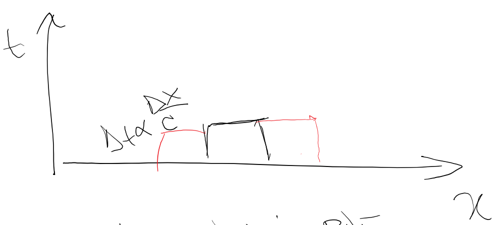
DG2020/02/25

Wednesday, March 25, 2020 11:41 AM

Last time we talked about the possibility of having explicit spacetime DG methods with extrusion of spatial mesh in time, and we discussed that there is a need for ghost elements

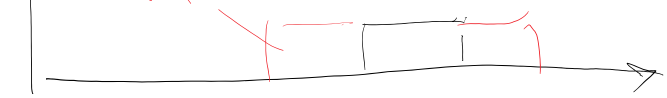


hyperbolic PDE

$\rho \ddot{u} - C u_{,xx} = 0$

$c = \sqrt{C/\rho}$

Parabolic

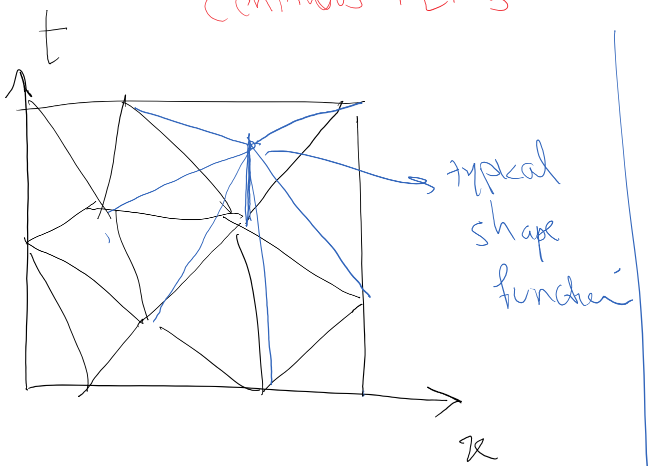


$\Delta t \propto \frac{\Delta x^2}{2\nu}$

$\dot{u} - \nu u_{,xx} = 0$

Spacetime Continuous Finite Element Methods

Continuous FEMs



The entire spacetime domain is solved simultaneously.
1D x time problem becomes a 2D globally coupled problem (Is this a bad thing)? Probably not.

Dyja_2018_Parallel-In-Space-Time, Adaptive Finite Element Framework for Nonlinear Parabolic Equations.pdf

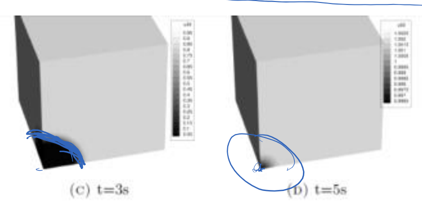


FIGURE 16. Results of computations for the 3D Allen-Cahn problem. Values of u for initial condition (a), after 1s (b), 3s (c) and 5s (d)

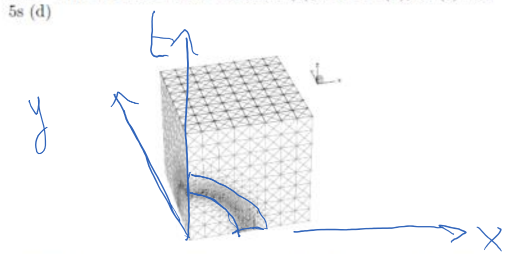
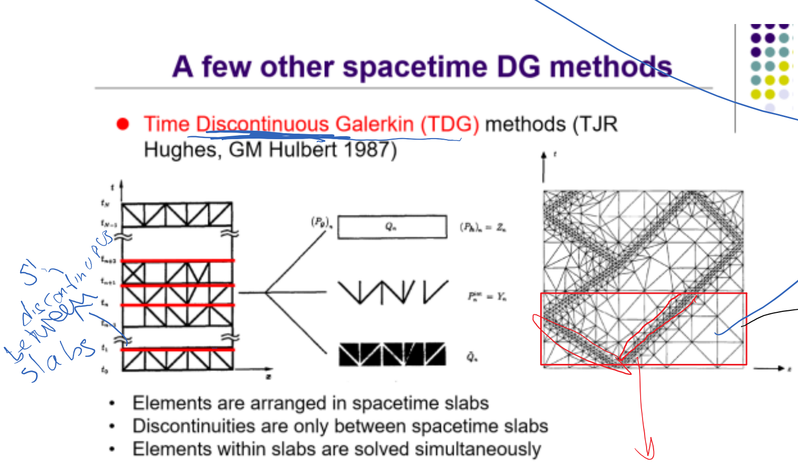


