DG2020/04/22

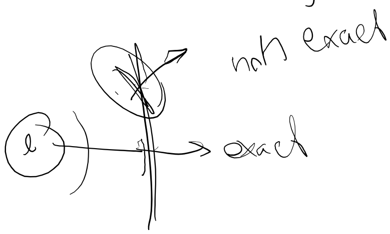
Wednesday, April 22, 2020 11:37 AM

For linear hyperbolic PDEs if the impedances match between two media, the reflection coefficient is zero:

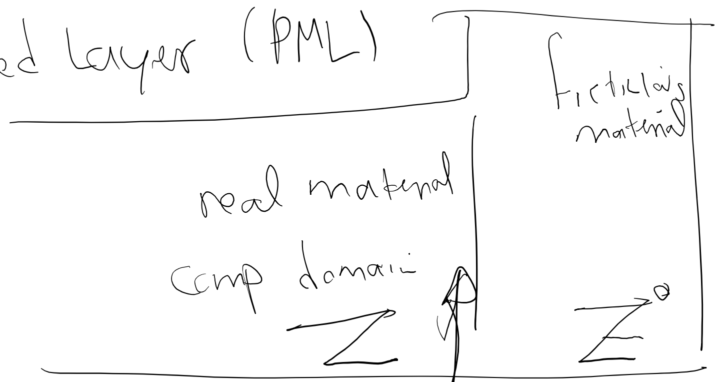
$$v^* = \frac{(\sigma^+ - \sigma^-) + (Z^+ v^+ + Z^- v^-)}{Z^- + Z^+}$$

$$\sigma^* = \frac{(Z^- \sigma^+ + Z^+ \sigma^-) + Z^- Z^+ (v^+ - v^-)}{Z^- + Z^+}$$

from last time Silver–Muller transmitting BC is exact only for normal incidence & 1D setting.



Perfectly Matched Layer (PML)



if $Z = Z^0$ no reflection here

Elastodynamics isotrop $\quad Z_d = \sqrt{(\lambda + 2\mu)\rho}$ normal

$Z_s = \sqrt{\mu \rho}$ shear

Acoustic $\quad Z = \sqrt{K\rho}$ normal
(K: bulk modulus)

Electromagnetics $\quad Z = \sqrt{\mu / \varepsilon}$ (μ: magnetic permeability, ε: electric permittivity)

Or lossless material $\quad \varepsilon, \mu$ real for a lossless material ε, μ real

$$Z = \sqrt{\mu/\varepsilon} \qquad \overset{\text{out}}{Z^\circ} = \sqrt{\frac{\mu^\circ}{\varepsilon^\circ}}$$

real material

$\mu^\circ = \mu\, s(\omega)$ ← frequency

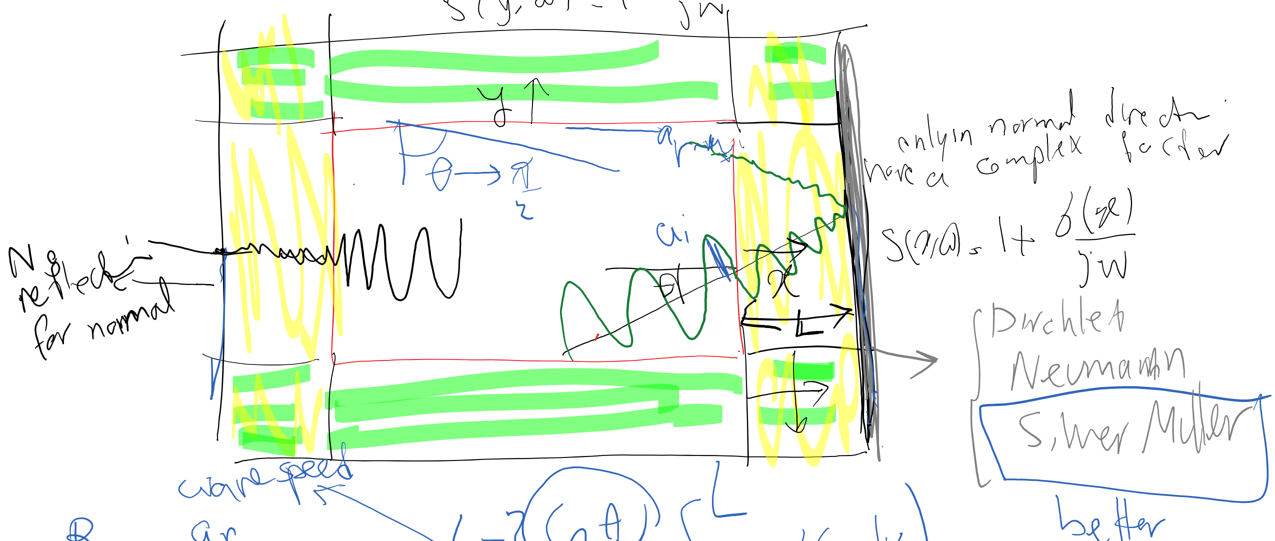
x $\varepsilon^\circ = \varepsilon\, \underbrace{s(\omega)}_{\text{stretch factor}}$

