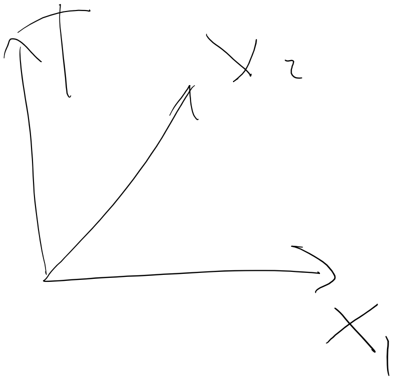
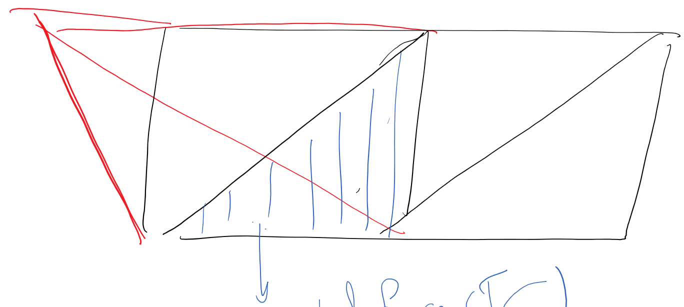
DG2020/04/27

Monday, April 27, 2020 11:24 AM

other examples



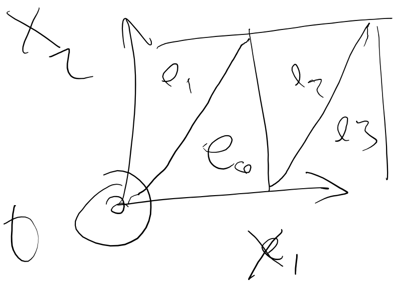


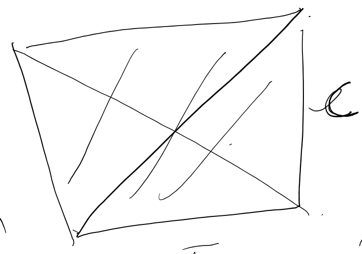
initial face (IC)

interior integrations (#I) 0

face integrations (#F) 1

inflow face of e_0





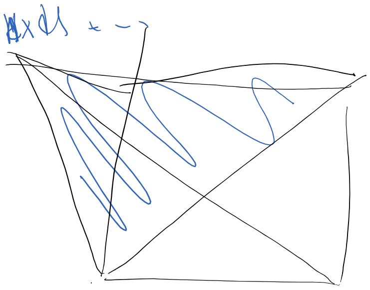
tetrahedron

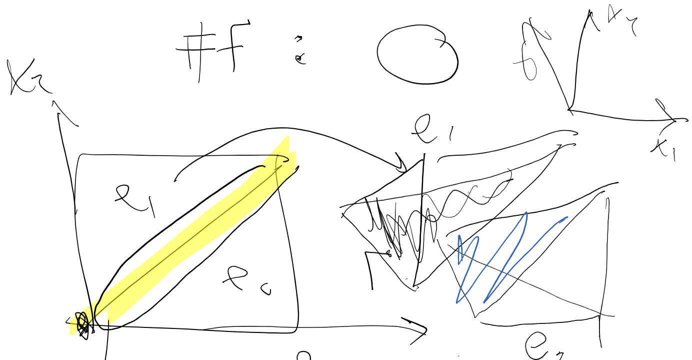
#I : 1

#F : 0

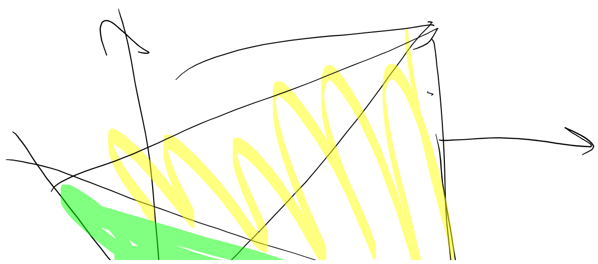
$\int_{\partial e_1 \cap \Gamma} \omega^{e_1} (\delta^{*} - \delta^{e_1}) \, dx\, ds + \dots$

$\int_{\partial e_2 \cap \Gamma} \omega^{e_2} (\delta^{*} - \delta^{e_2}) \, dx\, ds + \dots$





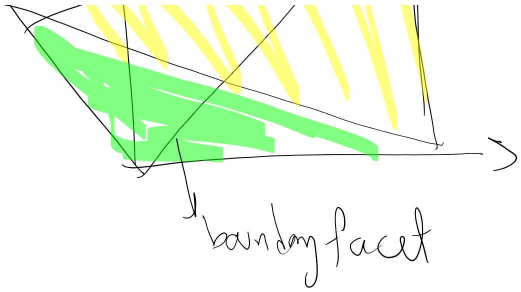
integration of interior face between e_0 & e_1 with no fracture



