

Discontinuous versus Continuous Galerkin Finite Element Methods for Dynamic Problems

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This focus of this presentation is the assessment of various continuous and discontinuous Galerkin (DG) finite element methods (FEMs) for dynamic problems. Specifically, we compare the so-called *spacetime discontinuous Galerkin* (SDG) method versus several continuous FEMs in terms of their solution scaling, error convergence, and efficiency. The SDG method is a novel FEM that replaces a separate temporal integration by direct and unstructured discretization of spacetime. When employed for hyperbolic equations, several novel properties of the SDG method—such as its linear solution scaling, and local and asynchronous solution scheme—yield an extremely efficient and versatile method. We investigate how each of these metrics is influenced by the choice of the finite element method, interpolation order in space, order of accuracy in time, and integration type, *i.e.* implicit versus explicit. Apart from efficiency and solution scaling, the locality and asynchronicity of the solution play important roles in both adaptive and parallel simulations. We briefly compare the SDG method versus continuous FEMs for highly adaptive and multiscale grids. We also demonstrate the superiority of the DG methods over conventional FEMs for problems with singularities, shocks, and other nonsmooth features in solid, fluid, and impact/fracture problems.

Many exceptional efficiency, scalability, and locality properties of the SDG method rely on the fact that information cannot propagate faster than the wave speeds implied by the hyperbolicity of a given PDE. For parabolic equations all points of the domain are mathematically coupled; direct discretization of these systems using the SDG method requires extruded meshes that preserve the global coupling in the discrete setting. The extension of the local solution and asynchronous properties of the SDG method to parabolic equations is a radical advance in computational science. We present promising results for uniform meshes by looking at hyperbolic approximations that preserve the method's inherent order of accuracy.