$\rightarrow \boxed{Z^\circ = Z}$

to make this lossy
$s(\omega)$ is complex

s(w) being complex is the whole idea of complex stretching (Chow).

$$s(y,\omega) = 1 + \frac{\sigma(y)}{j\omega}$$



$$R = \frac{a_r}{a_i} \approx \exp\left(\frac{-2\cos\theta}{c}\int_0^L \sigma(x)\,dx\right)$$

Dirichlet for end

$$\theta = 0 \Rightarrow R = \exp\left(-\frac{2}{c}\int_0^L \sigma(x)\,dx\right)$$

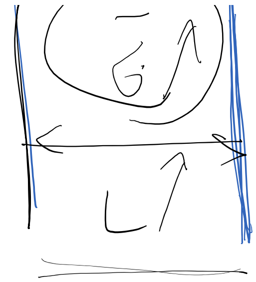
as $\theta \to \frac{\pi}{2}$ $R \to 1$
$\cos\theta \to 0$

another point $\sigma(x) = \text{const}$ $\rightarrow$ $\cos\theta$

another point $\sigma(x) = \text{const}$

$\sigma = \bar{\sigma}$

$\bar{\sigma} = 0$



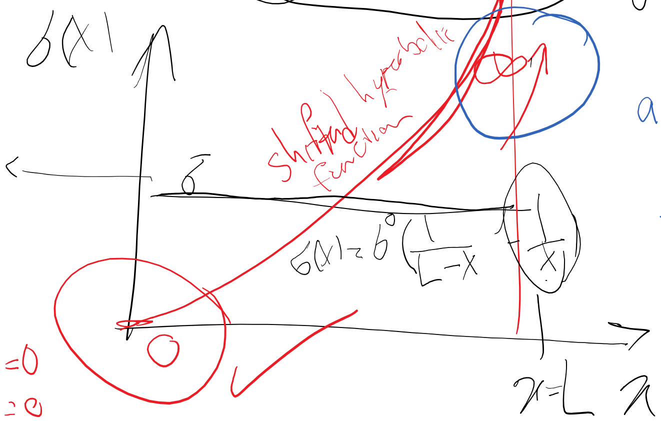
$$R = \exp\left(-\frac{2\cos\theta}{c}\,\bar{\sigma}\,L\right)$$

$\theta \to 0$:) smallest reflection

parameters $\bar{\sigma}$ & L ↗ again R ↘

$$\boxed{\int_0^L \sigma(x)\,dx}$$

how about a profile of $\sigma(x)$ for which goes to ∞



constant model

smooth transition from real to fictitious material :)

$\bar{\sigma}(x) = 0$ @ $x = 0$

allows $\int_0^L \sigma(x)\,dx \to \infty$:)

$\boxed{R \to 0}$

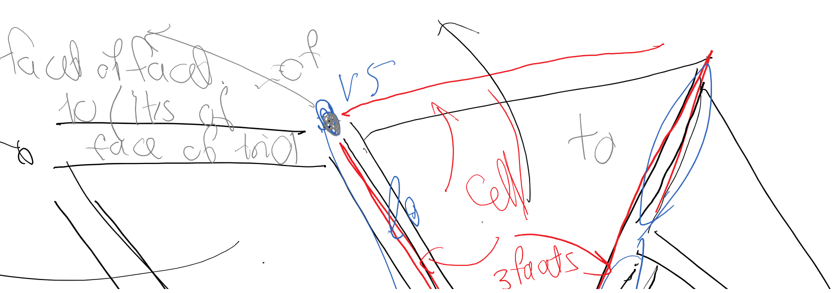
Implementation of a DG method / FEM:

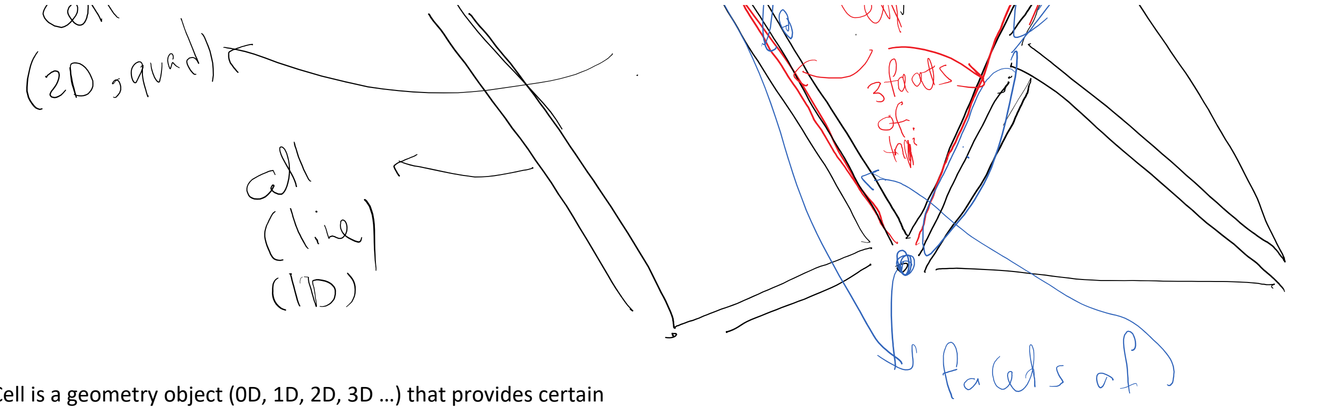
cell (2D, tri)

Geometry Cell

cell (vertex (0D))

cell (2D, quad)

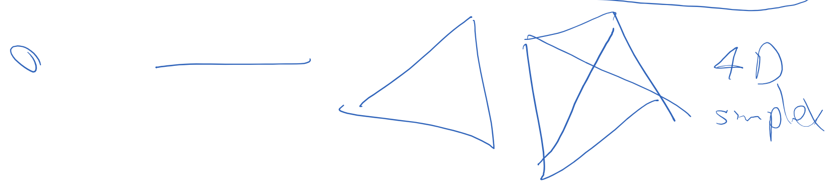




Cell is a geometry object (0D, 1D, 2D, 3D …) that provides certain functionalities.

{ f0 is a coface of l0
{ l0 is a facet of f0
f0 is a coface of v5
v5 is a face of f0

// simplex cells
class GCellSimplex : public GCell



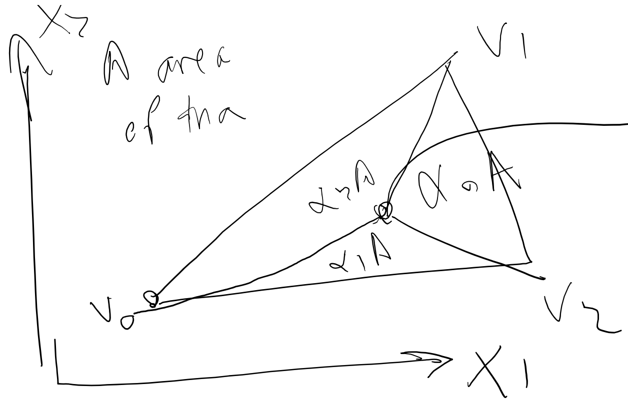

```
class GCellSimplex : public GCell
{
public:
    GCellSimplex(int geomCellOrderIn = 1, GCellID idIn = -1);
    // other functions used in initialization
    // the order of facets is important. It helps set the orver of vertices
    virtual void setFacetsReadingMesh(vector<GCellID>& facetIDs, GCellH mesh);

    virtual void TransferBaseFacetQuadCoord_2_BaseCofacetOrNbrQuadCoord(GCellH facetBase, const GQuadCoord& facetQuadCrd, GQuadCoord& cofacetOrNbrQuadCrd);
    virtual void ComputeX_dXdAlpha_BaseCell(GCellH actualCell_no_b2t, GQuadCoord& quadCrd, vector<double>& X, GCellGeomProp& geomPropOut, bool compute_dX_dAlpha, bool computeX);
    virtual void Compute_sdxFacet_from_sdxMatrix_in_Cofacet(GCellH facetBase, GCellGeomProp& geomPropOut);
};
```

$\alpha \longleftrightarrow X$

Main things about each geometry

— Natural coordinate for that element α (Alpha)

— transfer rules between $\alpha \longleftrightarrow X$ global coordinate



global coordinate

$X = (X_1, X_2)$ $\qquad 0 \le \alpha_i \le 1$

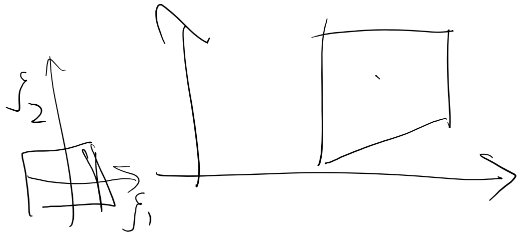
$\sum \alpha_i = 1$

$\alpha = (\alpha_0, \alpha_1, \alpha_2)$

most often we drop α_0 or last α

$\sum \alpha = 1$

$\alpha = (\alpha_1, \alpha_2)$ or (α_0, α_1)

