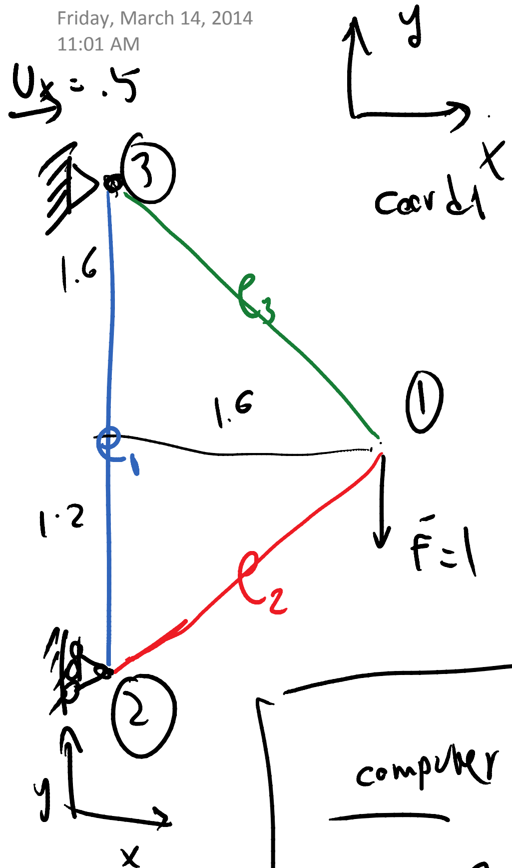
09/14/2014

Friday, March 14, 2014
11:01 AM



Quantities of interest

✓ - All the displacements

- Axial force in the bars T_1, T_2, T_3

- Reaction forces

computer code input for nodes

nn 3

1	1.6	1.2	
2	0	0	I
3	0	2.8	

element block

ne 3		material	nodes	
1	type 0 bar	1	2	3
2	0	1	2	1
3	0	1	3	1

II

prescribed dofs

npdof 3

#	Coordsys	n#	direction	value
1	1	3	1	.5

III

2	1	3	2	0
3	1	2	1	0

n nonzero free 1

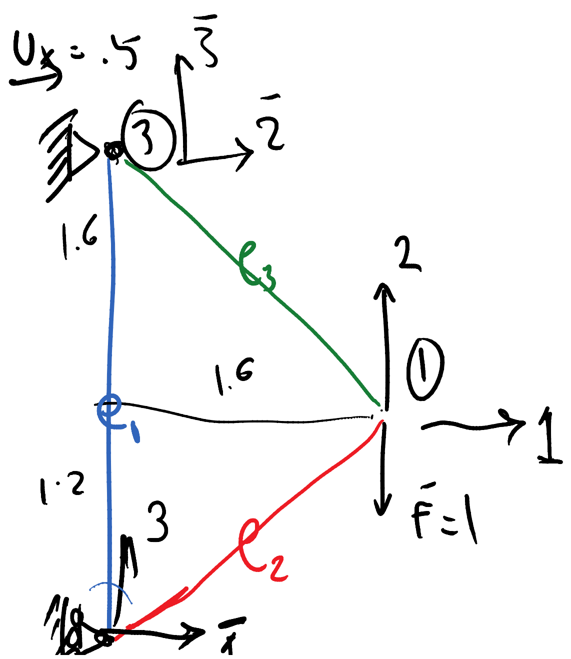
free d.o.f convention

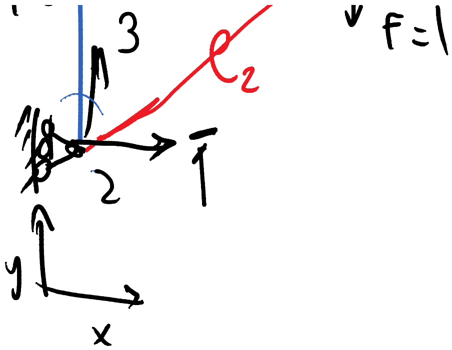
#	coordsys	n#	dir	value
1	1	1	2	-1

only enter free d.o.f with nonzero load

element property nproperties 1

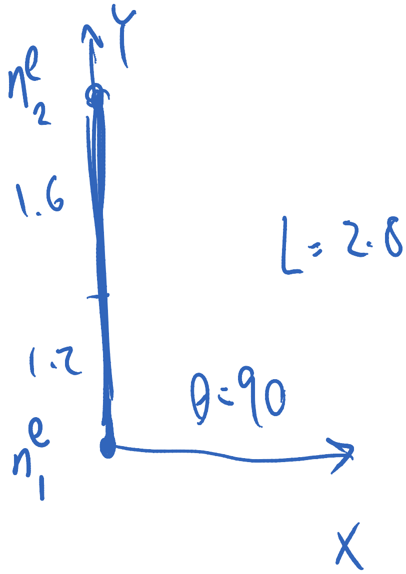
1	A=1	E=1





forming ~~stiffness~~ matrices & force vectors

element 1



$L = 2.8$

$\theta = 90$

$A = 1 \quad E = 1$

$c = \cos 90 = 0 \qquad s = \sin 90 = 1$

$$k^{e_1} = \frac{AE}{L}\left[\begin{array}{cc|cc} c^2 & cs & \rightarrow & \rightarrow \\ cs & s^2 & & \\ \hline - & & + & \end{array}\right]$$

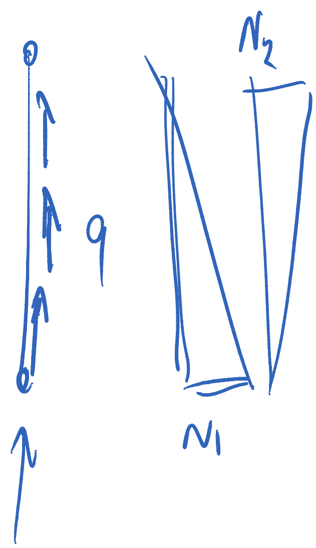
$$= \frac{(1)\cdot(1)}{2.8}\left[\begin{array}{cc|cc} 0 & 0 & 0 & 0 \\ & 1 & 0 & -1 \\ \hline & \text{sym.} & 0 & 0 \\ & & & 1 \end{array}\right]$$

sym.

