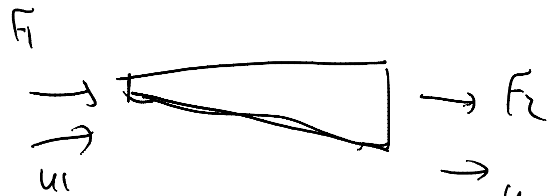
03/13/2014

Thursday, March 13, 2014
11:49 AM

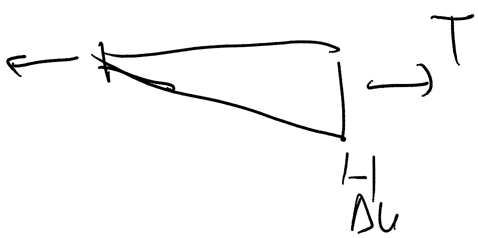


AE not constant

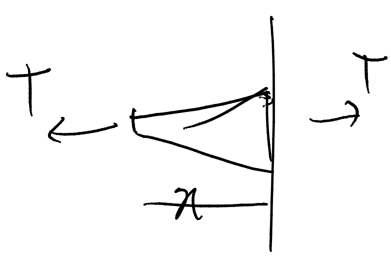
A changing.

$$A(x) = 1 + x$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$



T given what is ΔT



$$F(x) = T \Rightarrow \sigma = \frac{T}{A(x)} \Rightarrow \varepsilon = \frac{T}{A(x)E}$$

$$\varepsilon(x) = \frac{T}{A(x)E} \qquad \Delta L = \int_0^L \varepsilon(x)\,dx$$

we know thes K_{ff} K_{fp}

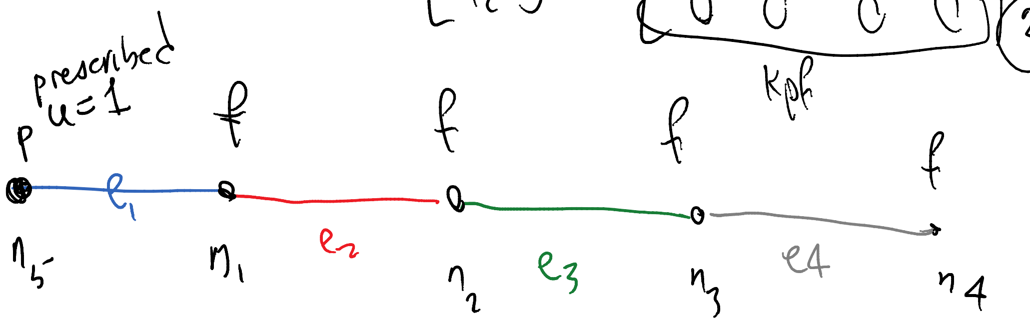
we know thes

$$\underbrace{\begin{bmatrix}F_1\\F_2\\F_3\\F_4\\ \hline F_5\end{bmatrix}}_{\text{Known } F_f \,/\, \text{unknown } F_p} = \begin{bmatrix}\frac{1}{2}\\ \frac{1}{12}\\ 0\\ \\ \frac{5}{12}\end{bmatrix} + \begin{bmatrix}4 & -2 & & & 0\\ -2 & 4 & -2 & & 0\\ & -2 & 4 & -2 & 0\\ & & -2 & 2 & 0\\ \hline 0 & 0 & 0 & 0 & 2\end{bmatrix}\begin{bmatrix}u_1\\u_2\\u_3\\u_4\\ \hline u_5\end{bmatrix}$$

