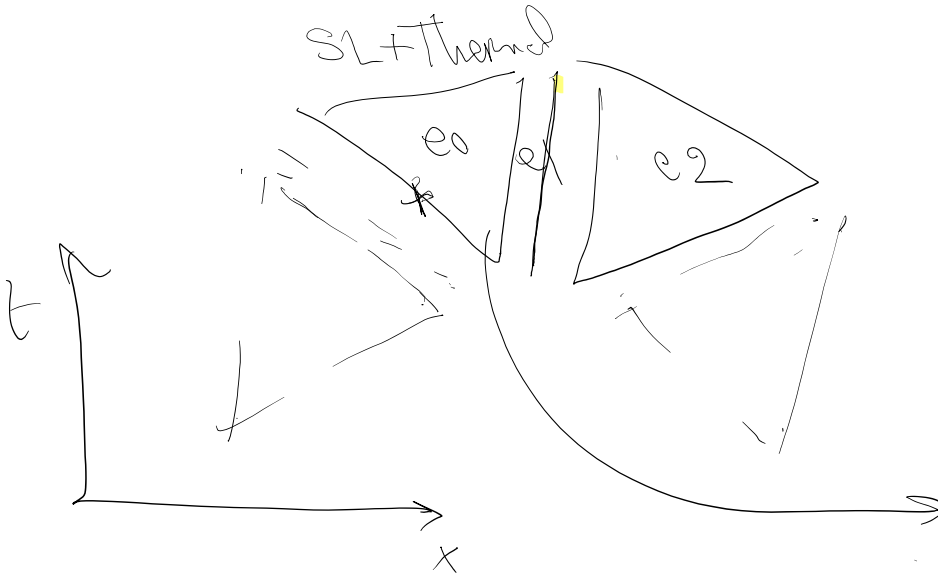


Second hierarchy: PhyPatch -> PhyElement -> PhyPhysics -> tensor fields -> component



1 patch

2 Element

3. Physics

$e_0: \left\{ \begin{array}{l} \text{Solid (3F)} \\ \text{thermal} \end{array} \right\} \left\{ \begin{array}{l} \text{SL} \\ \text{(1F)} \end{array} \right\}$

4. Tensor fields

$e_0 \rightarrow \text{solid } (U, V, E)$   
 $e_0 \rightarrow \text{thermal } (T, \rho)$

$e_1: \text{Solid} \rightarrow U$

5. tensor component

$e_0 \rightarrow \text{SL} \rightarrow U \rightarrow (U_0)$

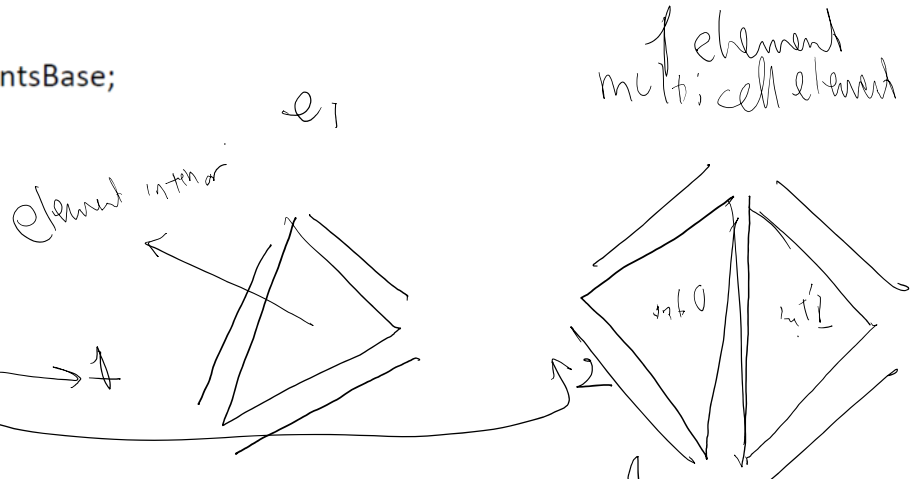
Level 1: PhyElement

In PhyPatch

```
vector<PhyElementBase*> phyElementsBase;
```

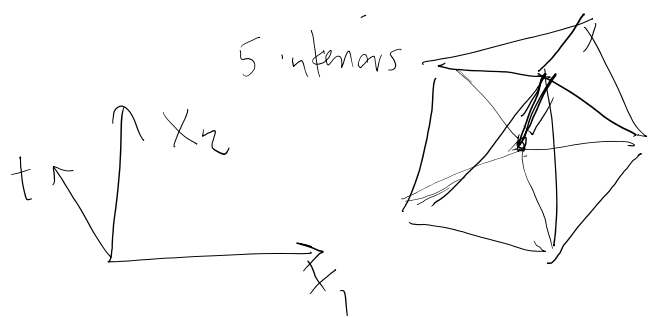
This is a list of elements in the patch.

```
class PhyElementBase
{
....
vector<PhyElementBaseInterior*> interiorCellsBase;
int numInteriorCellsBase; ///< number of Interior Cells.
vector<PhyElementBaseFacet*> facetCellsBase;
int numFacetCellsBase;
```



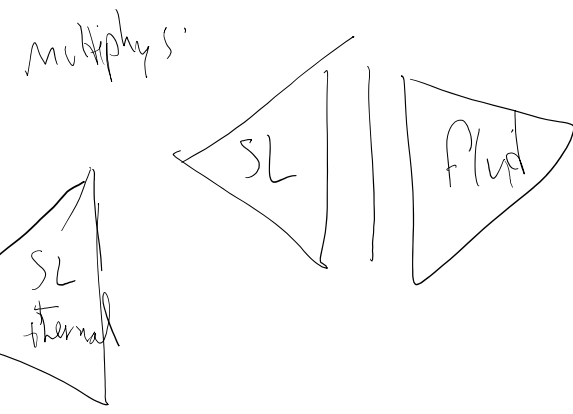
```
vector<PhyElementBaseFacet*> facetCellsBase;
int numFacetCellsBase;
```

Handwritten annotations: A bracket under the first line points to the number '3'. A bracket under the second line points to the number '4'. To the right, there are several arrows pointing towards a central point, and a curved arrow above them.



```
vector<PhyPhysics*> physics; // the physics in the element
int num_physics;
```

Handwritten annotations: An arrow points from 'SL, thermal' to the 'physics' variable. Another arrow points from '2' to the 'num\_physics' variable.



we have a lot of printers

→ Polymorphism & having virtual functions for different physics (e.g. diff. WR, ...)

