Spacetime Discontinuous Galerkin FEM: Spectral Response

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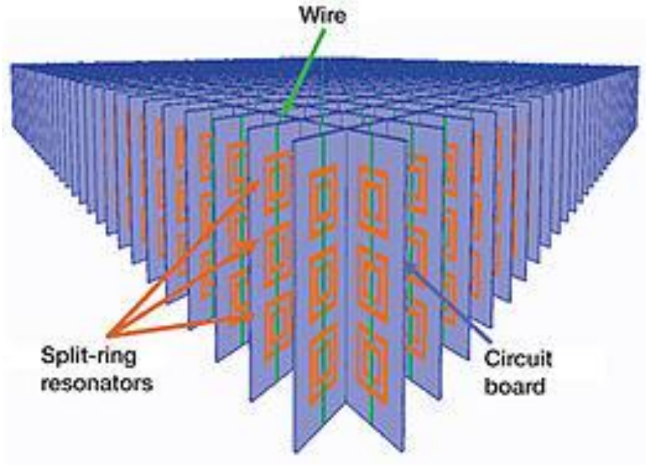
We present the novel spectral properties of metamaterials, discuss their highly multiscale and rich solution features, followed by challenges in their computational modeling. After elaborating on the advantages of time domain schemes , we elaborate on unique features of TD method of Spacetime Discontinuous Galerkin method for metamaterial and spectral analyses.

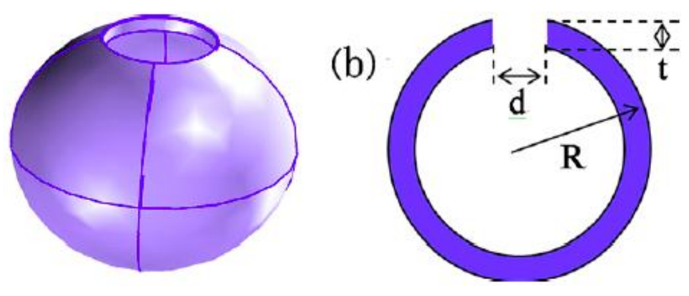
Metamaterials and photonic crystals



- Feature sizes of the system are:
  - comparable to (Photonic crystals) or
  - much smaller (metamaterials) than the wavelength of incident waves
- Metamaterials can be considered as effective media whose frequency-dependent properties are determined by building blocks

Properties and applications of metamaterials

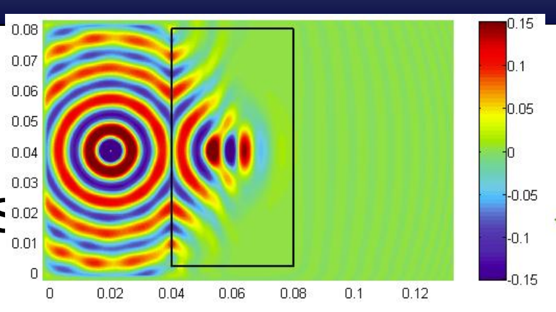
- Negative permittivity (Schurig et al 2006a), permeability (Shelby et al 2001), or both (negative index of refraction: Smith et al 2000) for electromagnetics
- Electromagnetic (Schurig et al 2006b) & acoustic cloaking
- Negative mass density and/or density for acoustics
- Unusually high refractive indices (Choi et al 2011)
- Perfect absorbers (Landy et al 2008, Tao et al 2008)
- Sub diffraction imaging (Fang et al 2005)
- Memory metamaterials (Driscoll et al 2009)



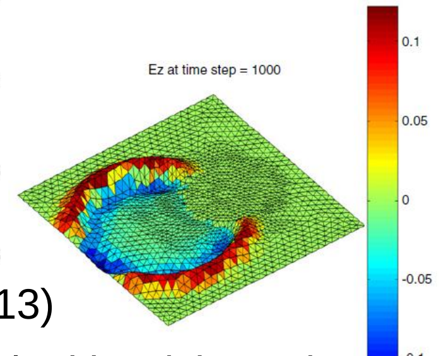
Split hollow sphere (SHS); microstructure for acoustic metamaterial

Numerical methods for spectral analysis

- Provide all spectral information
- Optimization (geometry & material)
- Information not available by experiments