K_{ff}, K_{fp}, K_{pf}, K_{pp}

$u_f = ?$ f

$u_5 = 1$ known p



$\vec{F_1} = \frac{1}{4}$ $\quad \vec{F_2} = 0$ $\quad \vec{F_3} = \frac{1}{2}$ $\quad \vec{F_4} = \frac{1}{4}$

known

$$\begin{bmatrix} F_f \\ F_p \end{bmatrix} = \begin{bmatrix} (K_{ff})_{n_f \times n_f} & (K_{fp})_{n_f \times n_p} \\ K_{pf}\,(n_p \times n_f) & K_{pp}\;{n_p \times n_p} \end{bmatrix} \begin{bmatrix} u_f \\ u_p \end{bmatrix}$$

uknown (F_p), unknown (u_f), known (u_p)

n_f : # of free d.o.f = 4

n_p : # of prescribed d.o.f = 1

Uknown

$$F_f = K_{ff}\, u_f + K_{fp}\, u_p \qquad \text{I}$$

$$F_p = K_{pf}\, u_f + K_{pp}\, u_p \qquad \text{II}$$

unknew $\qquad$ unknown

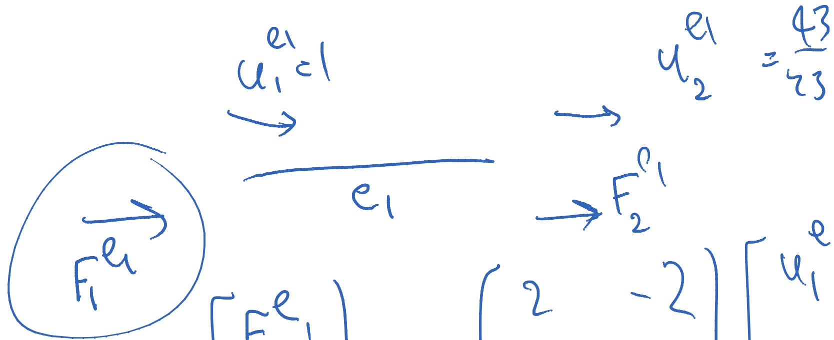
$$\text{I} \Rightarrow K_{ff}\, u_f = F_f - K_{fp}\, u_p$$

$$\boxed{u_f = K_{ff}^{-1}\left(F_f - \underbrace{K_{fp}\, u_p}\right)}$$

11 :

$$F_p = K_{pf}\, u_f + K_{pp}\, u_p$$

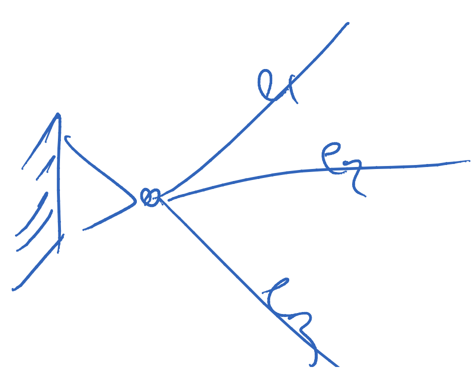
$$= K_{pf}\left(K_{ff}^{-1}\left(F_f - K_{fp} u_p\right)\right) + K_{pp} u_p$$