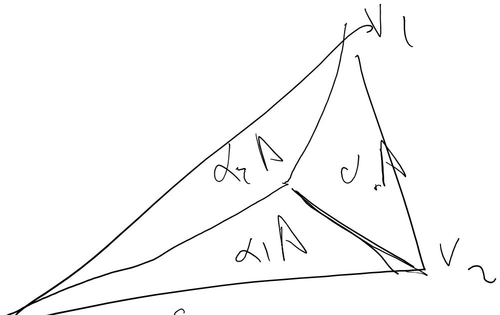


$\alpha = (\xi_1, \xi_2)$

$-1 \le \xi_i \le 1$

Coordinate transformation rules

$$X \longleftrightarrow A$$



V_0 $\quad X^0 = \begin{pmatrix} X_0^0 \\ X_1^0 \end{pmatrix}$

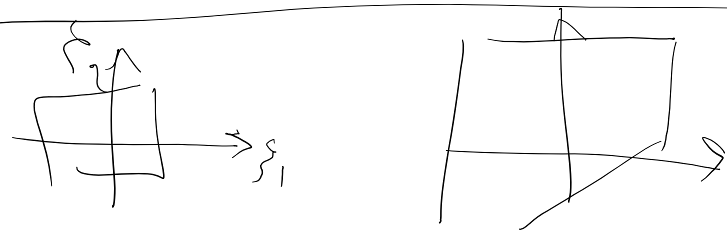
$$X = X^0 \alpha_0 + X^1 \alpha_1 + X^2 \alpha_2$$

$$= X^0 (1 - \alpha_1 - \alpha_2) + X^1 \alpha_1 + X^2 \alpha_2$$

$$= X^0 + \underbrace{\left[X^1 - X^0 \;\middle|\; X^2 - X^0 \right]}_{A} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$$

$\alpha \rightarrow X$ $\qquad \boxed{X = X^0 + A\alpha}$

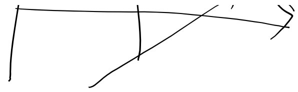
$X \rightarrow \alpha$ $\qquad \boxed{\alpha = A^{-1}(X - X^0)}$



$X = \sum X^i N^i(\xi_1, \xi_2)$

CFEM shape function

1 + 3,



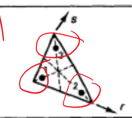
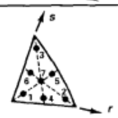
CFEM shape function

$A = (\xi, \eta) \to X$

$X \to A$ more difficult (N-Raphson) for high order element

Quadrature rules for each element

TABLE 5.8 *Gauss numerical integrations over triangular domains* $[\iint F\,dr\,ds = \frac{1}{2}\sum w_i F(r_i, s_i)]$

Integration order	Degree of precision	Integration points	r-coordinates	s-coordinates	Weights
3-point	2		$r_1 = 0.16666\ 66666\ 667$ $r_2 = 0.66666\ 66666\ 667$ $r_3 = r_1$	$s_1 = r_1$ $s_2 = r_1$ $s_3 = r_2$	$w_1 = 0.33333\ 33333\ 333$ $w_2 = w_1$ $w_3 = w_1$
7-point	5		$r_1 = 0.10128\ 65073\ 235$ $r_2 = 0.79742\ 69853\ 531$ $r_3 = r_1$ $r_4 = 0.47014\ 20641\ 051$ $r_5 = r_4$ $r_6 = 0.05971\ 58717\ 898$ $r_7 = 0.33333\ 33333\ 333$	$s_1 = r_1$ $s_2 = r_1$ $s_3 = r_2$ $s_4 = r_6$ $s_5 = r_4$ $s_6 = r_4$ $s_7 = r_7$	$w_1 = 0.12593\ 91805\ 448$ $w_2 = w_1$ $w_3 = w_1$ $w_4 = 0.13239\ 41527\ 885$ $w_5 = w_4$ $w_6 = w_4$ $w_7 = 0.225$
13-point	7		$r_1 = 0.06513\ 01029\ 022$ $r_2 = 0.86973\ 97941\ 956$ $r_3 = r_1$ $r_4 = 0.31286\ 54960\ 049$ $r_5 = 0.63844\ 41885\ 698$ $r_6 = 0.04869\ 03154\ 253$ $r_7 = r_5$ $r_8 = r_4$ $r_9 = r_6$ $r_{10} = 0.26034\ 59660\ 790$ $r_{11} = 0.47930\ 80678\ 419$ $r_{12} = r_{10}$ $r_{13} = 0.33333\ 33333\ 333$	$s_1 = r_1$ $s_2 = r_1$ $s_3 = r_2$ $s_4 = r_6$ $s_5 = r_4$ $s_6 = r_5$ $s_7 = r_6$ $s_8 = r_5$ $s_9 = r_4$ $s_{10} = r_{10}$ $s_{11} = r_{10}$ $s_{12} = r_{11}$ $s_{13} = r_{13}$	$w_1 = 0.05334\ 72356\ 088$ $w_2 = w_1$ $w_3 = w_1$ $w_4 = 0.07711\ 37608\ 903$ $w_5 = w_4$ $w_6 = w_4$ $w_7 = w_4$ $w_8 = w_4$ $w_9 = w_4$ $w_{10} = 0.17561\ 52574\ 332$ $w_{11} = w_{10}$ $w_{12} = w_{10}$ $w_{13} = -0.14957\ 00444\ 677$