outflow face

#I

#F



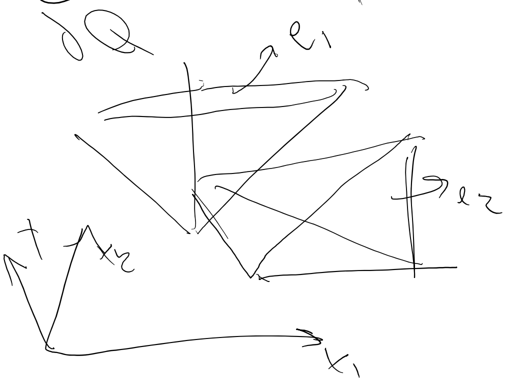
#I
#F

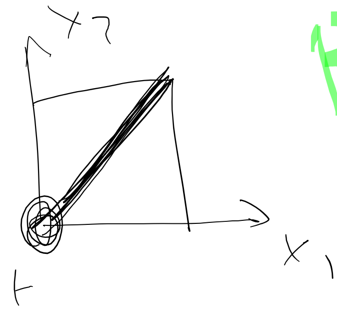
#I
#F

Interior

$$\int_Q \left(\hat{\dot{u}} p + \nabla \hat{\dot{u}} \sigma + \hat{\dot{u}} \rho b\right) dv \qquad I_{2D}$$

$$\int_{\partial Q} \left(-\hat{\dot{u}} p n_t + \hat{\dot{u}} \sigma n_x + [E] \hat{\sigma} n_t + [\dot{u}] \hat{\sigma} n_x + \hat{\dot{u}} [u] n_t\right) ds$$