$$k^{e_1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & .3571 & 0 & -.3571 \\ 0 & 0 & 0 & 0 \\ 0 & -.3571 & 0 & .3571 \end{bmatrix}$$

Forces for element 1

a) body force



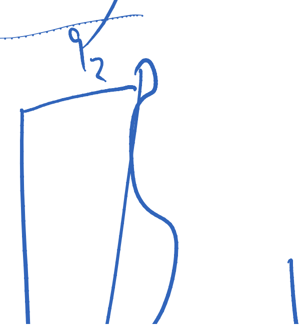
$$f_r^{e_1} = T_{Ff} \int_0^L N^T \cdot q \, dx$$

$$f_r^{e_1} = \begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix} \int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} q \, dx$$

$$q \simeq N_1 q_1 + N_2 q_2$$

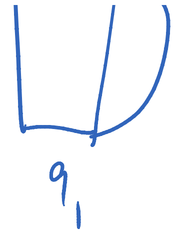
$$f_r^{e_1} = \begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix} \int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \begin{bmatrix} N_1 & N_2 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} dx$$

$$\left(\int_0^L \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} \begin{bmatrix} N_1 & N_2 \end{bmatrix} dx \right)$$



$$f^{e_1} \quad \begin{bmatrix} c & 0 \\ s & 0 \end{bmatrix} \quad \frac{L}{} \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} q_1 \end{bmatrix}$$

$$\begin{vmatrix} S & e \\ o & c \\ c & S \end{vmatrix} \quad \underbrace{\frac{L}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}} \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$$



FYI if we had q

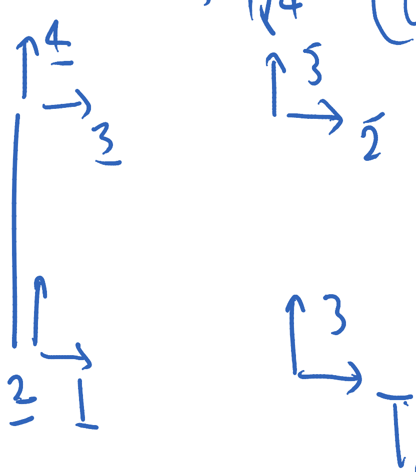
$f_N^{e_1}$ Natural boundary condition

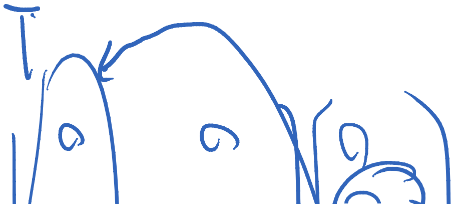
we don't have it.