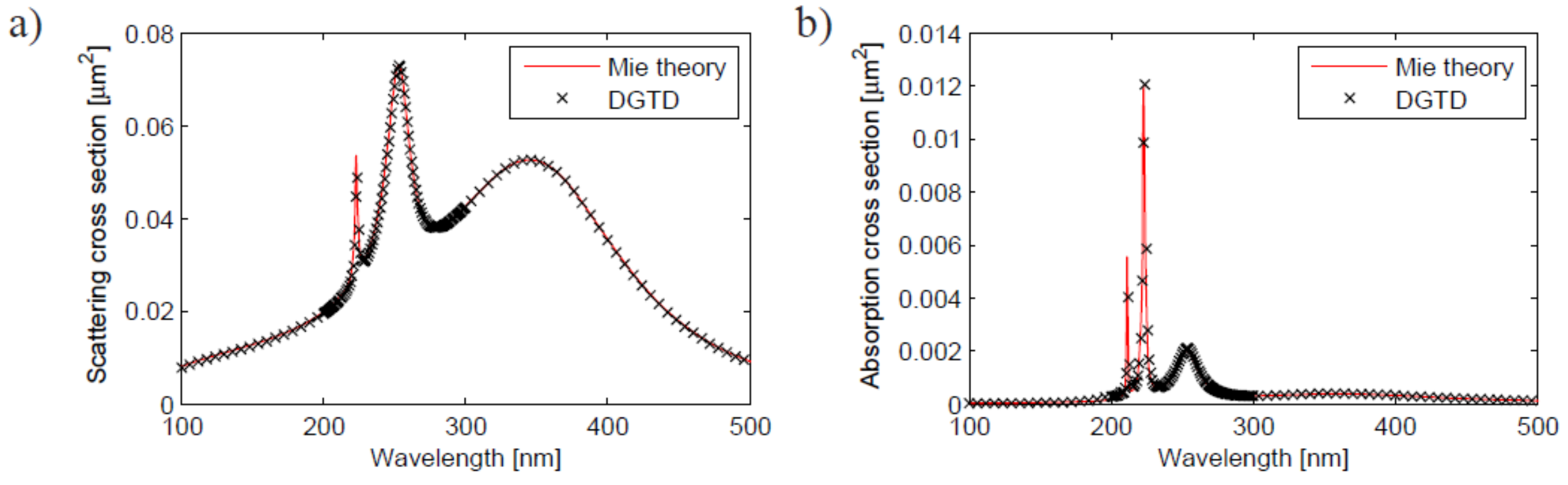


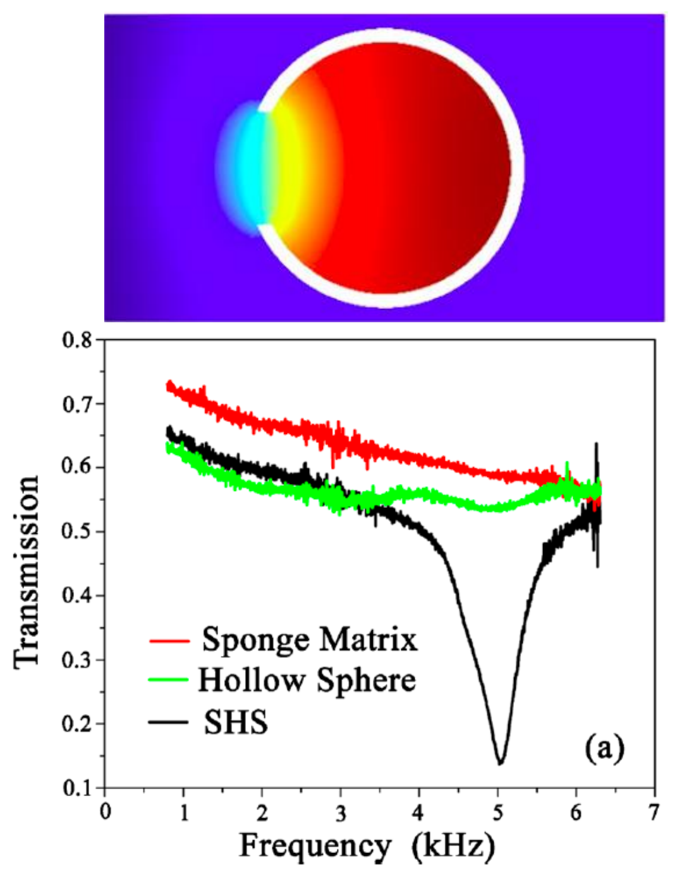
Reflection from metamaterial (Huang et al 2013)