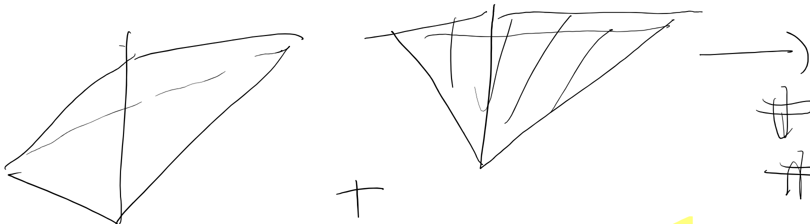




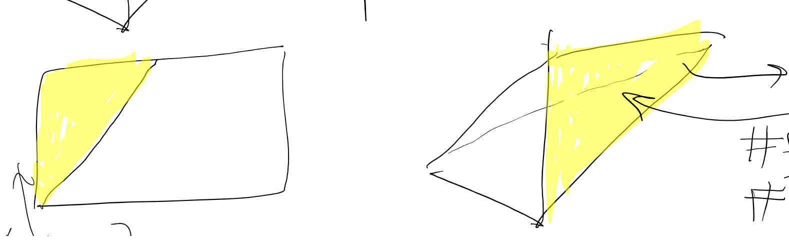
Boundary

Element centered integration

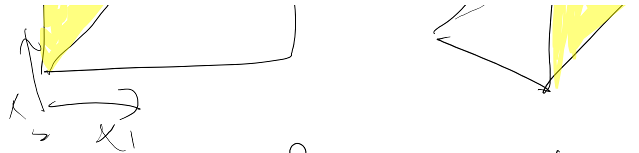
Take care of e_1 first



Interior integration
#F = 0
#I = 1



inflow face
#F → 1
#I → 0

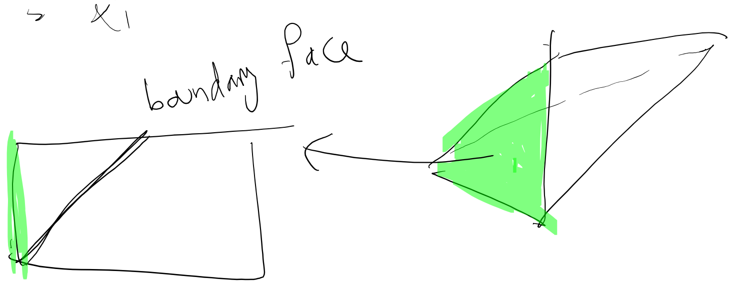


x_2 x_1

$\#+ \longrightarrow 1$

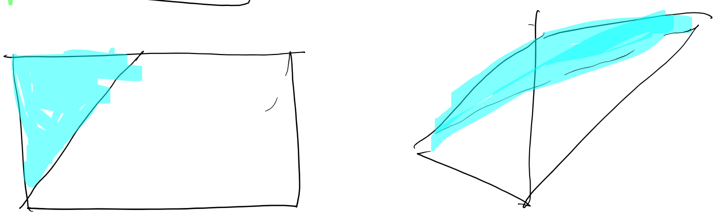
$\#I \longrightarrow 0$

boundary face



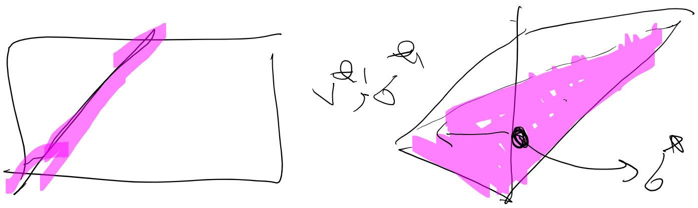
$\#F \longrightarrow 1$

$\#I \longrightarrow 0$



$\#F \rightarrow 1$

$\#I \longrightarrow 0$



v, σ^{e_1} e_1 σ^{e}

e_1 e_2

v, σ

$\boxed{\begin{array}{l}\#F = 2 \\ \#I = 0\end{array}}$

using PIC

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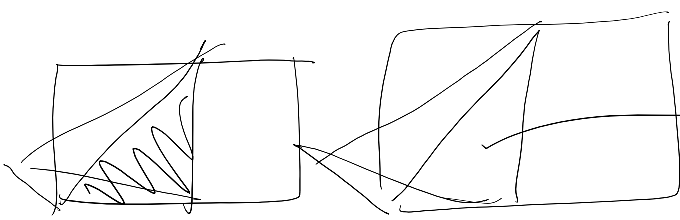
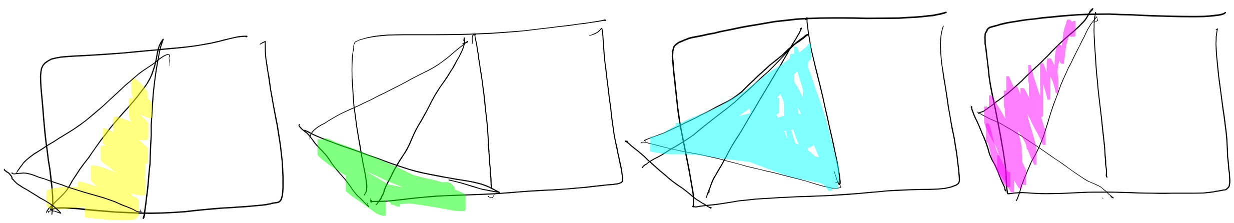
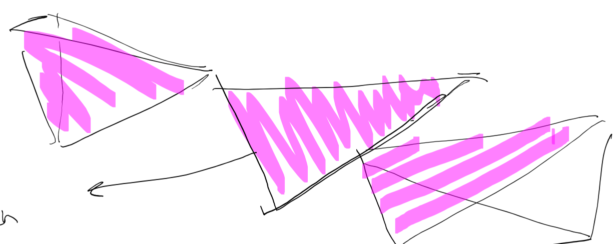
same for e_2