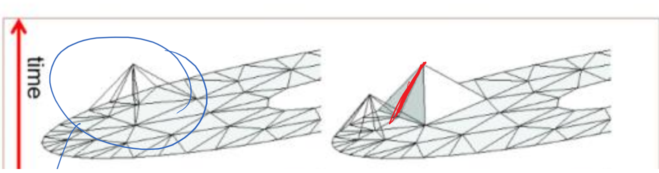
number of quadr.

needed order of integration

In our DG code
physics\PhyGauss.h

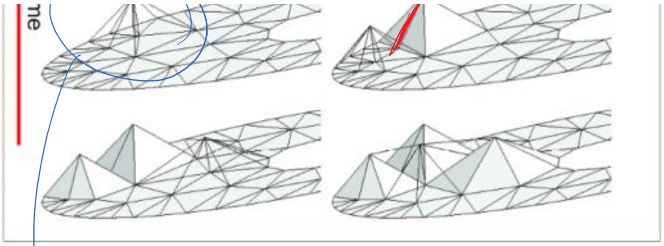
```
void getQuadPts(int p, V2TENSOR& gaussPts, VECTOR &gaussWts);
```

We'll have two hierarchies one is element hierarchy and one integration cell hierarchy.



elliptic

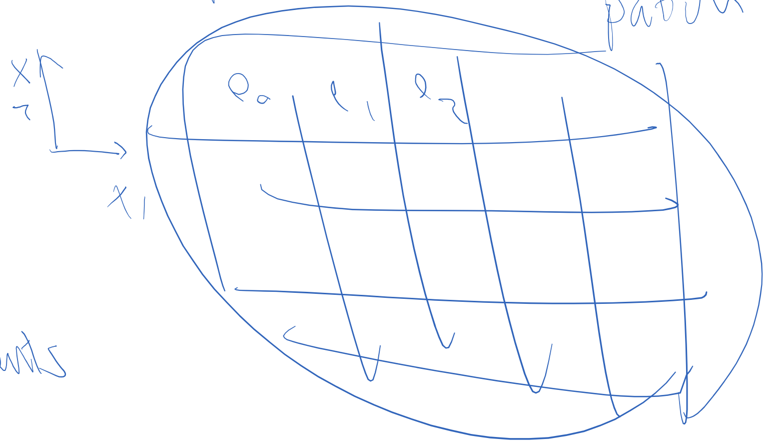
patch



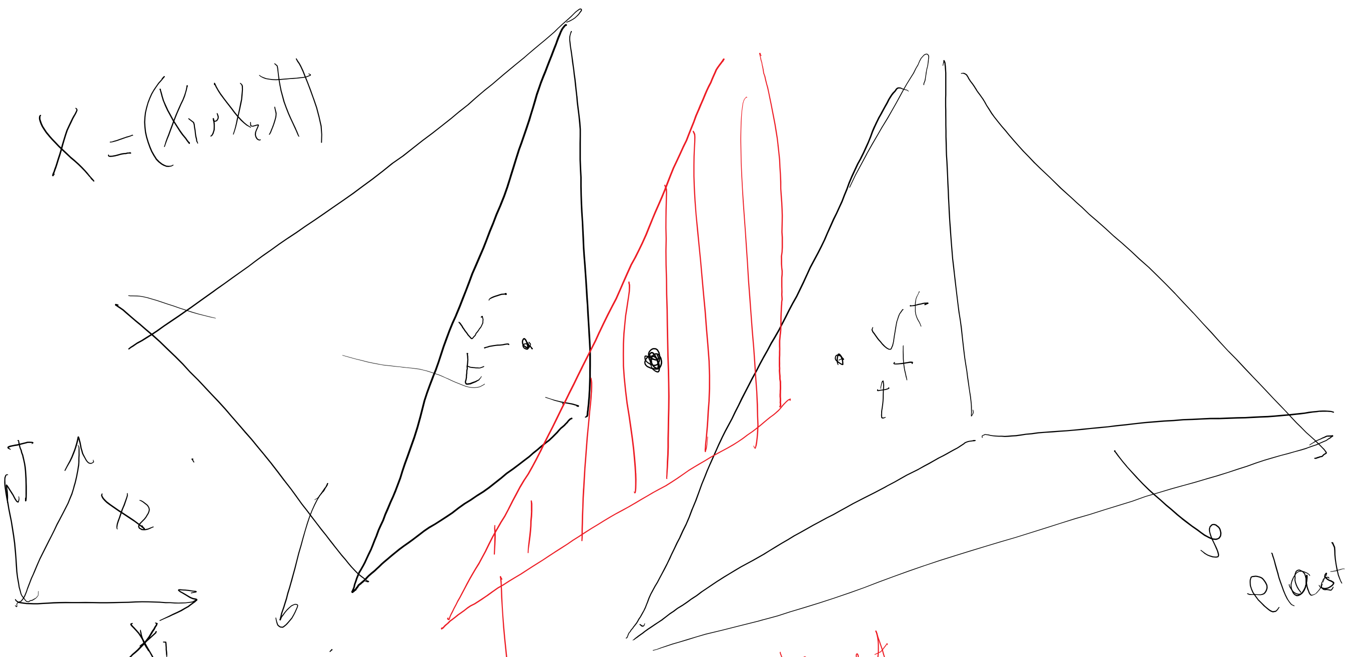
tent–pitching sequence

patch: 1 solution unit
contains coupled elements

elliptic



$X = (X_1, X_2, T)$



elastodynamics in the bulk