Cloaking (Li 2011)

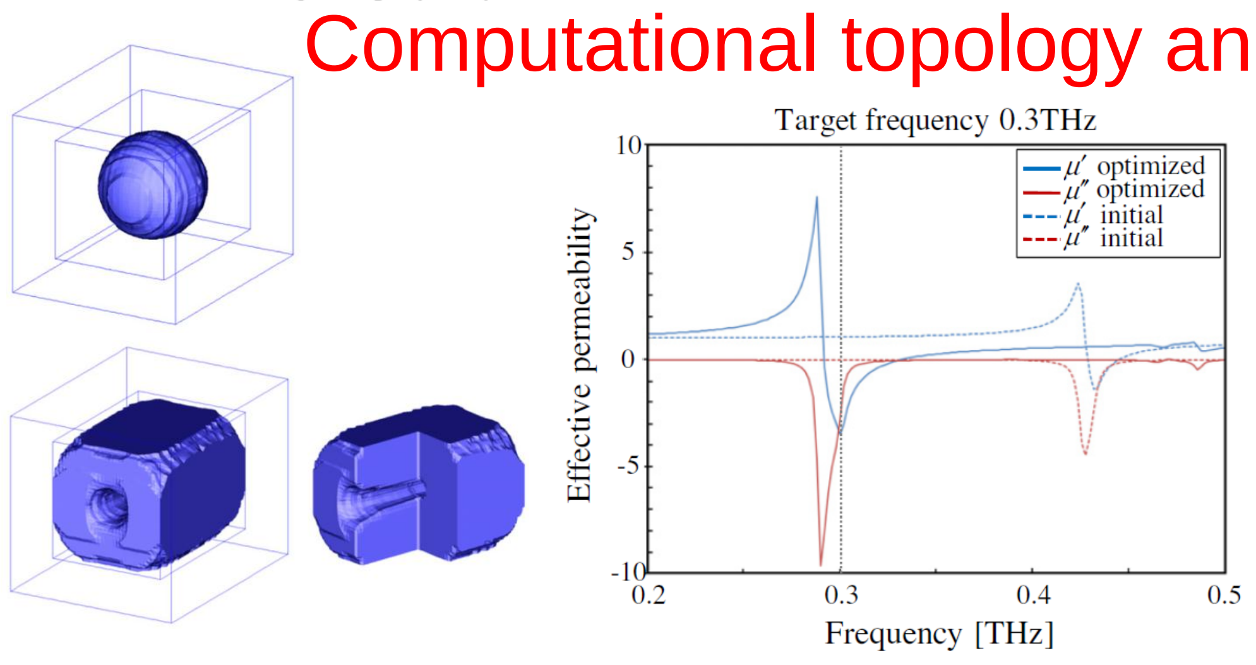
Scattering by metallic sphere (Stannigel et al 2009)



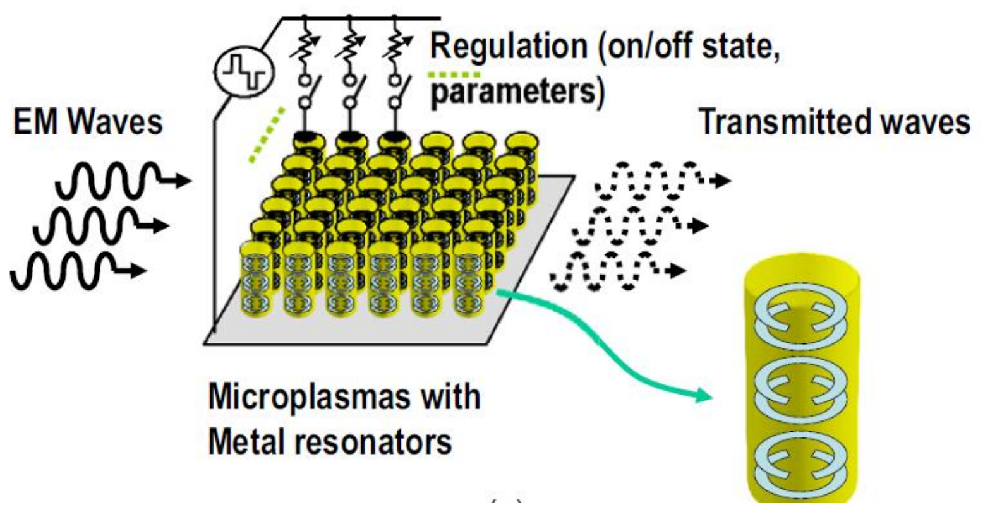


Double negative acoustic metamaterial: strong vibration of the sound medium at the resonant frequency (top); transmission results (bottom): (Ding et al 2010)