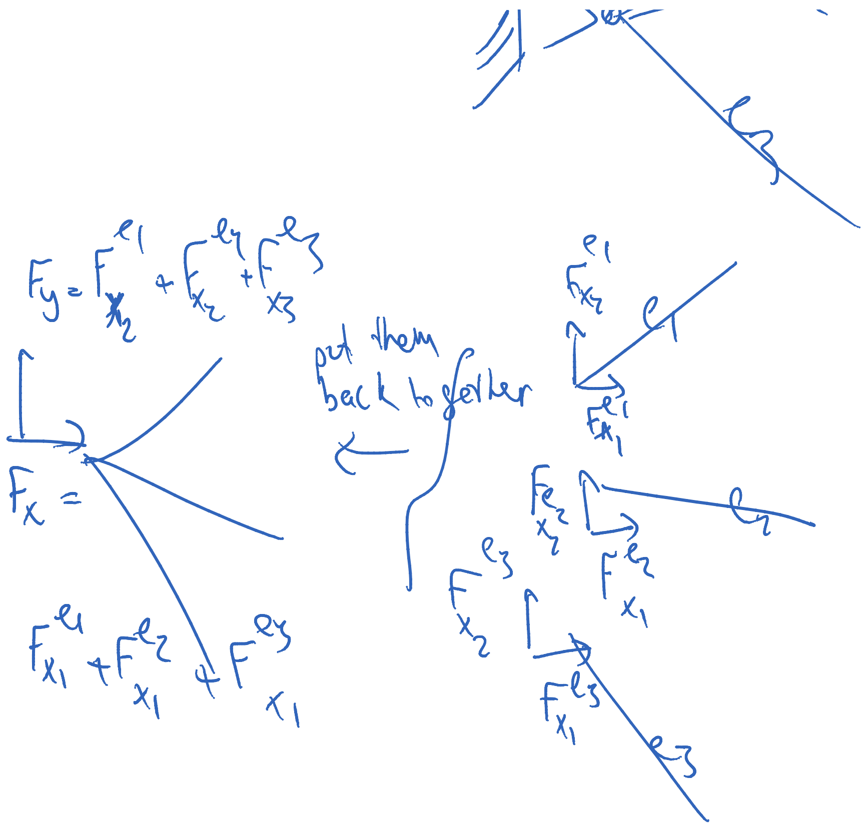


$$\begin{bmatrix} F_1^{e_1} \\ F_2^{e_1} \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} u_1^{e_1} \\ u_2^{e_1} \end{bmatrix}$$

$$\begin{bmatrix} F_1^{e_1} \\ F_2^{e_1} \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 43/23 \end{bmatrix}$$

