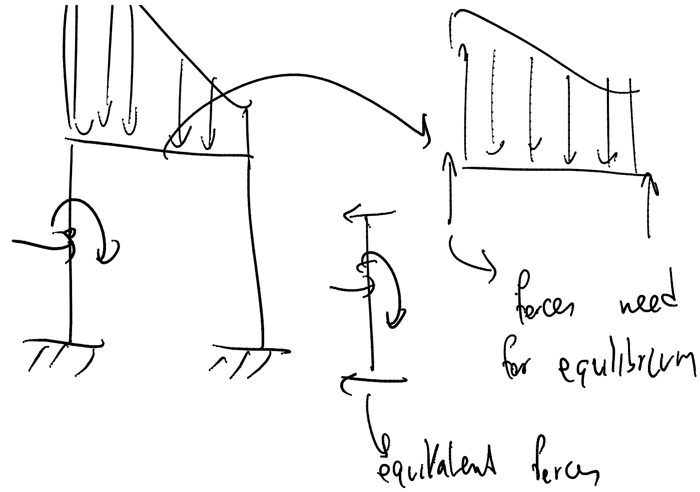
$\sigma_\varepsilon = 0$

$\varepsilon = -\alpha \Delta T$



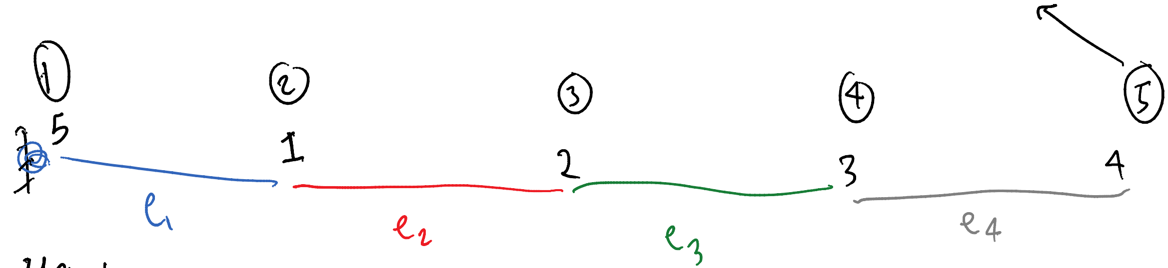
balancing forces





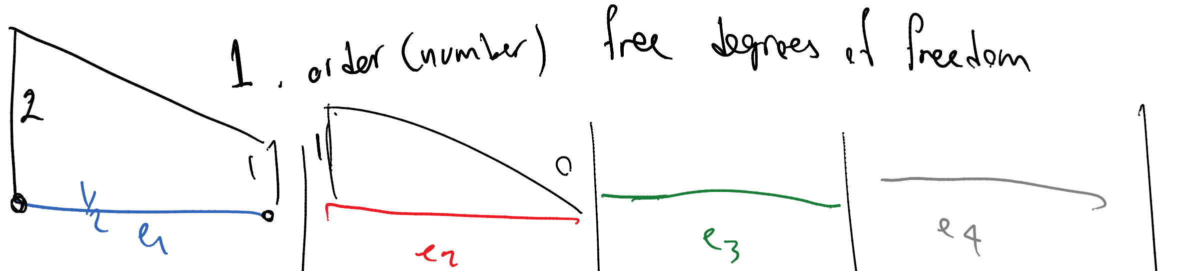
Assembl Continued

global numbering



$u_{1} = 1$

1. order (number) free degrees of freedom

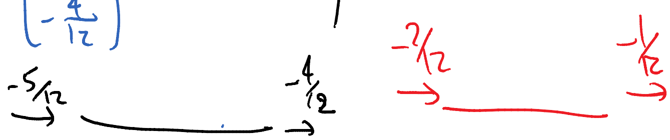


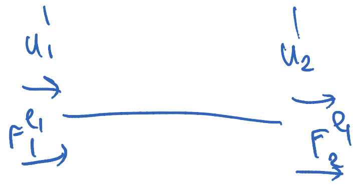
$$-\frac{L}{6}\begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}q_1^e\\q_2^e\end{bmatrix}$$

$$-\frac{L}{6}\begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}$$

$$-\frac{1/2}{6}\begin{bmatrix}2&1\\1&2\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix}$$

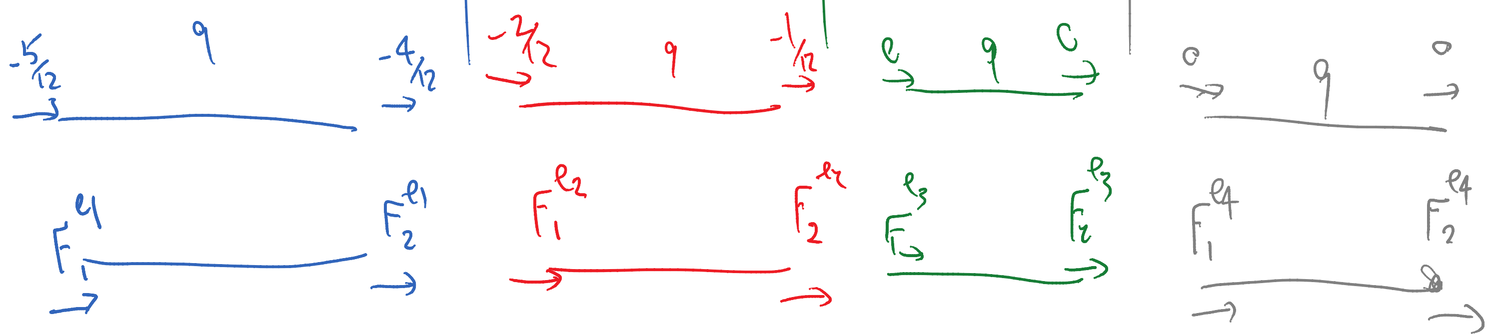
$$\begin{bmatrix}-\frac{5}{12}\\-\frac{4}{12}\end{bmatrix}$$