volume integration

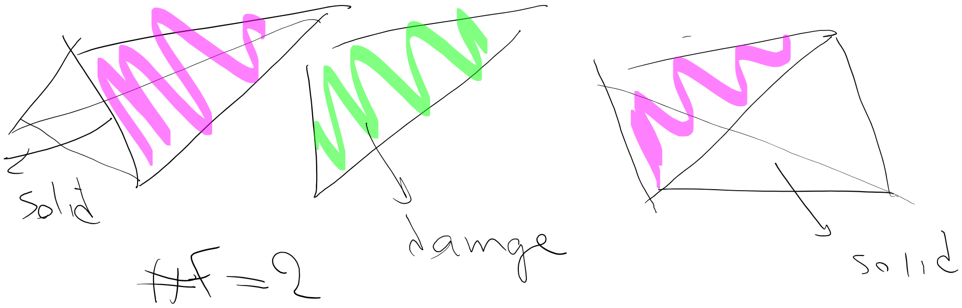




integration cell is a better idea

it integrates 2 facets of e_1 & e_2 here

○ interior integration

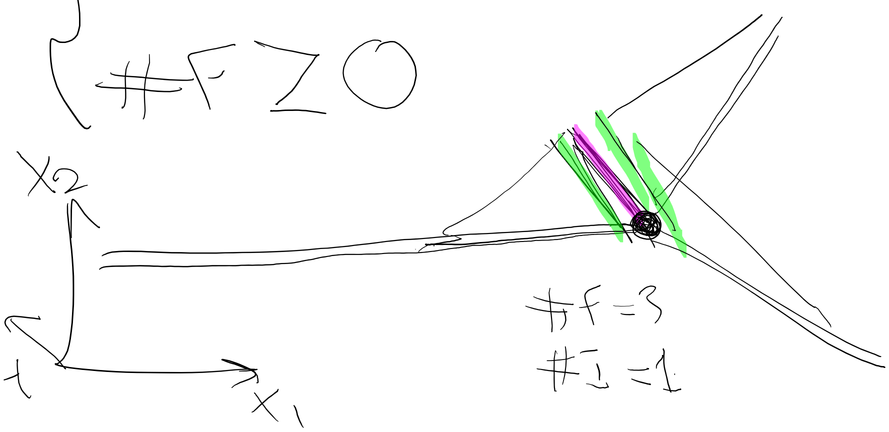


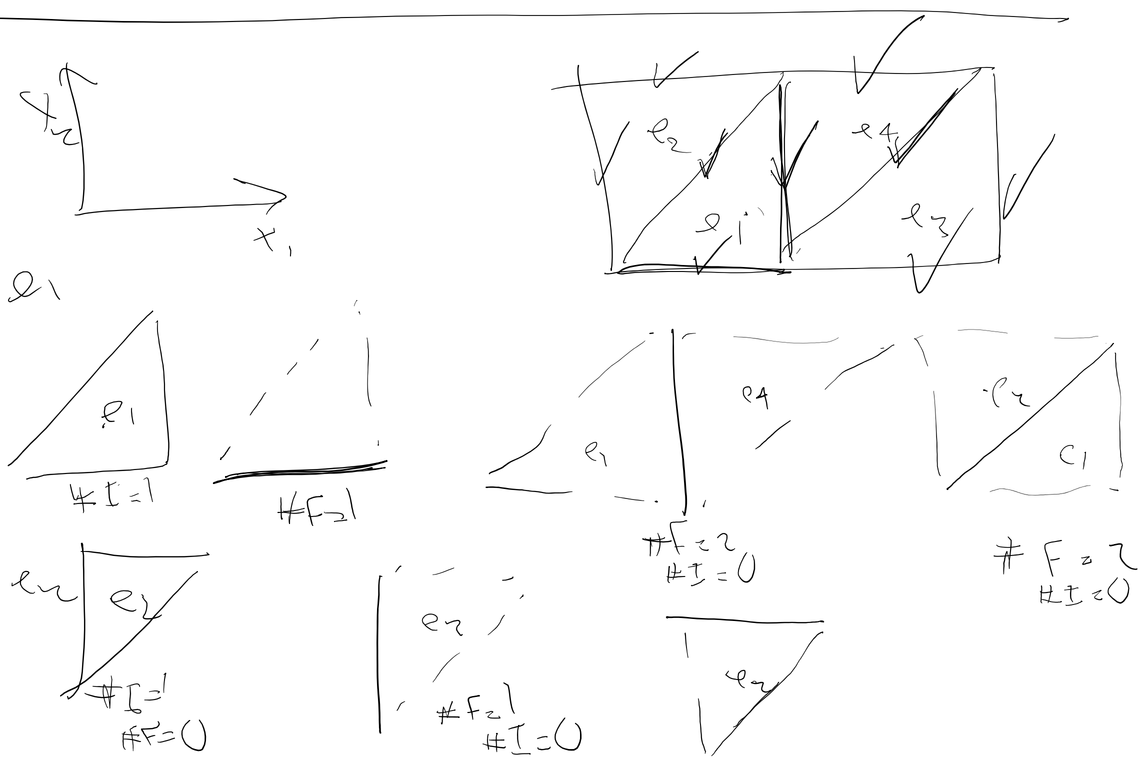
#F = 2

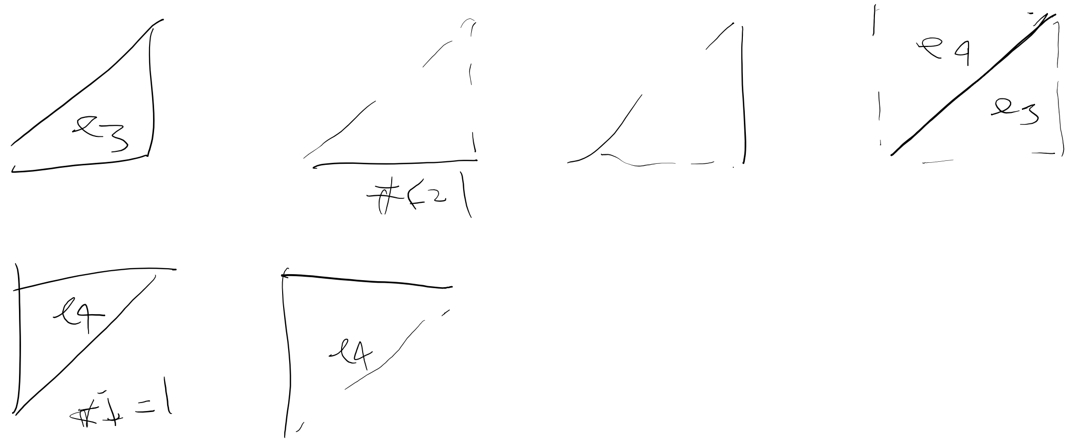
#I = 1

Most general integration cell for FEM implementation:

$$\begin{cases} \#I & 0 \text{ or } 1 \\ \#F \geq 0 \end{cases}$$

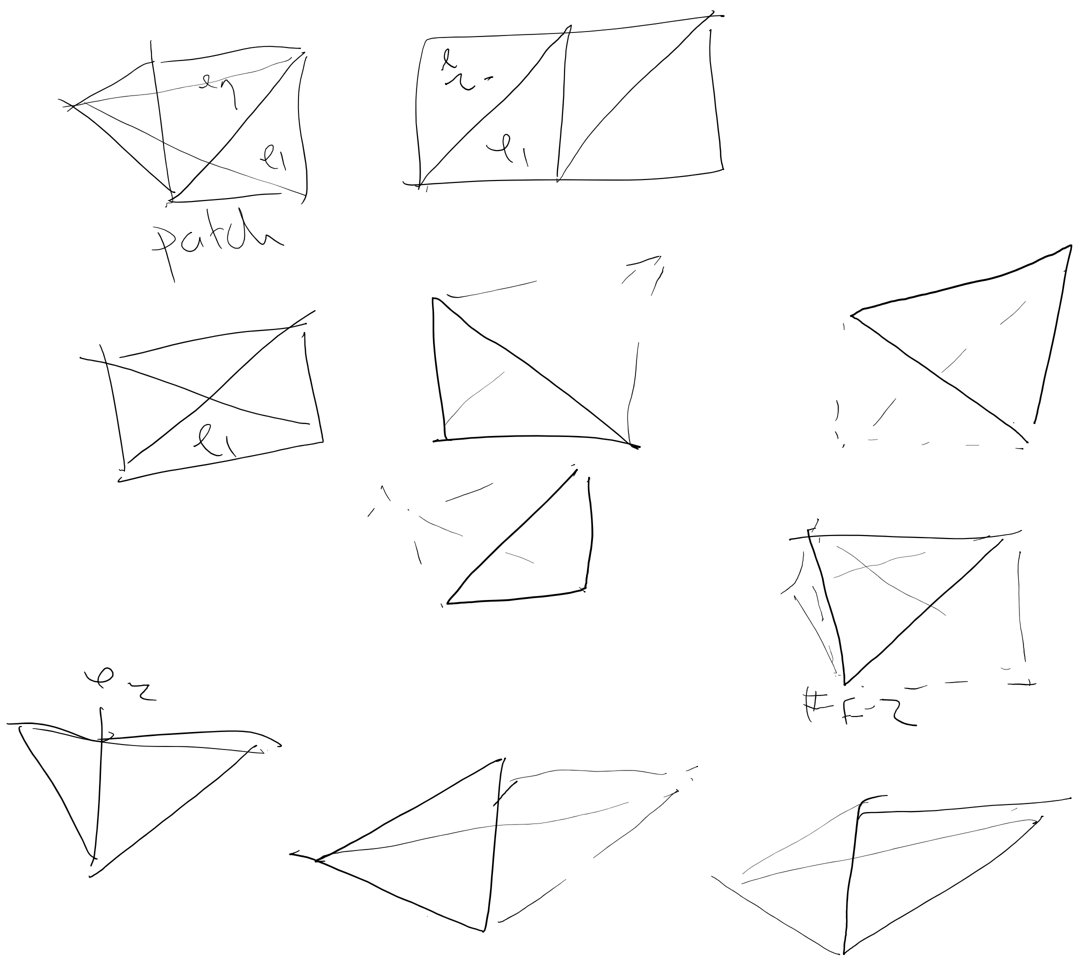


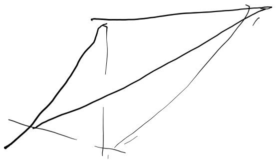




$\# PIC_3 = 13$

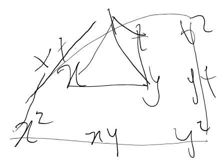
For hyperbolic PDE, patch (the unit of s/n is smaller)





Solving apotch

Loop over integration cells & integrate them



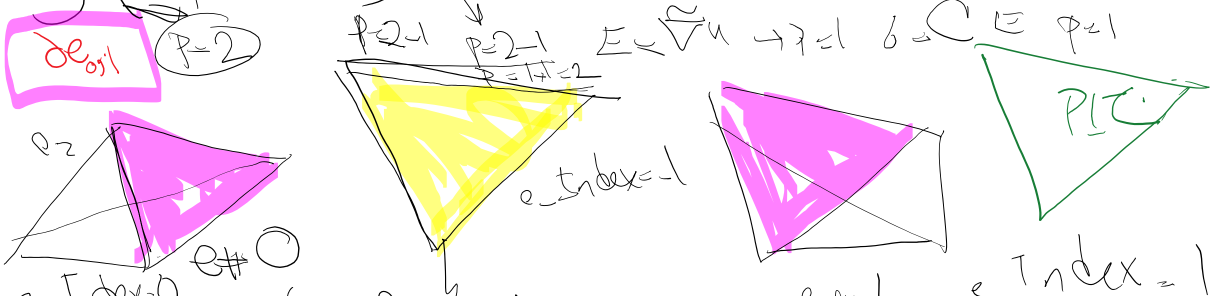
$u \to p=2$ polynomial

$$\int \left(\hat{u}p + \nabla\hat{u}\,q + \hat{u}\rho b\right) dv \qquad I_{2D}$$

order $= 2p-2$, $p_s=2 \to$ order $=2$

$$\int_{e_1}\left(-\hat{u}p\,n_1 + \hat{u}\,\sigma\, n_x + [E]\,\hat{\sigma}\,n_+ + [\hat{u}]\,\hat{\sigma}\,n_x + u_0[u]\,n_+\right) ds$$

$\partial e_{0,1}$ $\quad p=2 \quad p=2-1 \quad p=2-1 \quad p=1+1=2 \quad E = \tilde{\nabla}u \to p=1 \quad \sigma = C E \quad p=1$



$$\int \hat{\delta}(\dot{D} - D_{src})\,dv + \int_{\partial e_{-1}} \hat{\delta}(D^* - D)\,n_+\,ds$$

e_+

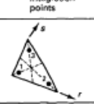
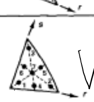
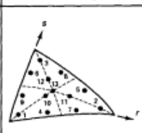
D $p=2$ $\quad p_d + p_d - 1 = 2p_d - 1$ $\qquad p_d = 2 \to$ order $=3$