$$F_1^{e_1} = 2 - 2 \cdot \frac{43}{23} =$$





Comparison

$K_{5\times5}$ system benefits

(1) — It's simpler

(2) — We don't need to obtain forces from essential boundary conditions

$F_p = -K_{fp} U_p$ is already taken care of by the equation

$$u_f = K_{ff}^{-1} (F_f - K_{fp} u_p)$$

③ — We directly obtained prescribed BC's F_p

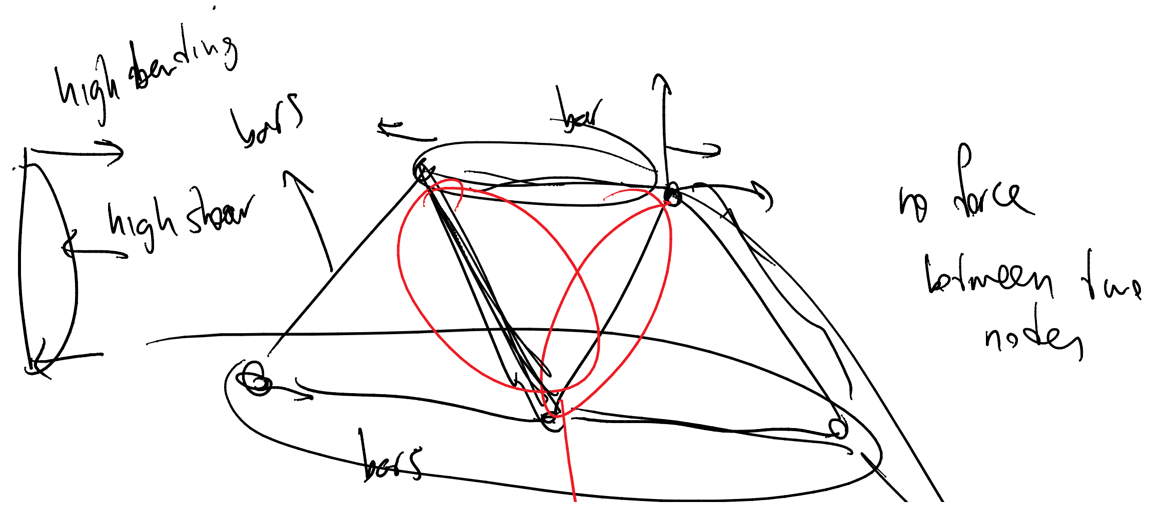
$$F_p = K_{pf} u_f + K_{pp} u_p$$

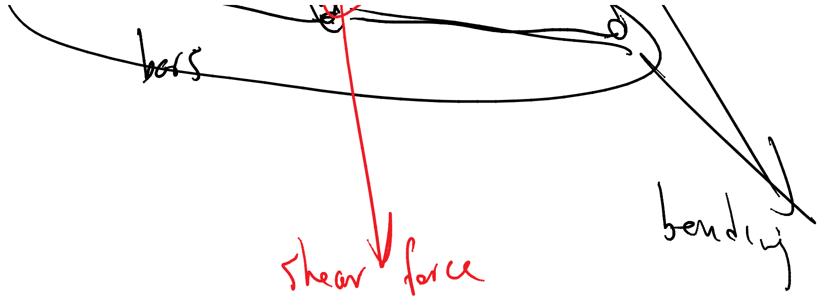
Disadvantage

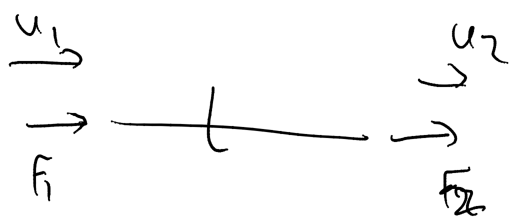
— Computationally more expensive & needs more memory

In practice we only form K_{ff}

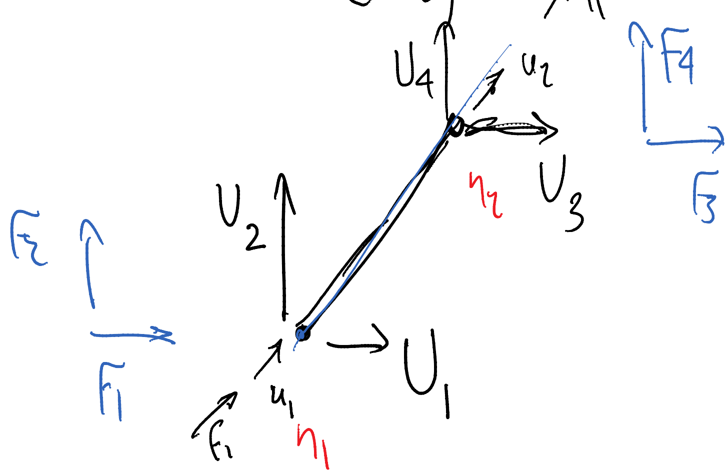
Trusses :





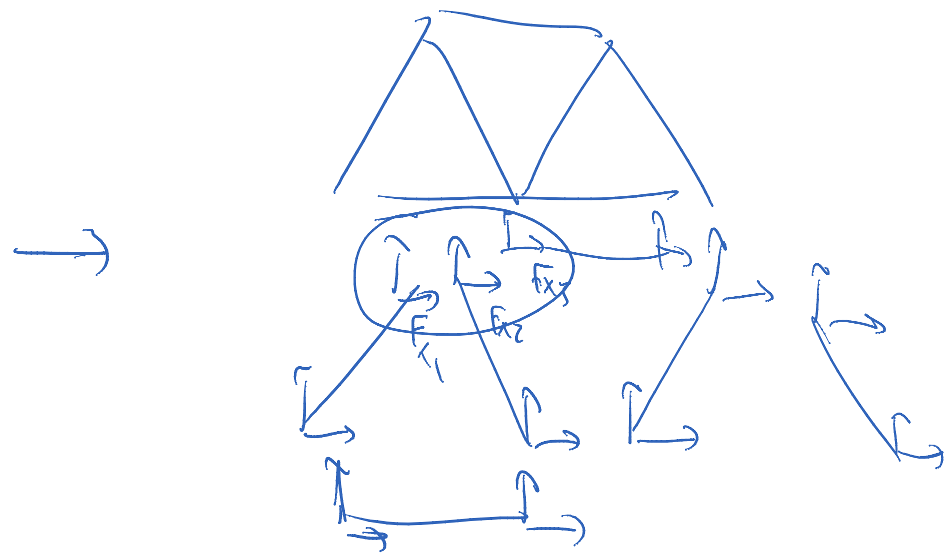


$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \frac{AE}{L} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