Computational topology and material optimization



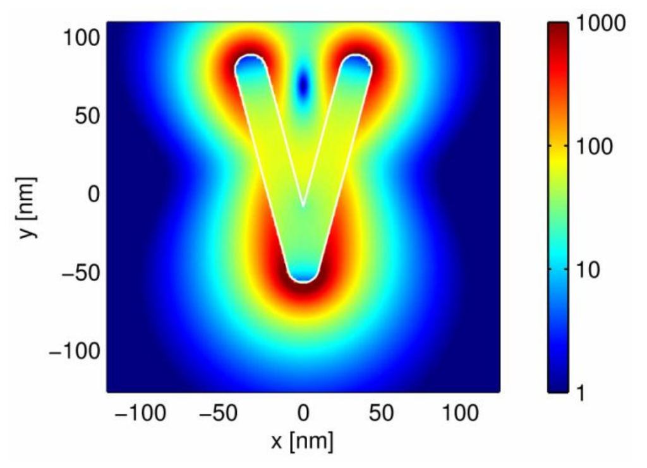
Topology optimization to minimize permeability (Otomori et al 2012)



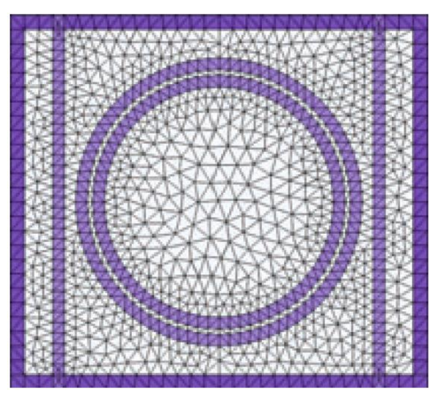
Alter/ enhance metamaterial properties using plasmas (Sakai & Tachibana 2012)

Computational challenges for metamaterials

- Most of literature on Maxwell solvers for free & dispersive media but not for metamaterials. Same applies to acoustic & electromagnetic metamaterial.
- Computation of metamaterials is much more challenging:
  - High gradient fields and sharp discontinuities Jumps in material properties and very strong fields at air/metamaterial interface call for highly dense and high order meshes
  - Multiscale domains: Very small grid spacing



Intensity distribution for a silver V shape excited at resonance (Stannigel et al 2009)



Two waveguides coupled to a slotted microresonator (Busch 2011)

Time Domain (TD) vs. Frequency Domain (FD)

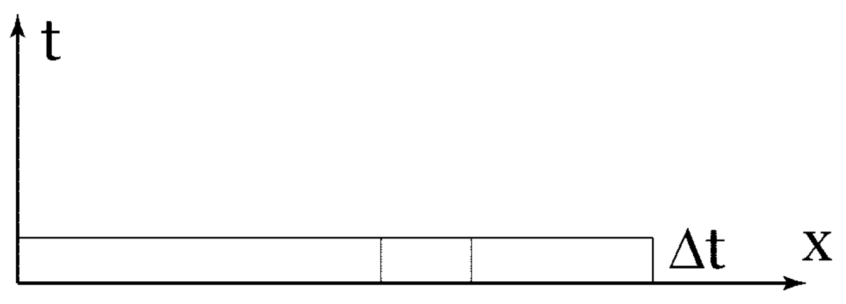
- Quasi-static FD solution may be sufficient (Ding et al 2010):
  - How can the field get to its stable state?
  - In there any strong scattering in the process?
  - How long is the relaxation process?
  - What is the system response to a pulse?
  - TD needed for dynamic topics such as temporal coherence, change in propagation, tuning of cavity frequency, and the nonlinear response
- Obtaining the entire spectra with one simulation: FD requires one simulation per point in spectra while in TD response and subsequent Fourier transformation to a broad-band signal is sufficient (Stannigel et al 2009)
- Nonlinearity: nonlinear phenomena cannot easily be treated in FD
- Efficiency, scaling: Unlike FD domains that entail a global coupling, some TD solution scales linearly versus number of unknowns/elements.

