Spacetime Discontinuous Galerkin FEM: Spectral Response

Reza Abedi^a, Omid Omidi^a, and Philip L. Clarke^a

^aDept. of Mechanical, Aerospace & Biomedical Engineering, University of Tennessee Space Institute, 411 B. H. Goethert Parkway, MS 21, Tullahoma, TN, 37388, USA, {rabedi, oomidi, pclarke}@utsi.edu

Materials in nature demonstrate certain spectral shapes in terms of their material properties. For example in electromagnetics, permittivity and/or permeability are either constant or follow a certain frequency dependent response. Since successful experimental demonstrations about a decade ago, metamaterials have provided a means to engineer materials with desired spectral shapes for their material properties. They are formed by structured building blocks whose size and periodicity are much smaller than the wavelengths of interest. Under these conditions the material behaves as homogeneous at continuum level and can even exhibit effective negative permittivity, permeability, mass density, and/or stiffness in electromagnetics or elastodynamics over a range of frequencies.



Left: Negative index metamaterial constructed of split-ring resonators; Center: Scattering of a silver sphere obtained by time domain Discontinuous Galerkin method [1]. Right: A spacetime finite element discretization for dynamic analysis using SDG method [2].

Derivation of spectral shapes for various frequency dependent properties of metamaterials such as scattering and absorption coefficients is quite a challenging problem. Multiscale geometric features as well as high gradient and discontinuous solution fields necessitate the use of high order and multiscale/adaptive solution schemes for reliable computation of spectral response. In addition while spectral response is represented in frequency domain, time domain (TD) methods have several advantages over frequency domain (FD) methods: 1. FD schemes require a separate solution for each point of the spectrum while in TD approach the entire spectrum can be obtained by solution to only one frequency rich signal and subsequent Fourier transformation; 2. TD methods easily handle material nonlinearities; 3. Some TD methods, such as SDG below, can break the global solution coupling in FD and are far more efficient. We present the Spacetime Discontinuous Galerkin (SDG) finite element method [2], a novel TD method with exceptional multiscale/adaptive, parallel, and solution scaling properties. We present the roadmap to apply the SDG method to obtain metamaterial spectral response.

References

- [1] Stannigel K, Konig M, Niegemann J, Busch K; 2009, Optics Express, 17, 14934-14947.
- [2] Abedi R, Petracovici B, Haber R; 2006, Comput Method Appl M, 195, 3247-3273, 2006.