17th U.S. National Congress on Theoretical and Applied Mechanics
Michigan State University, June 15-20, 2014

Abstract: C-06-826

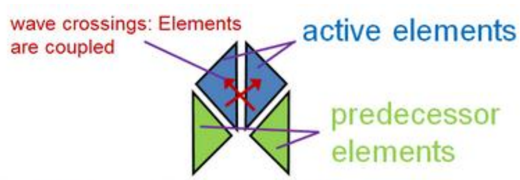
A Riemann-solver-free Discontinuous Galerkin Method for Hyperbolic Systems

Reza ABEDI, Dept. of Mechanical, Aerospace, and Biomedical Engineering, University of Tennessee Space Institute, rabedi@utk.edu
Rajkumar PAL, Dept. of Mechanical Science and Engineering, University of Illinois Urbana-Champaign, pal3@illinois.edu
Ian MCNAMARA, Dept. of Mechanical Science and Engineering, University of Illinois Urbana-Champaign, imcnama2@illinois.edu
Robert B. HABER, Dept. of Mechanical Science and Engineering, University of Illinois Urbana-Champaign, r-haber@illinois.edu

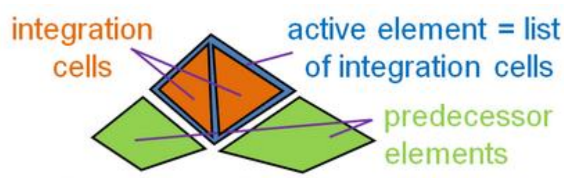
Keywords: Riemann problems, discontinuous Galerkin, finite element method, spacetime.

Riemann fluxes, or their numerical approximations, play an important role in high-resolution computational methods for solving systems of hyperbolic equations by finite volume or discontinuous Galerkin finite element methods. Especially in nonlinear problems, incorporation of Riemann fluxes improves accuracy, preserves characteristic structure, and may be essential for correct prediction of shock dynamics. Unfortunately, the solution of the Riemann problems from which these fluxes derive becomes more expensive and more difficult as the spatial dimension and the complexity of the physical system increase. Various approximate Riemann solvers and fluxes have been developed in response, as well as a limited number of Riemann-solver-free methods, such as those by Kurganev and Tadmor.

We have previously described asynchronous spacetime discontinuous Galerkin solution schemes in which we solve hyperbolic systems on unstructured, simplicial meshes that are organized into small patches of elements, such that the facets between patches are space-like manifolds. This enables a patch-by-patch solution scheme where the Riemann solution is trivial between patches (the fluxes on the predecessor element always control). However, we still enforce Riemann fluxes via jump terms on the time-like facets between simplex elements within each patch, and here we face the same challenges as in other methods. In this presentation, we explore the possibility of eliminating all non-trivial Riemann solutions by treating each patch as a single element (i.e., with patch-wise-continuous basis functions), using simplex cells only for purposes of numerical quadrature. We present results to demonstrate that this approach can both simplify the implementation of our SDG method and improve computational efficiency in the context of various linear and nonlinear conservation laws.



Single-cell elements: Riemann or numerical fluxed needed at interior facets



Multi-cell elements: All element facets are causal ⇨ Riemann-free scheme