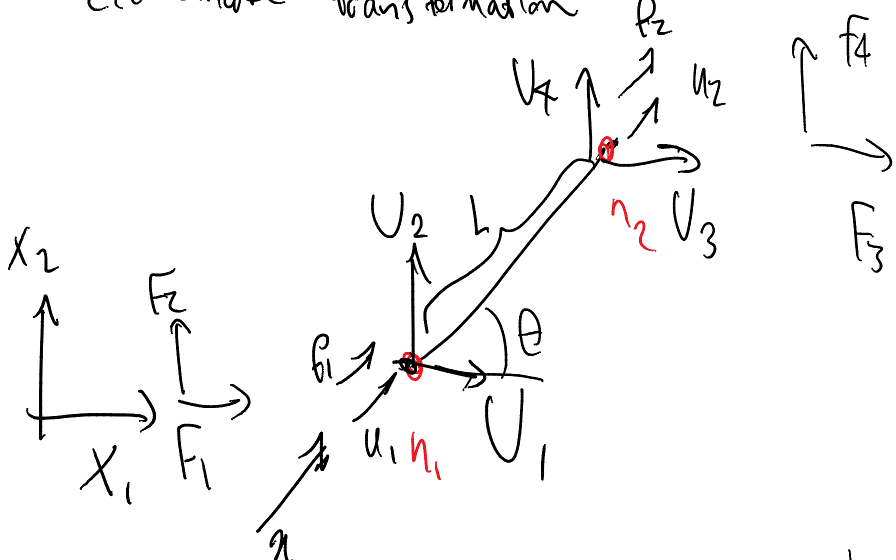


$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = K^{e}_{4 \times 4} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

$F_{x_3} = f_{x_1} + f_{x_2} + f_{x_3}$



Coordinate transformation



$$\text{I} \qquad \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\text{II} \qquad \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} \quad & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}_{4\times 4} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \qquad (T_{\cdot}) \begin{bmatrix} f_1 \end{bmatrix}$$

$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \left(T_{Ff}\right)_{4\times 2} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \left(T_{uU}\right)_{2\times 4} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$

$$\frac{AE}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

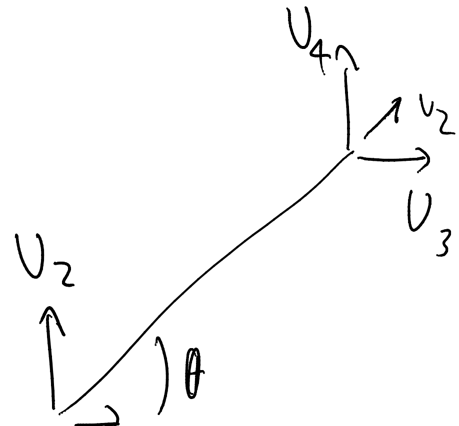
$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = T_{Ff} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \left(T_{Ff}\right)_{4\times 2} k_{2\times 2} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \Rightarrow$$

$$\boxed{\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \left(T_{Ff}\right)_{4\times 2} \; k_{2\times 2} \left(T_{uU}\right)_{2\times 4} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}$$

$$K_{4\times 4} = \left(T_{Ff}\right)_{4\times 2} \; k_{2\times 2} \; \left(T_{uU}\right)_{2\times 4}$$

T_{uU} :

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \underbrace{\begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix}}_{T_{uU}\;\; 2\times 4} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}$$