Solid needs order 2

Mat Damage " " 3

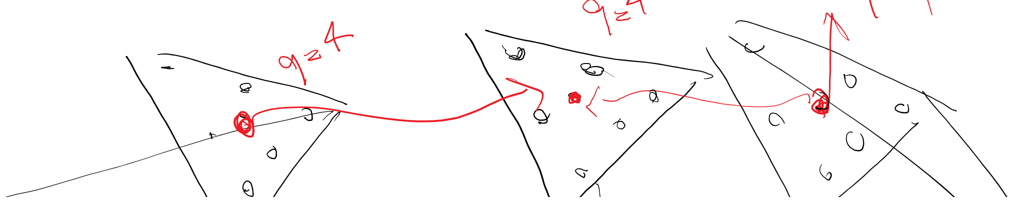
$\boxed{\text{Order} = 3}$

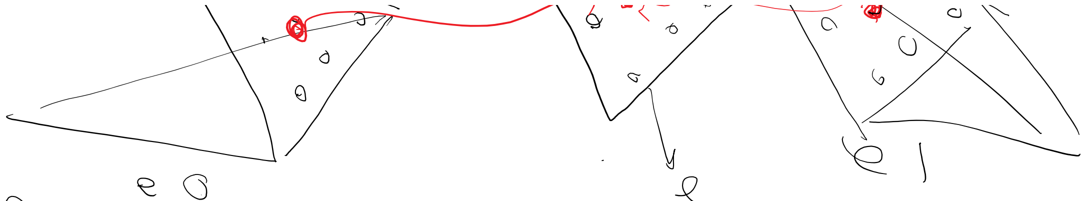
TABLE 5.8 *Gauss numerical integrations over triangular domains* $[\iint F\,dr\,ds = \frac{1}{2}\sum w_i F(r_i, s_i)]$

Integration order	Degree of precision	Integration points	r-coordinates	s-coordinates	Weights
3-point	2		$r_1 = 0.16666\ 66666\ 667$ $r_2 = 0.66666\ 66666\ 667$ $r_3 = r_1$	$s_1 = r_1$ $s_2 = r_1$ $s_3 = r_2$	$w_1 = 0.33333\ 33333\ 333$ $w_2 = w_1$ $w_3 = w_1$
7-point	5		$r_1 = 0.10128\ 65073\ 235$ $r_2 = 0.79742\ 69853\ 531$ $r_3 = r_1$ $r_4 = 0.47014\ 20641\ 051$ $r_5 = r_4$ $r_6 = 0.05971\ 58717\ 898$ $r_7 = 0.33333\ 33333\ 333$	$s_1 = r_1$ $s_2 = r_1$ $s_3 = r_2$ $s_4 = r_6$ $s_5 = r_4$ $s_6 = r_4$ $s_7 = r_7$	$w_1 = 0.12593\ 91805\ 448$ $w_2 = w_1$ $w_3 = w_1$ $w_4 = 0.13239\ 41527\ 885$ $w_5 = w_4$ $w_6 = w_4$ $w_7 = 0.225$
13-point	7		$r_1 = 0.06513\ 01029\ 022$ $r_2 = 0.86973\ 97941\ 956$ $r_3 = r_1$ $r_4 = 0.31286\ 54960\ 049$ $r_5 = 0.63844\ 41885\ 698$ $r_6 = 0.04869\ 03154\ 253$ $r_7 = r_5$ $r_8 = r_4$ $r_9 = r_6$ $r_{10} = 0.26034\ 59660\ 790$ $r_{11} = 0.47930\ 80678\ 419$ $r_{12} = r_{10}$ $r_{13} = 0.33333\ 33333\ 333$	$s_1 = r_1$ $s_2 = r_1$ $s_3 = r_2$ $s_4 = r_6$ $s_5 = r_4$ $s_6 = r_5$ $s_7 = r_6$ $s_8 = r_5$ $s_9 = r_4$ $s_{10} = r_{10}$ $s_{11} = r_{10}$ $s_{12} = r_{11}$ $s_{13} = r_{13}$	$w_1 = 0.05334\ 72356\ 088$ $w_2 = w_1$ $w_3 = w_1$ $w_4 = 0.07711\ 37608\ 903$ $w_5 = w_4$ $w_6 = w_4$ $w_7 = w_4$ $w_8 = w_4$ $w_9 = w_4$ $w_{10} = 0.17561\ 52574\ 332$ $w_{11} = w_{10}$ $w_{12} = w_{10}$ $w_{13} = -0.14957\ 00444\ 677$

3 > X

3 <



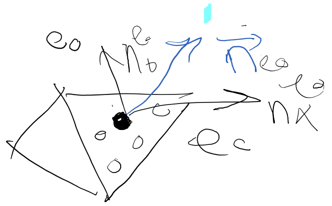


$n_{qd} = 7$

Same process for solid

$$\int_{e_1} \hat{D}(\dot{D} - D_{src})\,dV$$

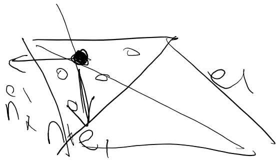
$$= \sum_{q=0}^{n_{qd}} \hat{D}(X_q)\,(\dot{D}(X_q) - D_{src}(X_q))\,\underbrace{w_q\,|J|_{base}}$$