FIGURE 17. Finite element mesh after 20 refinement iterations with superimposed solution at time 1s at the dense part of mesh



within a slab we have space time continuous FEM

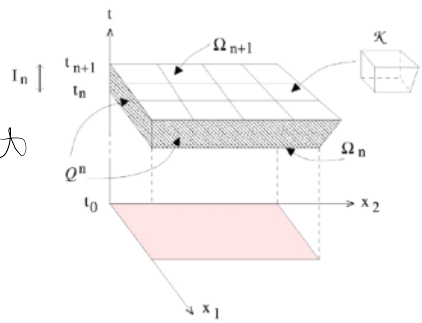
All the elements within the slab are solved simultaneously

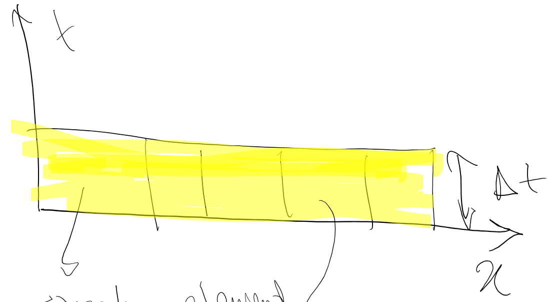
Advantage: We can do adaptive operations in spacetime to track moving wave fronts
Disadvantage: The entire slab of elements need to be solved simultaneously.

A few other spacetime DG methods

- Spacetime discontinuous Galerkin method (JJW Van der Vegt, H Van der Ven, *et al*)

DG





this whole slab need to be solve simultaneously

Implicit

No limit on Δt

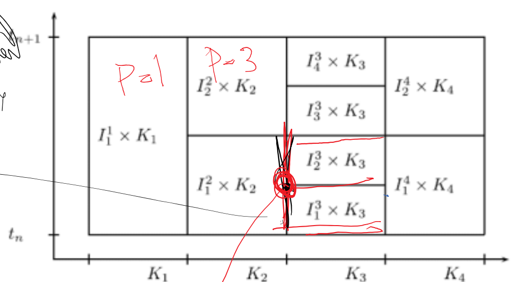
- Elements are arranged in spacetime slabs
- Discontinuities are across all element boundaries
- Elements within slabs are solved simultaneously as this method is implicit

- One shared advantage of all spacetime finite element and DG methods is that the order of accuracy in time can very easily be increased. In contrast, for time marching schemes, increasing the order of accuracy in time is difficult.

Some more advanced extruded DG meshes in spacetime

- hp-adaptive Spacetime discontinuous Galerkin method, Discontinuous in space, continuous in time (M. Lilienthal, S.M. Schnepp, and T.Weiland. Non-dissipative space-time hp-discontinuous Galerkin method for the time-dependent Maxwell equations. Journal of Computational Physics, 275:589 –607, 2014.)

h-refinement can easily be done with DG methods (space & spacetime)



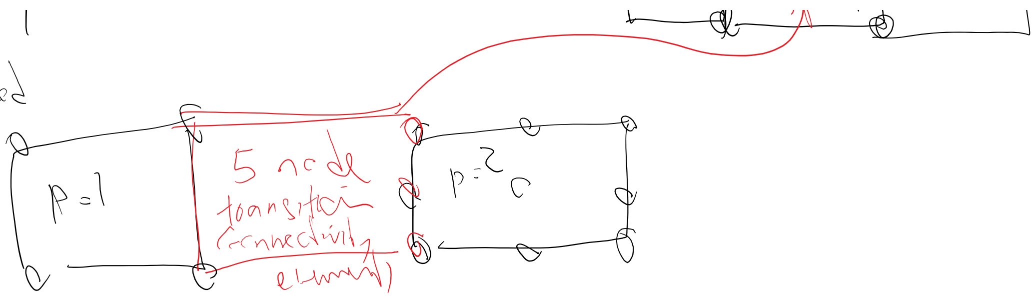
extruded DG meshes in spacetime but with nested refinement (similar to subcycling with time marching schemes)

hanging node:
Because this is a DG formulation, hanging nodes are fine & no connection elements are needed