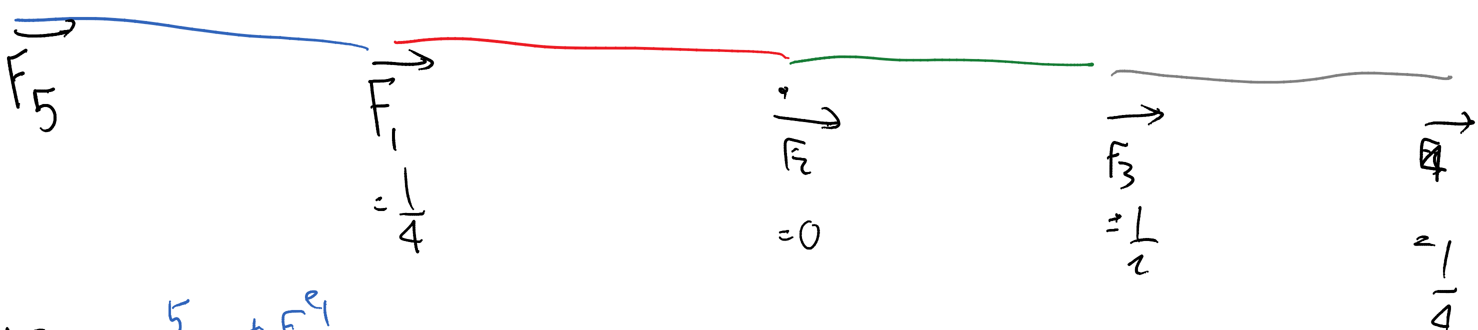




$$\begin{bmatrix} F_1^{e_1} \\ F_2^{e_1} \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u_1^1 \\ u_2^1 \end{bmatrix}$$

$$\begin{bmatrix} F_1^{e_1} \\ F_2^{e_1} \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u_5 \\ u_4 \end{bmatrix} \qquad \begin{bmatrix} F_1^{e_2} \\ F_2^{e_2} \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad \begin{bmatrix} F_1^{e_3} \\ F_2^{e_3} \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} \qquad \begin{bmatrix} F_1^{e_4} \\ F_2^{e_4} \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u_3 \\ u_4 \end{bmatrix}$$





$$\begin{cases} F_5 = -\frac{5}{12} + F_1^{e_1} \\ F_1 = -\frac{4}{12} + F_2^{e_1} + \frac{-2}{12} + F_1^{e_2} \\ F_2 = \frac{-1}{12} + F_2^{e_2} + F_1^{e_3} \\ F_3 = F_2^{e_3} + F_1^{e_4} \\ F_4 = F_2^{e_4} \end{cases}$$

$$\begin{bmatrix} F_1 \\ \vdots \end{bmatrix} \quad \begin{bmatrix} \frac{1}{2} \\ \vdots \end{bmatrix} \quad \overbrace{\begin{bmatrix} 2+2 & -2 & \cdots \\ \vdots & & \end{bmatrix}}^{\text{stiffness matrix}} \begin{bmatrix} u_1 \\ \vdots \end{bmatrix}$$

$$
\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix}
= - \begin{bmatrix} \frac{1}{2} \\ \frac{1}{12} \\ 0 \\ 0 \\ \frac{5}{12} \end{bmatrix}
+ \begin{bmatrix} 2+2 & -2 & & & \\ -2 & 2+2 & -2 & & 0 \\ & -2 & 2+2 & -2 & \\ & & -2 & 2 & \\ 0 & & & & 2 \end{bmatrix}
\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \end{bmatrix}
$$

$$
\begin{array}{l} \text{known } F_f \left\{ \begin{matrix} \\ \\ \\ \\ \end{matrix} \right. \\ \text{unknown } F_p \left\{ \begin{matrix} \\ \end{matrix} \right. \end{array}
\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix}
= \begin{bmatrix} \frac{1}{2} \\ \frac{1}{12} \\ 0 \\ 0 \\ \hline \frac{5}{12} \end{bmatrix}
+ \left[\begin{array}{cccc|c} 4 & -2 & & & \\ -2 & 4 & -2 & & \\ & -2 & 4 & -2 & \\ & & -2 & 2 & \\ \hline & & & & 2 \end{array}\right]
\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ \hline u_5 \end{bmatrix}
\begin{array}{l} \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} u_f = ? \\ = 1 \text{ known} \end{array}
$$