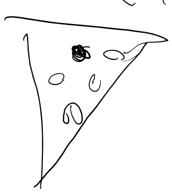
$A = [.5, .33]$

$$\left[-\hat{u}_{e_0} P n_t^{e_0} + \hat{u}_{e_0} \dot{\sigma} n_x^{e_0} + [E]\,\hat{\sigma}_{e_0} n_t^{e_0} + [\dot{u}]\,\hat{\sigma}_{e_0} n_x^{e_0} + \hat{u}_{e_0}[u]\,n_t^{e_0}\right] \underbrace{w_q\,|J|_{base}}_{\text{area of triangle}}$$

$[E] = E - E_{e_0}$, $[\dot{u}] = \dot{u} - \dot{u}_{e_0}$, $[u] = (u - u_{e_0})$



$$\left[\hat{u}_{e_1} P n_t^{e_1} + \hat{u}_{e_1} \dot{\sigma} n_x^{e_1} + [E]\,\hat{\sigma}_{e_1} n_t^{e_1} + [\dot{u} - \dot{u}_{e_1}]\,n_x^{e_1} + \hat{u}\,(u - u_{e_1})\,n_t^{e_1}\right) \times w_q\,|J|_{base}$$



$$\hat{D}_{e_{-1}}(\dot{D}_{e_{-1}} - D_{src})\; w_q\,\underbrace{|J|_{base}}_{dV(A_q)}$$

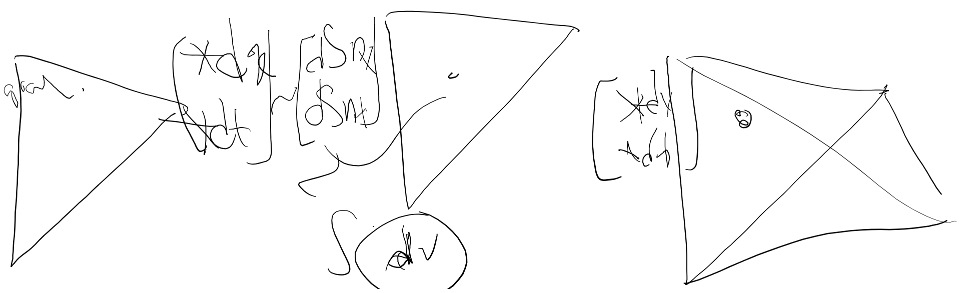
At each quadrature point we have the following data:

1. Coordinates (X: global coordinate, A: integration coordinate, <x> of basis coordinates)
2. Quadrature information (w_q, A_q)
3. Geometry information:

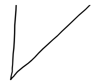
values for

dV → volume integral.

<xd x>, <xd t> } facets

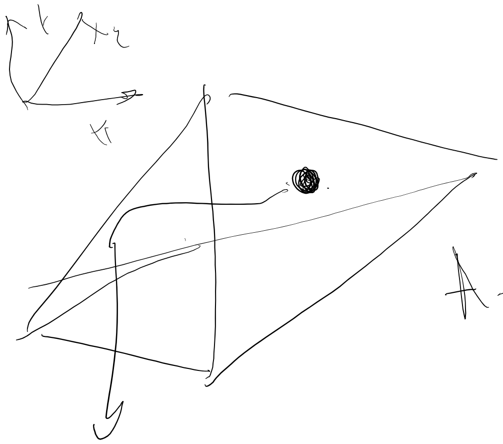


$\langle xdf \rangle \int^{\text{Tacots}}$



$\int^{V} dV$

4. Mechanical fields ($U_{e_0 e_1}$, $\sigma^{*}_{e_0} = \sigma^{*}_{e_1}$, D_{e_1}, ...)



$\vec{u}$ at this point

$A = [.5, 35] \longrightarrow X$ global coordinate

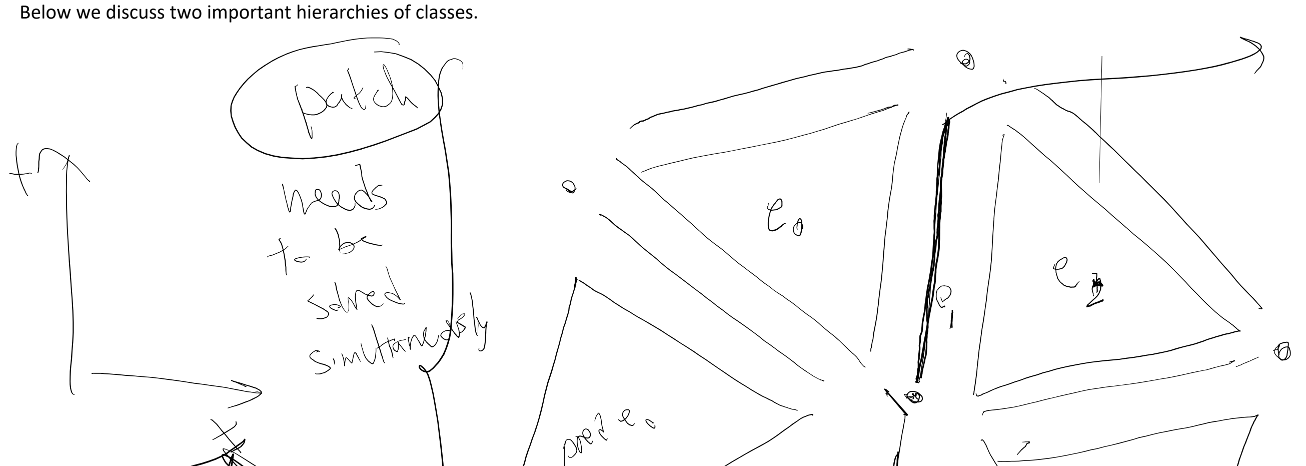
$$X = \begin{bmatrix} 6.5 \\ 3.2 \\ 4.2 \end{bmatrix}$$