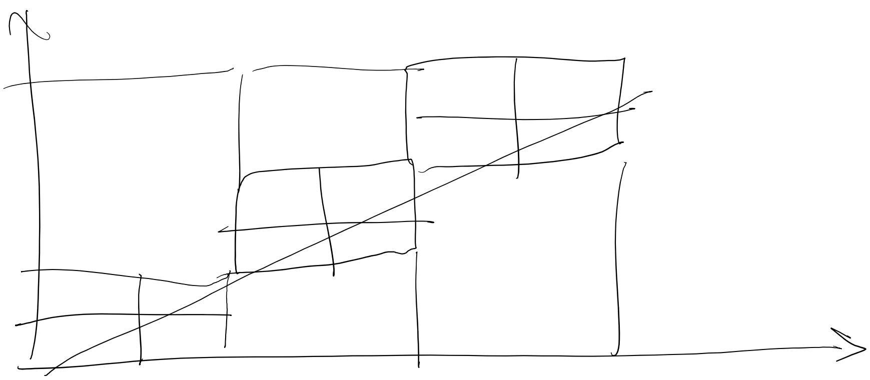
p-enrichment can also be done easily with DG method

Remember CFEM

easily with DG method



Potentially we can do



Similar to

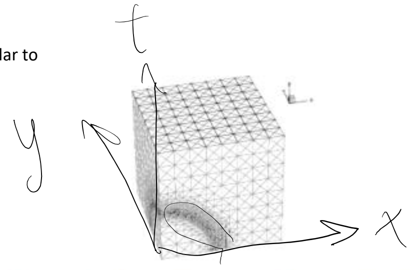


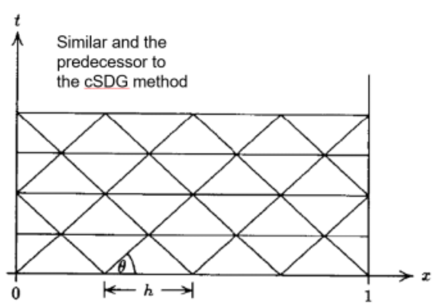
FIGURE 17. Finite element mesh after 20 refinement iterations with superimposed solution at time 1s at the dense part of mesh