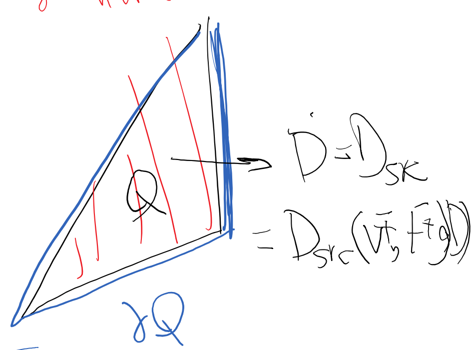
interfacial element
for example we can have damage here

elastodynamics

Damage weighted residual

$$\underbrace{\int_Q \hat{D}(\dot{D} - D_{SK})\, dV}_{I_{1D}} + \underbrace{\int_{\partial Q} \hat{D}(D^* - D)\, n_t\, ds}_{B_{1D}} = 0$$

it's only on ODE



$\dot{D} = D_{SK} = D_{SK}(\bar{v}_t, \bar{F}, D)$

~~weak eqn~~ for elastodynamics

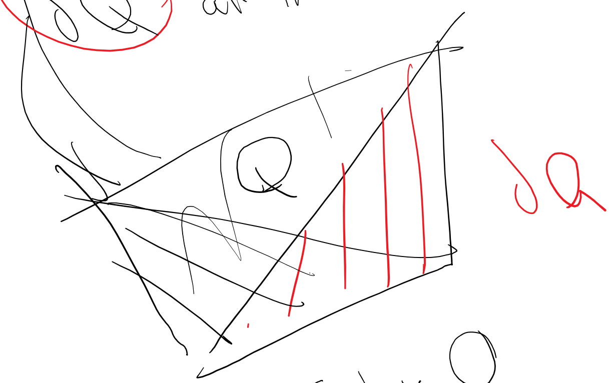
$$\int_Q \left(\hat{\dot{u}}\, \dot{p} + \nabla \hat{u} \cdot \sigma + \hat{u}\, \rho b \right) dV \qquad I_{2D}$$