$$U = \begin{bmatrix} u_1(x) \\ u_2(x) \end{bmatrix} = \begin{bmatrix} a_0^{u_1} & \cdots & a_{ndf-1}^{u_1} \\ a_0^{u_2} & \cdots & a_{ndf-1}^{u_2} \end{bmatrix} \begin{bmatrix} \phi_1(x) \\ \vdots \\ \phi_{ndf-1}(x) \end{bmatrix}$$

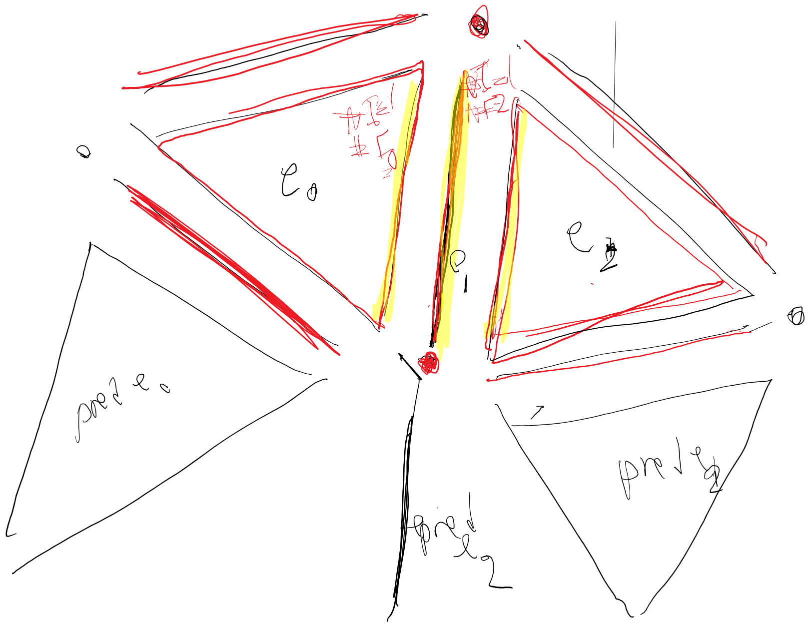
basis coordinate

$X \rightarrow e_0$ basis coordinate

Above was a very brief overview of what assembly would involve.
Below we discuss two important hierarchies of classes.



element if we solve physics on it.



hierarchy 1

$\boxed{\text{integration cells}}$

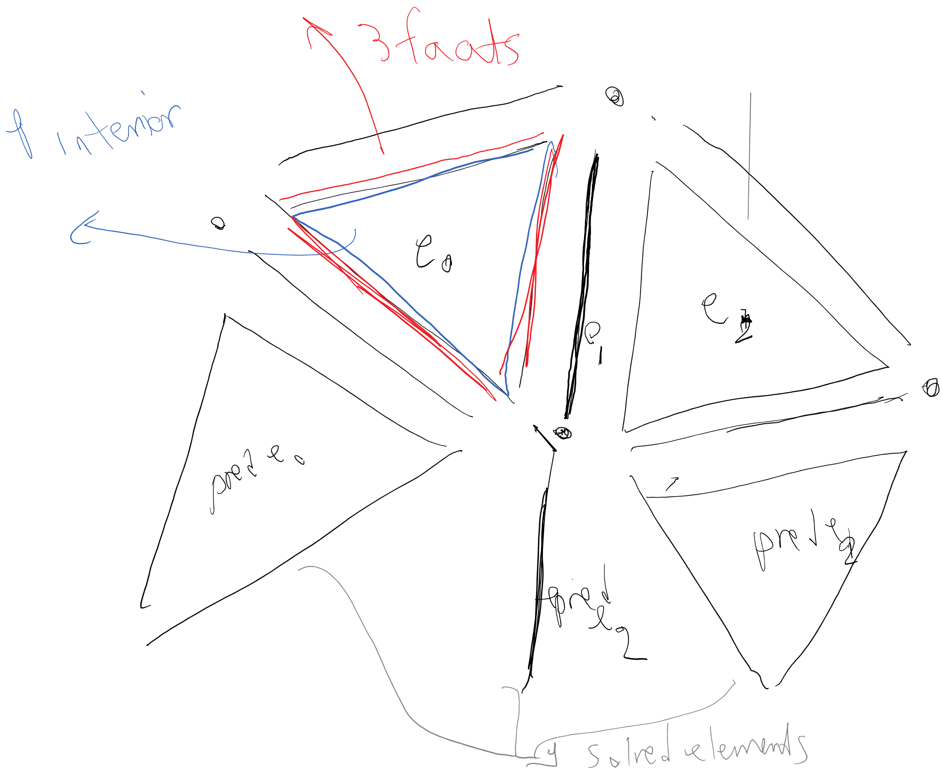
patch

↓

<integration cells>

↓

< quad pts >



patch

↓

< elements > (e_0, e_1, e_2)