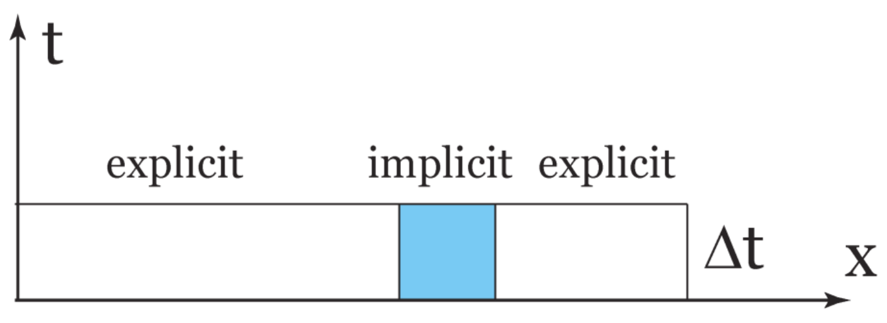
Caveats with TD methods

1. Frequency-dependent material properties: In TD method, frequency dependent properties that are more easily handles in FD are pulled back to a convolution; e.g. for acoustic equations:

$$\int_{-\infty}^{\infty} \rho_{0(x,t-t')} \cdot \frac{\partial v}{\partial t'} dt' + \nabla p = 0$$

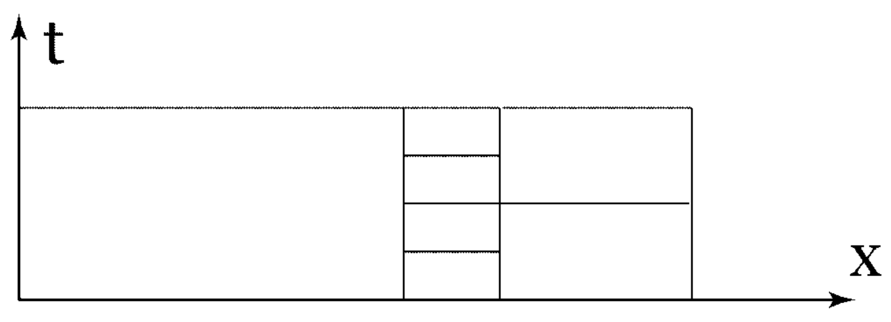
2. Global coupling: Most TDs, e.g. implicit method pose a global coupling.
3. Time integration problems from multiscale domains:





Implicit/Explicit (MIX)

- Explicit methods eliminate the global coupling of implicit methods
- For multiscale meshes time step is limited by minimum element size
- IMEX and subcycling method partially alleviate the problem and are generally limited to low orders



Multiscale explicit methods

Frequency-dependent properties in TD

There can be two approaches to eliminate the convolution op. in item 1:

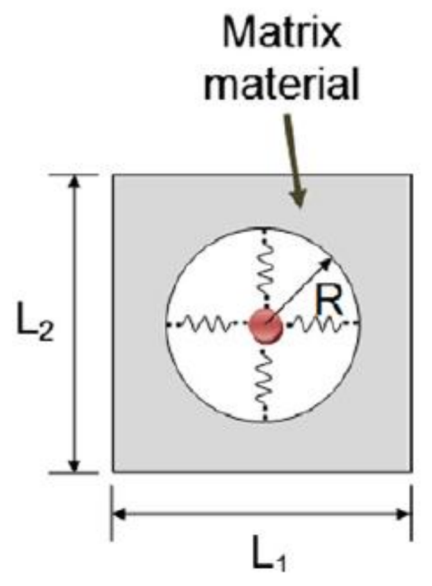
1. Auxiliary Differential Equations (ADE): By “auxiliary fields” (*cf.* Rodriguez 2005 for treatment of Debye & plasma materials in EM) ω is eliminated from acoustic equation) $m_i^{eff} = m_1 + \frac{m_2(\omega_0)_i^2}{(\omega_0)_i^2 - \omega^2} \quad (i = 1, 2)$