$f_D^{e_1} = K^{e_1} a^{e_1}$ $\boxed{F_D = K_{fp}\, a_p}$

global

$$= \Big[\quad \Big]_{4\times 4} \begin{bmatrix} 0 \\ X \\ .5 \\ 0 \end{bmatrix} \begin{matrix} U_1 \\ U_3 \\ U_2 \\ U_3 \end{matrix}$$





f^{e_1} $\Big[\, 0 \quad 0 \quad \Big| \quad 0 \quad 0 \quad 0$

$$f_D^{e_1} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & .3571 & 0 & -.3571 \\ 0 & 0 & 0 & 0 \\ 0 & -.3571 & 0 & .3571 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}$$

put zero for free dof

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$f^{e_1} = f_r^{e_1} + f_N^{e_1} - f_P^{e_1} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

important

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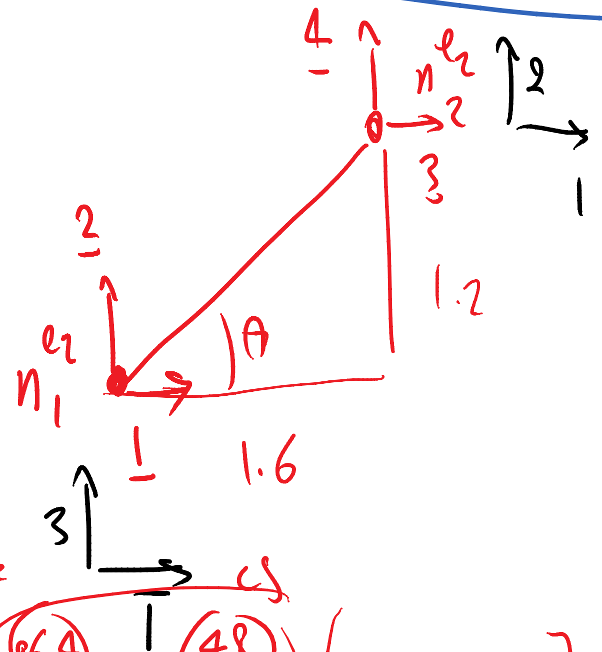
e2

$$L^{e_2} = \sqrt{1.6^2 + 1.2^2} = 2$$

$$C = \frac{1.6}{2} = .8$$

$$S = \frac{1.2}{2} = .6$$



c^2 $\quad$ cs

$, e_n \quad (1)(1) \quad \begin{bmatrix} .64 & .48 & \\ .48 & .36 & s^2 \end{bmatrix}$

$$k^{e_2} = \frac{(1)(1)}{2} \begin{bmatrix} .48 & .37 & & - \\ & & & \\ - & & & + \end{bmatrix}$$

$$k^{e_2} = \begin{bmatrix} .32 & .24 & -.32 & -.24 \\ & .18 & -.24 & -.18 \\ & & .32 & .24 \\ \text{sym} & & & .18 \end{bmatrix}$$

$$f_r^{e_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad f_N^{e_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$f_D^{e_2} = ?$$

$$f_D^{e_2} = k^{e_2} a^{e_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$a^{e_2} = \begin{bmatrix} 0 \\ X \\ X \\ X \end{bmatrix} \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

$$f^{e_2} = f_I^{e_2} + f_N^{e_2} - f_D^{e_2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

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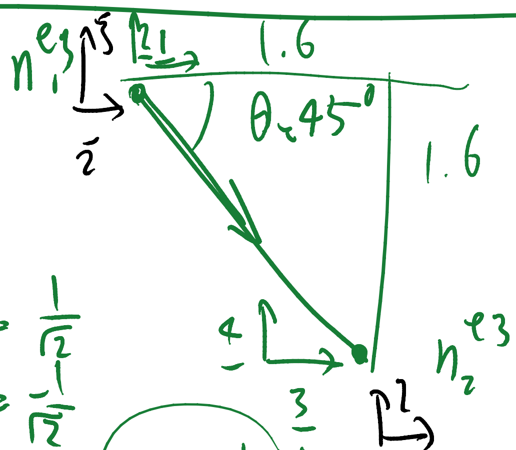
e_3