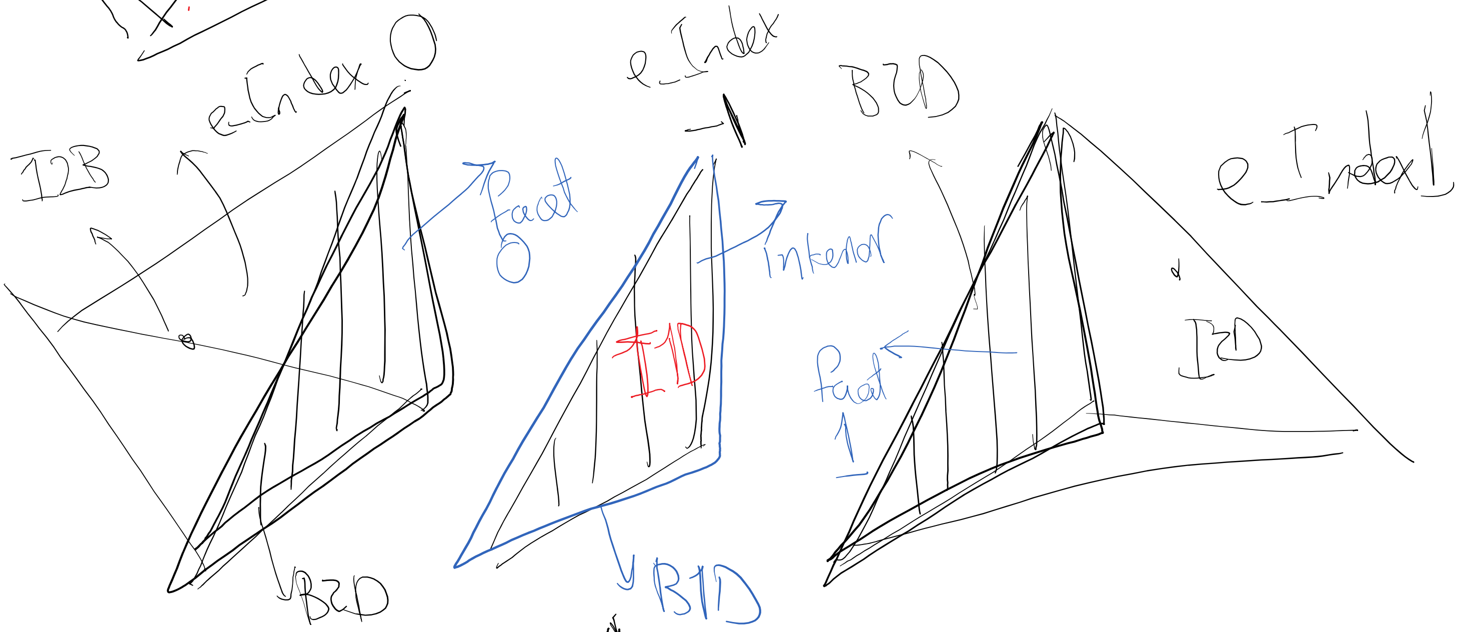
$$\int \left(\ddot{u} p + \nabla u q + u g b \right) dV \quad + \cdots 2D)$$

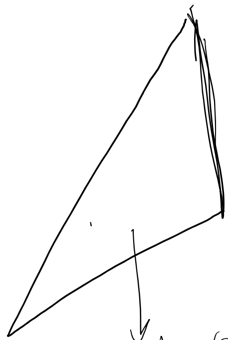
$$\int_{\partial Q} \left(-\hat{u} p n + \hat{u} \sigma^{*} n_x + \underbrace{[E] \hat{\sigma} n_t + [\dot{u}] \hat{\sigma} n_x}_{\text{comptability between}} + \hat{u} [u] n_t \right) ds$$

∂Q all the facets

compatibility between $E = \tilde{\nabla} u$ & $v = \dot{u}$ and $\underbrace{B2D}$

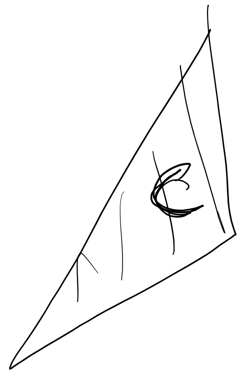






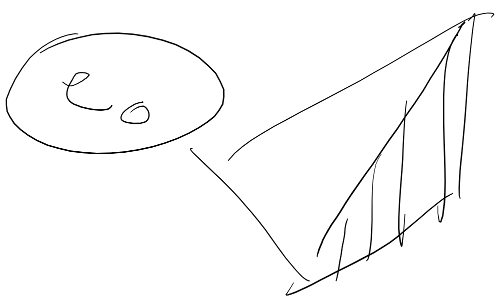
integration cell

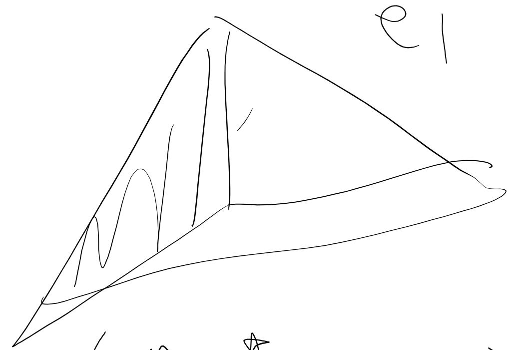
Integrates $\underline{I}$ interior of the element