2. Multi-field (e.g. displacement) continuum formulation:

$$\bar{\rho}_m \frac{\partial^2 U_i}{\partial t^2} + \frac{\Sigma'_{\underline{ii}}}{R} - \frac{\partial \Sigma_{ij}}{\partial X_j} = 0 \text{ (macro)}$$

$$\frac{m_2}{A} \frac{\partial^2 U_{2i}}{\partial t^2} - \frac{\Sigma'_{\underline{ii}}}{R} = 0 \text{ (micro) displacements}$$

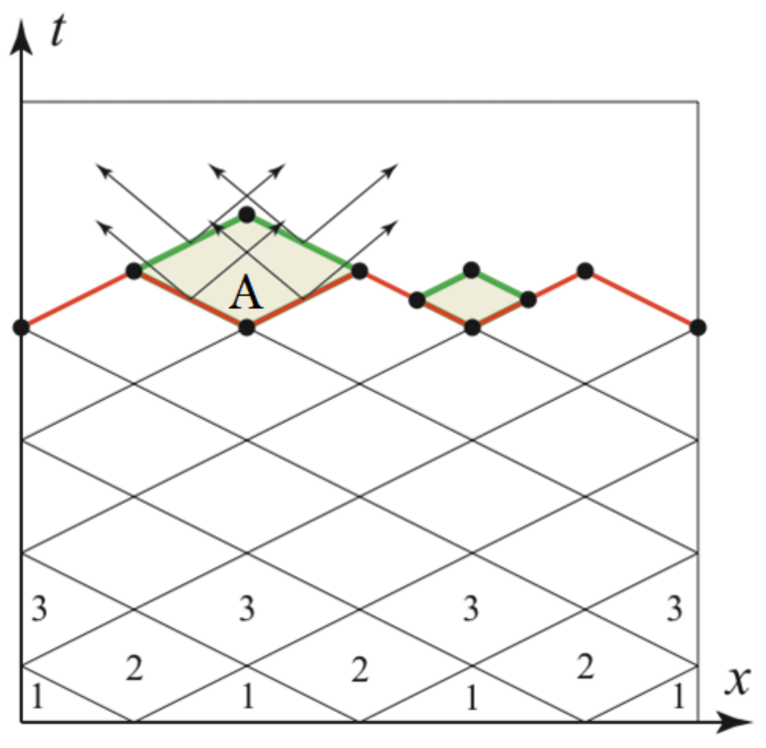
Huang & Sun 2012

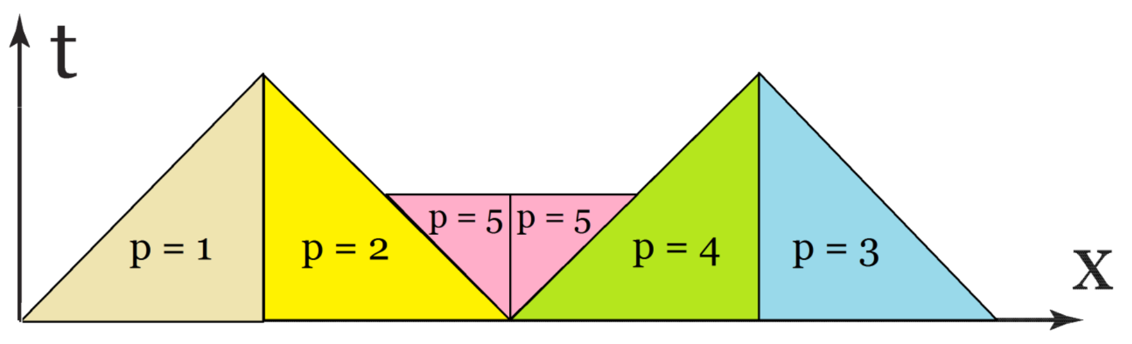


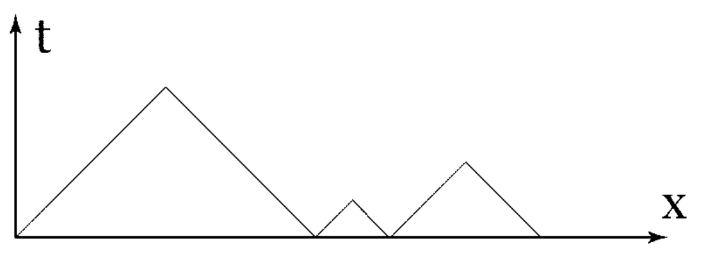
Spacetime Discontinuous Galerkin method

Direct discretizion of spacetime, enforcing causality property in discrete setting, and discontinuous basis functions yields these distinct advantages:

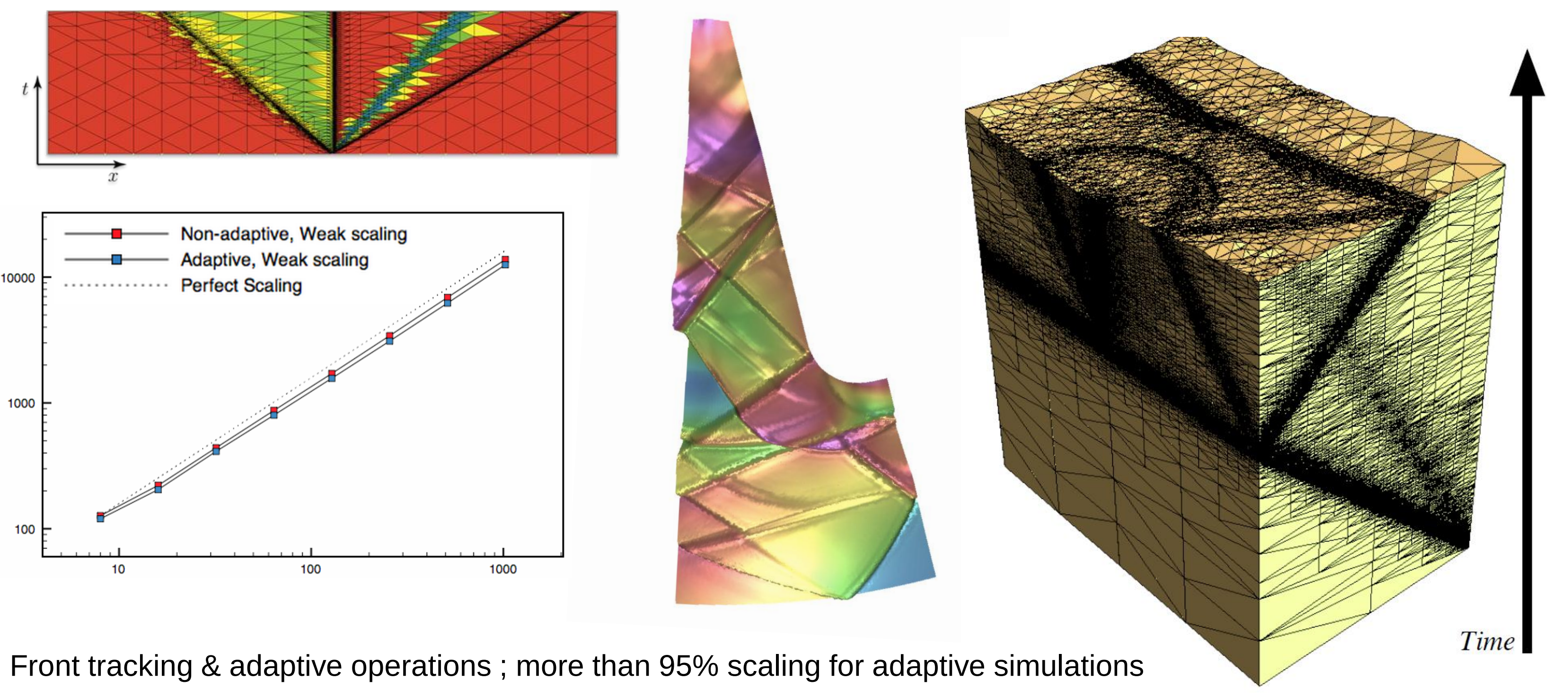
1. Local solution property
2. O(N) cost vs. number of elements
3. Excellent resolution of high gradient fields and discontinuities
4. Arbitrary element size (h) and polynomial order (p) adjustment (below)
5. Arbitrary high order in time (below)







6. Excellent for multiscale domains; local time step not affected by smallest size
7. Local & asynchronous features ideal for adaptive & parallel simulations



Front tracking & adaptive operations ; more than 95% scaling for adaptive simulations

Refernces

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