$$L = \sqrt{2}\, 1.6$$

$$c = \cos -45° = \frac{1}{\sqrt{2}}$$

$$s = \sin -45° = \frac{-1}{\sqrt{2}}$$

$$k^{e_3} = \frac{(1)(1)}{1.6\sqrt{2}} \left[\begin{array}{cc|cc} 1/2 & -1/2 & - & \\ -1/2 & 1/2 & & \\ \hline & & & \\ - & & + & \end{array}\right]$$

$$k^{e_3} = \left[\begin{array}{cc|cc} .221 & -.221 & -.221 & .221 \\ & .221 & .221 & -.221 \\ \hline & & .221 & -.221 \\ \text{sym} & & & .221 \end{array}\right]$$

$$f_I^{e_3} = \vec{0} \qquad f_N^{e_3} = \vec{0}$$

$$a^{e_3} = \begin{bmatrix} 0.5 \\ 0 \\ \hline x \\ x \end{bmatrix} \begin{matrix} \\ \\ \to \text{put zero} \\ \to \end{matrix}$$

$$f_D^{e_3} = \left[\begin{array}{cc|cc} .221 & -.221 & - & \\ -.221 & .221 & & \\ \hline -.221 & & + & \\ .221 & & & \end{array}\right] \begin{bmatrix} .5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$f_D^{e_3} = \begin{bmatrix} .1105 \\ -.1105 \\ -.1105 \\ .1105 \end{bmatrix}$$

$$f^{e_3} = f_r^{e_3} + f_N^{e_3} - f_D^{e_3}$$

$$\boxed{f^{e_3} = \begin{bmatrix} -.1105 \\ .1105 \\ .1105 \\ -.1105 \end{bmatrix}}$$

$$k^{e_1} = \begin{array}{c} \\ \bar{1} \\ 3 \\ \bar{2} \end{array} \begin{array}{c} \begin{array}{cccc} \bar{1} & 3 & \bar{2} & \bar{3} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & .3571 & 0 & -.3571 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

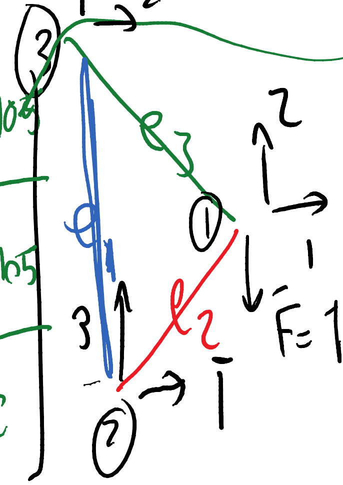
$$k^{e_2} = \begin{array}{c} \\ \bar{1} \\ 3 \\ 1 \end{array} \begin{array}{c} \begin{array}{cccc} \bar{1} & 3 & 1 & 2 \end{array} \\ \left[\begin{array}{cc|cc} .32 & .24 & -.32 & -.24 \\ .24 & .18 & -.24 & -.18 \\ \hline -.32 & -.24 & & \end{array}\right] \end{array}$$

$$k^{e_3} = \begin{array}{c} \\ \bar{2} \\ \bar{3} \\ 1 \end{array} \begin{array}{c} \begin{array}{cccc} \bar{2} & \bar{3} & 1 & 2 \end{array} \\ \left[\begin{array}{ccc|c} .221 & -.221 & -.221 & .221 \\ -.221 & .221 & .221 & -.221 \\ -.221 & .221 & .221 & \end{array}\right] \end{array}$$

2

$$f^{e_1} = \begin{array}{c} \bar{1} \\ \bar{3} \\ \bar{2} \\ \bar{3} \end{array}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad f^{e_2} = \begin{array}{c} \bar{1} \\ \bar{3} \\ 1 \\ 2 \end{array}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad f^{e_3} = \begin{array}{c} \bar{2} \\ \bar{3} \\ 1 \\ 2 \end{array}\begin{bmatrix} -.1105 \\ .1105 \\ .1105 \\ -.1105 \end{bmatrix}$$

$$K = \begin{bmatrix} .32 + .221 & .24 - .221 & -.24 \\ .24 - .221 & .18 + .221 & -.18 \\ -.24 & -.18 & .3571 + .18 \end{bmatrix} \qquad F = \underbrace{\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}}_{F_1} + \underbrace{\begin{bmatrix} .1105 \\ -.1105 \\ 0 \end{bmatrix}}_{F \text{ from elements}}$$