$$\int_e \hat{D}(D - D_{src})\,dv = 0$$

+ 2 face integrations





$\int_e (\hat{u}\, \zeta\, n_x^* \ldots\ldots)$ $\qquad$ $\int_e (\hat{u}\, \zeta\, n_x^* \ldots\ldots)$

e_0

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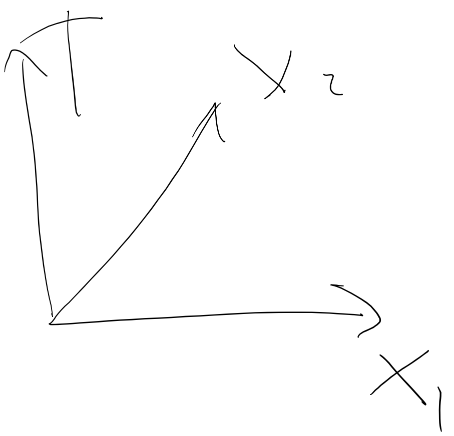
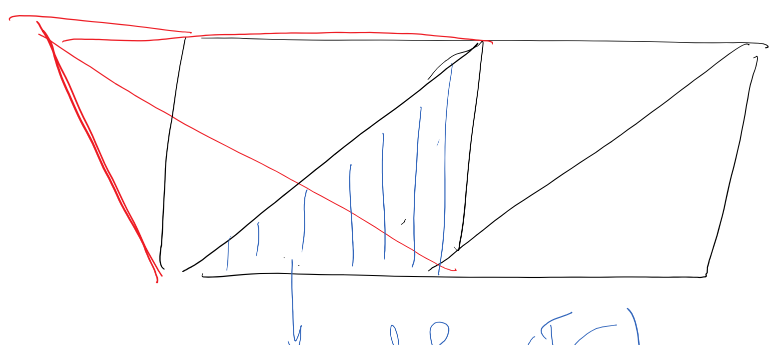
other examples

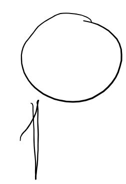




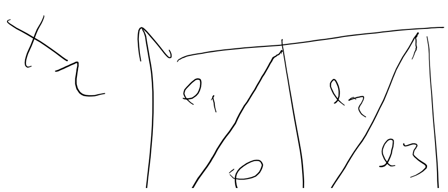
initial face (IC)

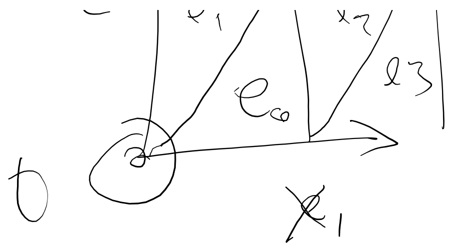
interior integrations (#I)

face integrations (#F)

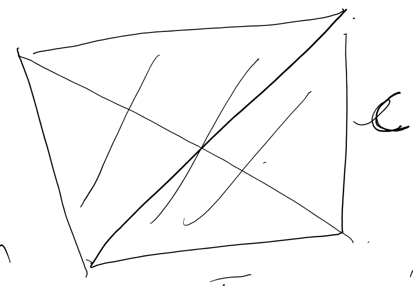


interior face of e..



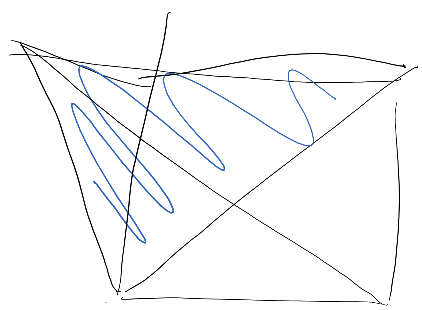


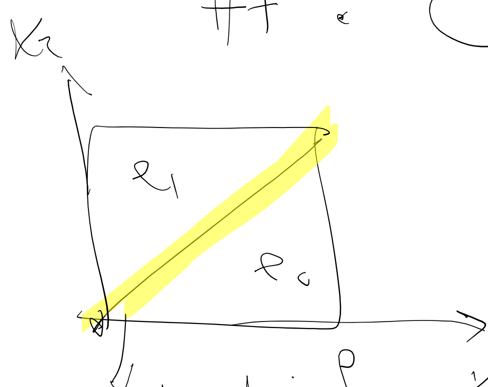
tetrahedron



#I : 1

#F : 0





integration of interior face between e_0 & e_1 with no fracture

outflow face

#I

#F

boundary facet

#I

#F