Implicit spacetime methods

Explicit-like spacetime methods

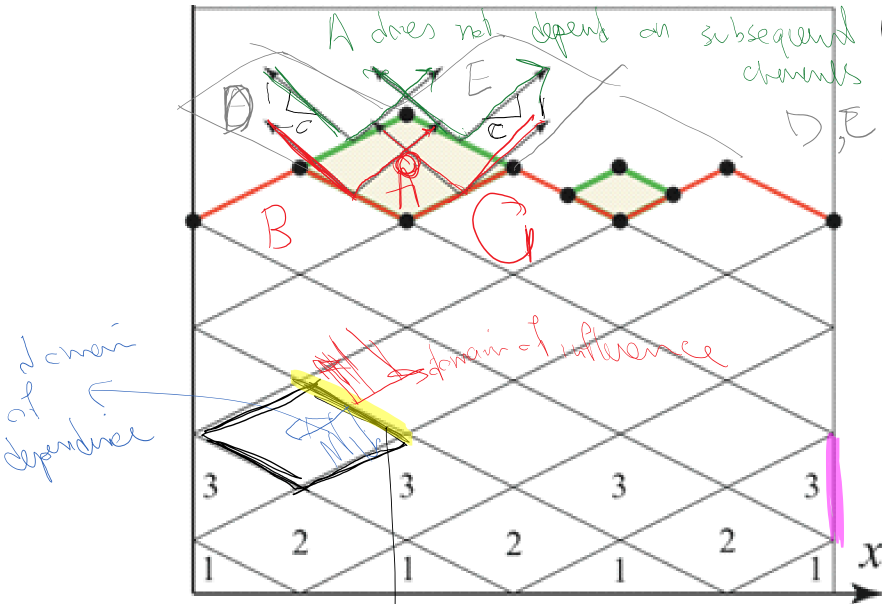
Causal spacetime meshing

- **Causal spacetime meshing** (Richter, Falk, Lowrie, Roe, Leer, Monk, Schoberl, Gopalakrishnan, etc.)
  Gerard R. Richter, An explicit finite element method for the wave equation, Applied Numerical Mathematics 16 (1994) 65-80



Richter, Falk, ... 1990

(1) – DG (~~C~~FEM)



① — DG (~~CFEM~~)

② — hyperbolic PDE

$c = \sqrt{C/\rho}$

$\rho \ddot{u} - C u_{,xx} = 0$

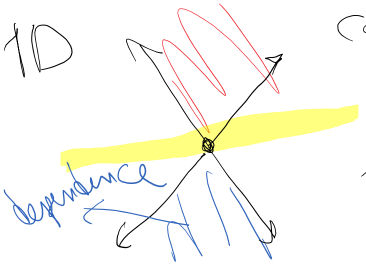
③ — Causal Mesh

Causal mesh: We can arrange elements in patches of the elements so that the boundaries of each patch are causal except when they are on the spatial boundary of the domain.