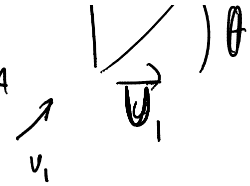
$c = \cos(\theta)$

$s = \sin(\theta)$

$$u_1 = c\,U_1 + s\,U_2 + 0U_3 + 0U_4$$

$v_1 = c\,U_1 + s\,U_2 + 0U_3 + 0U_4$

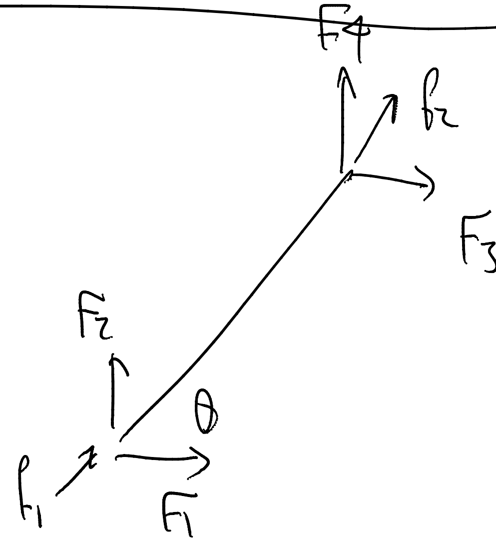
$v_2 = 0\,U_1 + 0U_2 + c\,U_3 + s\,U_4$



$s = \sin(\theta)$

T_{Ff} :

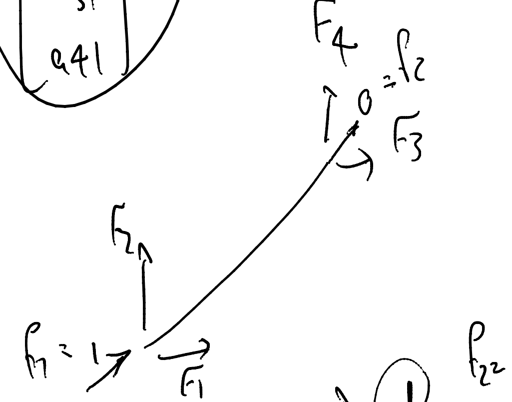
$$\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \underbrace{\begin{bmatrix} \; & \; \\ & \\ & \\ & \end{bmatrix}}_{4\times 2} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

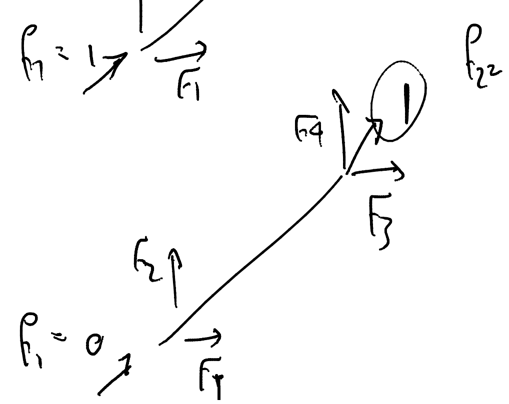


first column $= \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$ for $f_1 = 1$ & $f_2 = 0$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \\ a_{41} & a_{42} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \\ a_{41} \end{bmatrix}$$

$$\begin{bmatrix} c & \\ s & \\ 0 & \\ 0 & \end{bmatrix}$$





$$\begin{bmatrix} & 0 \\ & 0 \\ & c \\ & s \end{bmatrix}$$

$$L \begin{vmatrix} c \\ s \end{vmatrix}$$

$f_1 = 0, \quad F_T$

$$F_{ff} = \begin{pmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{pmatrix} = T^T$$

$$T = T_v U = \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix}$$

$$K = T^T \frac{AE}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} T =$$

$$\frac{AE}{L} \begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix}$$

$$\boxed{K = \frac{AE}{L} \left[\begin{array}{c|c} \overbrace{\begin{matrix} c^2 & cs \\ cs & s^2 \end{matrix}}^{B} & -B \\ \hline -B & B \end{array}\right]}$$