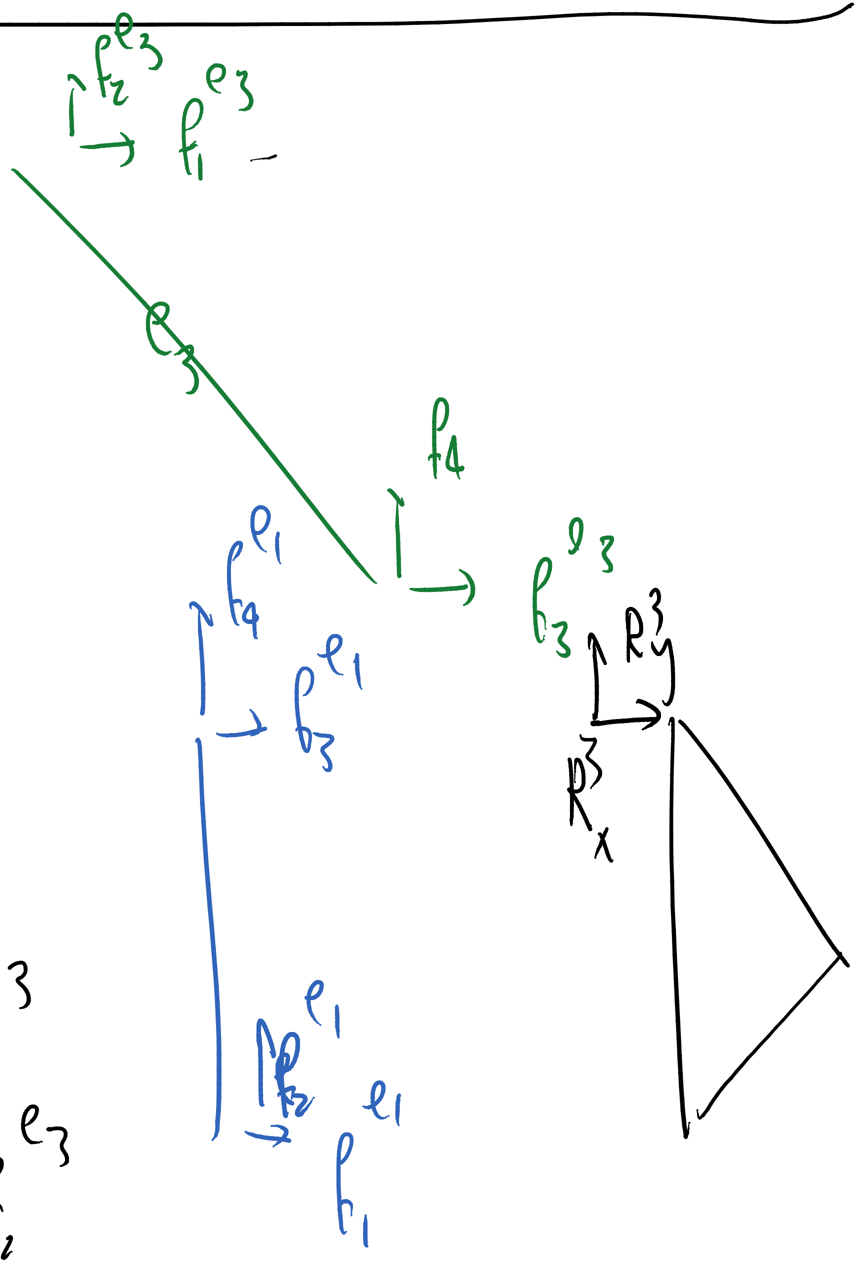


$$K = \begin{bmatrix} .5410 & .019 & -.24 \\ & .401 & -.18 \\ \text{sym} & & .5371 \end{bmatrix} \qquad F = \begin{bmatrix} .1105 \\ -1.1105 \\ 0 \end{bmatrix}$$

$$a = K^{-1}F = \begin{bmatrix} -.2123 \\ -3.2980 \\ -1.200 \end{bmatrix}$$

$$f_T^{e_3} = \begin{bmatrix} K^{e_3} \end{bmatrix} \begin{bmatrix} a_1^{e_3} \\ a_2^{e_3} \\ a_3^{e_3} \\ a_4^{e_3} \end{bmatrix}$$

$=$



$$f_T^{e_1} = K^{e_1} a^{e_1}$$

$$R_x^3 = f_3^{e_1} + f_1^{e_3}$$

$$R_y^3 = f_4^{e_1} + f_2^{e_3}$$