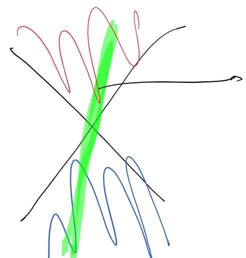
influence

causal face separates domain of influence & dependence

1D

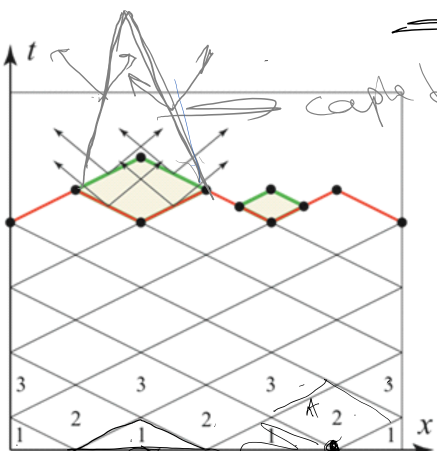


causal "space-like" face

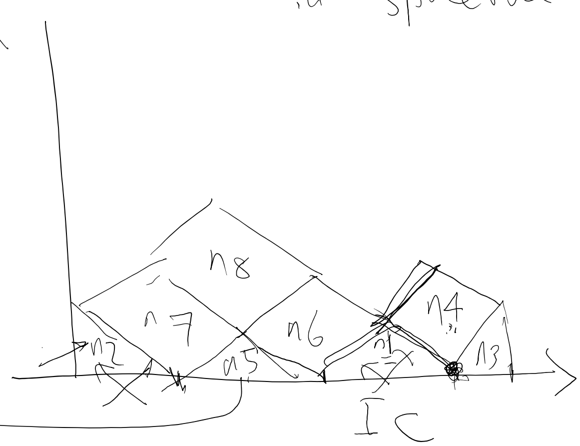


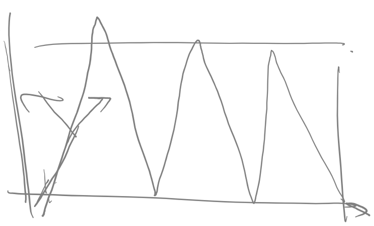
non causal "time-like" face

1 example of meshing in spacetime

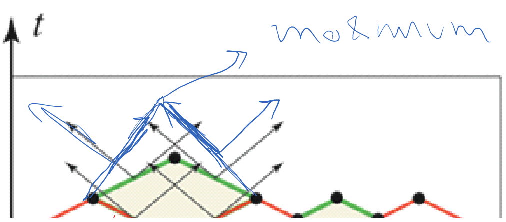


coupled to unsolved elements

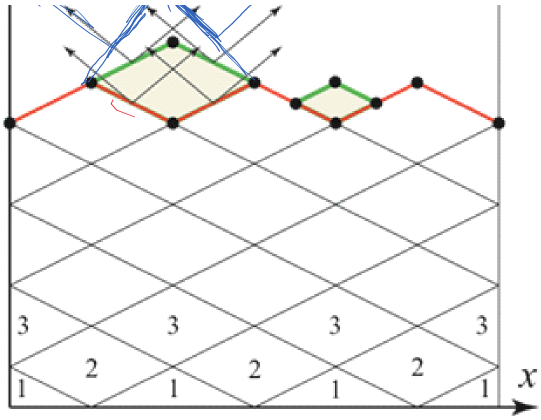




solve the whole spacetime simultaneously



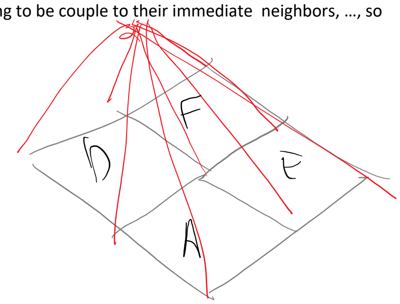
maximum time advance so that the face remain

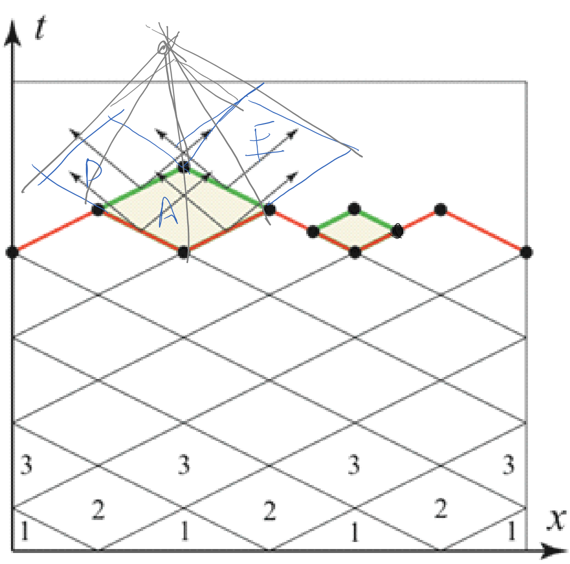


so that
the face remain
causal

the method is similar to
an explicit time marching scheme
(1 element at a time
& has stability limit)

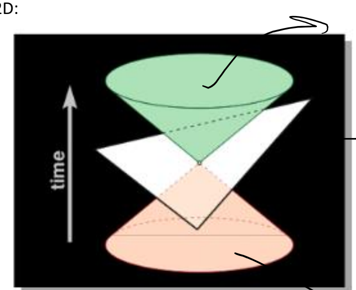
The condition of having a spacetime DG method is necessary as if CFEM was used every single element was coupled to its immedi ate neighbors (through its nodes) because of using continuous shape functions and those elements were going to be couple to their immediate neighbors, …, so the entire spacetime domain will become coupled!





influence

2D:



causality constraint

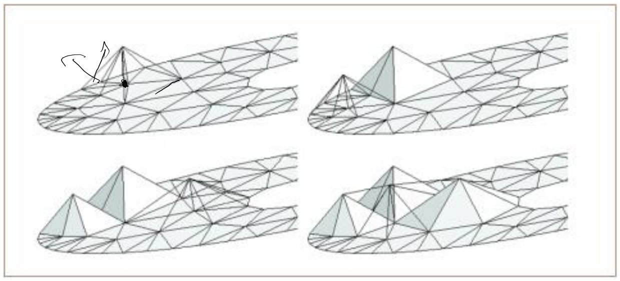
d.
dependence

causal face
separating d. influence
& d. of dependence

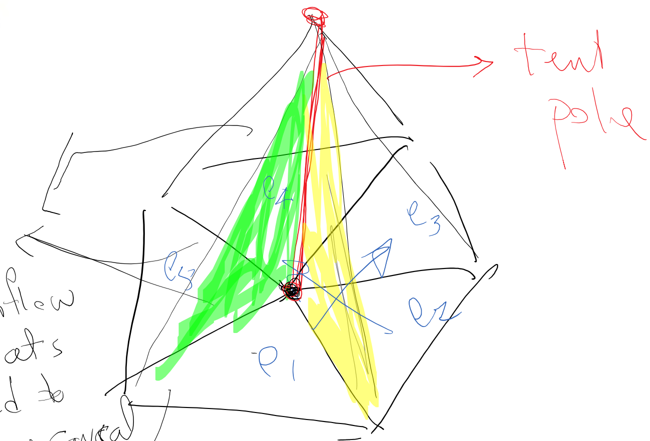
Tent Pitcher:
Patch–by–patch meshing

- meshing and solution are interleaved
  - patches ('tents') of tetrahedra are solve immediately ⇨ O(N) property
  - rich parallel structure: patches can be created and solved in parallel

t



tent–pitching sequence



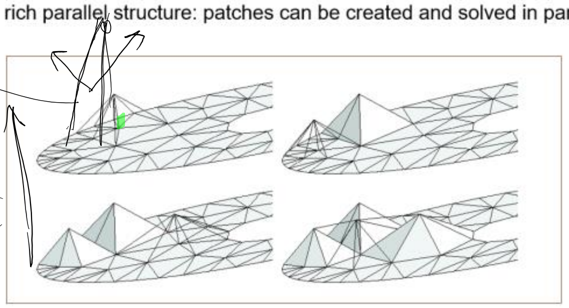
patch of 5 tetrahedral elements

Tent Pitcher:
Patch–by–patch meshing

- meshing and solution are interleaved
  - patches ('tents') of tetrahedra are solve immediately ⇨ O(N) property
  - rich parallel structure: patches can be created and solved in parallel

non causal

t



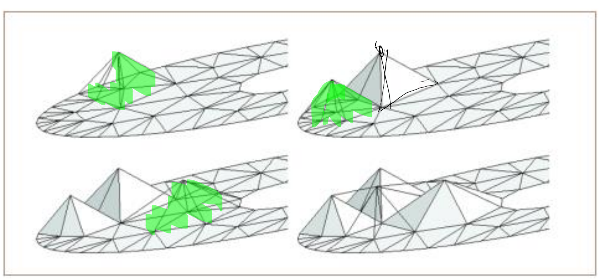
tent–pitching sequence

Why the method looks like an explicit time marching scheme?
- The time advance of the vertices is limited so that the outflow facets of the patch remain causal (except those on the spatial boundary of the domain).
- Like explicit time marching schemes a small problem rather than the whole domain is solved at a time (here a patch of elements).

What does the method share with implicit solvers?
- The solution within a patch is implicit.

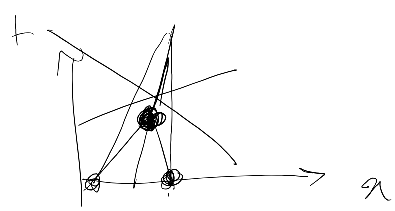
Tent Pitcher:
Patch–by–patch meshing

- meshing and solution are interleaved
  - patches ('tents') of tetrahedra are solve immediately ⇨ O(N) property
  - rich parallel structure: patches can be created and solved in parallel



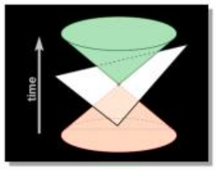
tent–pitching sequence

patches are created from local minimum

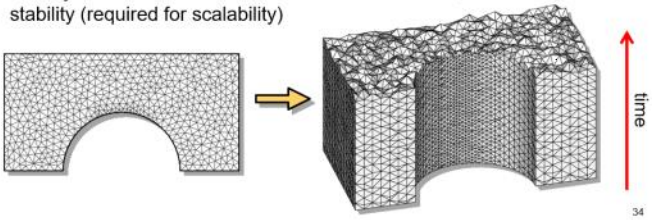


- Given a space mesh, Tent Pitcher constructs a spacetime ~~mesh such~~

- Given a space mesh, Tent Pitcher constructs a spacetime mesh such that the slope of every facet on a sequence of advancing fronts is bounded by a causality constraint
- Similar to CFL condition, except entirely local and not related to stability (required for scalability)



causality constraint



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There is also a progress constraint that ensures meshing in spacetime never gets to a lock situation where no more progress can be made.