

1. For steel the elastic modulus $E = 210$ GPa. For a given specimen material strength $\sigma_f = 250$ MPa. **(40 Points)**
 - (a) Provide an estimate for theoretical strength σ_c based on atomistic potential.
 - (b) What is the ratio σ_f/σ_c and what is the reason for discrepancy?
 - (c) Obtain an estimate for the ratio $r = a/x_0$ for $a =$ representative defect size in the material and $x_0 =$ atomistic lattice length scale.
 - (d) If $x_0 \approx 10^{-10}$ m obtain an estimate for surface energy γ_c .

2. For a specimen which can be characterized by a point force P and displacement u we have two measurements of (u_1, P_1) for crack length a_1 and (u_2, P_2) for crack length a_2 . The stiffness is defined as $K = P/u$. Recalling that $G = -\frac{\partial \Pi}{\partial A} = -\frac{\partial \Pi}{\partial aB}$ for A crack surface, a crack length, and B crack (a) obtain an equation for G in terms of u and $\frac{\partial K}{\partial a}$, (b) Given that compliance $C = 1/K$ show that $G = \frac{P^2}{2B} \frac{\partial C}{\partial a}$. **(70 Points)**
 Hint: Note that $K = \tan(\theta)$, $G \approx -\frac{\Pi_2 - \Pi_1}{aB} = -\frac{U_{e2} - U_{e1} - W_{12}}{aB} = \frac{\text{Shaded area}}{aB}$. Finally, express shaded area using $\Delta\theta$.

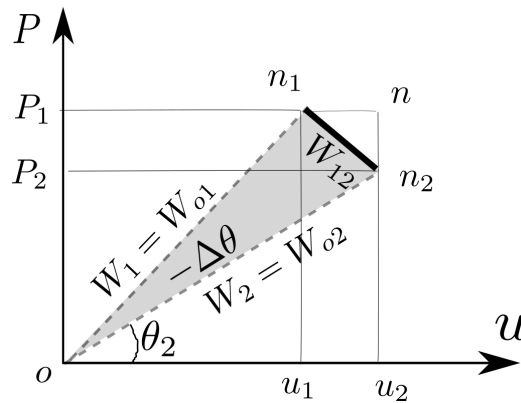


Figure 1: Force displacement relation for a point force system.

3. A composite laminate is made by bonding two long thin strips of steel with an adhesive epoxy, as shown in figure 2. A patch of the adhesive was intentionally left out in the central section in order to create a central crack of length $2a$ in the bilayer plate. The joined plates are pulled apart by equal and opposite tensile forces, P . From beam theory, the deflection of a double cantilever beam of length L (half the crack length) under load P is $\Delta = \frac{PL^3}{192EI} \Rightarrow$

$$\frac{\delta}{2} = \frac{P(2a)^3}{192EI} \Rightarrow \delta = \frac{PL^3}{12EI}$$

where the crack opening δ is the displacement of load P . Since there are two crack front we have,

$$2G = \frac{P^2}{2B} \frac{\partial C}{\partial a}$$

for $C = \delta/P$. The parameters for the problem are: initial crack length $2a_0 = 60$ mm, $E = 200$ GPa, $H = 0.97$ mm and $B = 10.1$ mm.

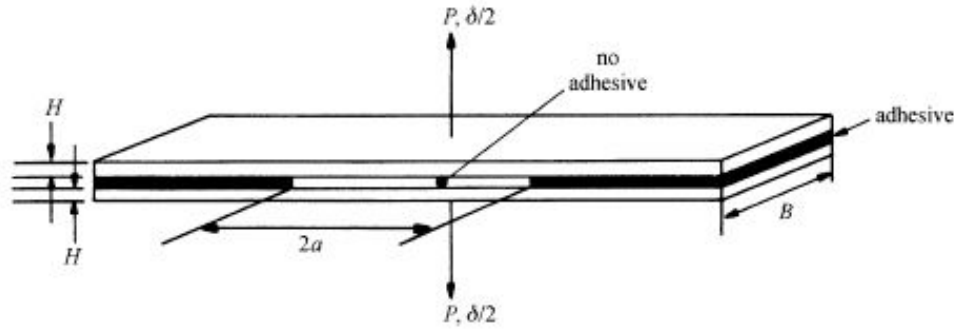


Figure 2: Geometry and loading of joined plates (image source: Suresh, Fatigue of materials)

Consider two different R (crack propagation resistance) equations:

$$R_1(a) = R_0 \quad (1a)$$

$$R_2(a) = \begin{cases} R_0 & a = a_0 \\ R_0 + \Delta R \left[1 - \left(\frac{a - a_0 - l}{l} \right)^2 \right] & a_0 < a < a + a_0 \\ R_0 + \Delta R & a + a_0 < a \end{cases} \quad (1b)$$

Basically R_1 corresponds to a constant resistance and R_2 is for a case that R increases from R_0 from $a = a_0$ to $R_0 + \Delta R$ at $a = a_0 + l$ by a parabolic equation that smoothly (zero slope at $a = a_0 + l$) transitions to constant $R = R_0 + \Delta R$ for larger crack lengths. The parameters ΔR and l are strengthening of crack resistance due to crack propagation and a characteristic length respectively. Let $R_0 = 300 \text{ Pa}\cdot\text{m}$, $\Delta R = 1200 \text{ Pa}\cdot\text{m}$ and $l = a_0 = 30 \text{ mm}$. Answer the following **(140 Points)**:

- Plot R-Curve for R_1 and R_2 (Resistance R versus crack length a).
- Consider load control and displacement control methods to initiate and cause crack propagation. In load control we increase P until at P_{ini} crack starts propagating. If a critical value P_{cr} exists once $P = P_{cr}$ crack propagation becomes unstable, *i.e.*, propagates without increasing P . Same concepts apply for displacement control to δ_{ini} and δ_{cr} (if it exists). Add G curves for load control $P = 45, 105, 130 \text{ N}$ and displacement control for $\delta = 1 \text{ mm}$ and 3 mm to the same R plot that included R_1 and R_2 curves.
- For constant resistance given by R_1 obtain P_{ini}^1, P_{cr}^1 (if for any a crack becomes unstable) and for load control δ_{ini}^1 and δ_{cr}^1 (if for any a displacement control method results in unstable crack propagation).
- For increasing resistance case R_2 obtain P_{ini}^2, P_{cr}^2 and for load control δ_{ini}^2 and δ_{cr}^2 . Again, note that the critical values may not exist.
- Schematically add G curves for $P_{ini}^2, \delta_{ini}^2, P_{cr}^2$ (if exists), and δ_{cr}^2 (if exists) to G, R plot from previous steps. Which of loadings eventually result in unstable crack propagation?
- For stable crack growth the relation between a, P , and δ is obtained by $G = R$ (crack force equal to crack resistance). For load control and displacement control solve for both P and δ as functions of a for two different cases of resistance R_1 and R_2 (2×2 solutions) and generate the following plots: 1) P (vertical axes) plotted versus δ (horizontal), 2) a versus P , and 3)(optional) a versus δ . The explicit form of these relations are not needed and only the plots need to be correct.
- Based on all the generated plots compare load control and displacement control and also constant R versus increasing R cases.