**Members of PhyPhysics and creation of PhyPhysics**

For specific physics we need to derive them from a base PhyPhysics class.

There are many specific physics implementations. We use the notation of factory to create them.

PhyPhysics are created by a factory:

Physics/PhysicsFactory.h

```

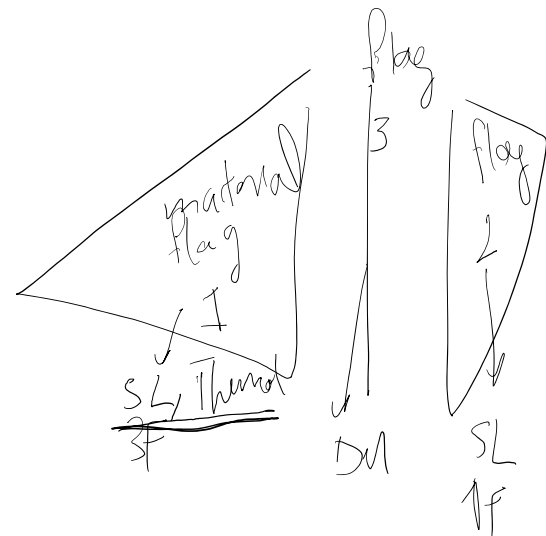
PhyPhysics* createPhysics(subConfigRef subConRef)
{
    PhyPhysics* pp;
    int subConfigIndex = subConRef.subConfigIndex;
    int option;
    switch(subConRef.formulationT)
    {
        case CL: (0) → enumeration
            option = phyConf->subConf[subConfigIndex]->
                physics_options(0);
            pp = createCLInstance(option);
            break;
        case SL: (1)
            pp = new SLPhysics();
            break;
    }
}

```

```

// the use of the factory in PhyElement
void PhyElementBase::setPhysics()
{
    num_physics = descProp.subConfigRefs.size();
    physics.resize(num_physics);
    for(int i = 0; i < num_physics; i++)
    {
        physics[i] =
            createPhysics(descProp.subConfigRefs[i]);
        physics[i]->phyLoInElement = i;
        physics[i]->peParent = this;
        // physics[i]->patch = patch;
    }
}

```



By using this function, we have created the vector of PhyPhysics inside the element.

Level 2: PhyPhysics  
 What is inside PhyPhysics?

```

class PhyPhysics
{
    virtual bool IntegrandFacet_intSAssembly_inflowT(double factor, int e_Index, ptCoords& crds, PhyIntCellBase* pic, int quadPN, PhyFieldVals& fldVals);
    virtual bool IntegrandFacet_intSAssembly_outflowT(double factor, int e_Index, ptCoords& crds, PhyIntCellBase* pic, int quadPN, PhyFieldVals& fldVals);
    virtual bool IntegrandFacet_intSAssembly_interiorT(double factor, int e_Index, ptCoords& crds, PhyIntCellBase* pic, int quadPN, PhyFieldVals& fldVals);
    virtual bool IntegrandFacet_intSAssembly_boundaryT(double factor, int e_Index, ptCoords& crds, PhyIntCellBase* pic, int quadPN, PhyFieldVals& fldVals);
    posDof physicsDof; // the dof for PhyPhysics
}

```

```

vector<PhyTensorField> pTFields; //tensor fields interpolating the element

```

vector<PhyTensorField> pTFields; //tensor fields interpolating the element



Physics 0 → SL3F : →  $\underline{U}, \underline{V}, \underline{E}$   
 Physics 1 → Thermal2F →  $\underline{T}, \underline{q}$

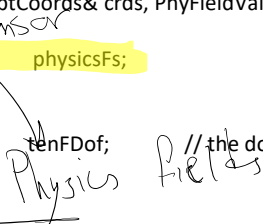
Phy0 → SL3F →  $\underline{U}$

Level 3: PhyTensorField

```
class PhyTensorField
{
void ComputeDivHDX(ptCoords& crds, PhyFieldVals& fldVals, IntHStorage& basisShapes, int e_Index, vsT cvH, rotT rT);
void ComputeCurl(ptCoords& crds, PhyFieldVals& fldVals, IntHStorage& basisShapes, int e_Index, vsT cvH, rotT rT);
```

```
vTensor<phyField> physicsFs;
int num_physicsFs
```

```
posDof
}
tenFDof; // the dof and pos for the tensor field;
```



Level 4:  
PhyField

$$U_i = \left[ \underbrace{\left[ \Phi_0(\mathcal{X}), \dots, \Phi_{\text{dof}_i}(\mathcal{X}) \right]}_{\text{basis function: shape}} \right] \begin{bmatrix} a_{0i} \\ \vdots \\ a_{\text{dof}_i} \end{bmatrix}$$

$\mathcal{X}$

basis coordinate

basis function shape

$\underbrace{[a^T]_{dof_i}}_{\text{coefficients}}$   
 $P_{Coef}$

Basis functions

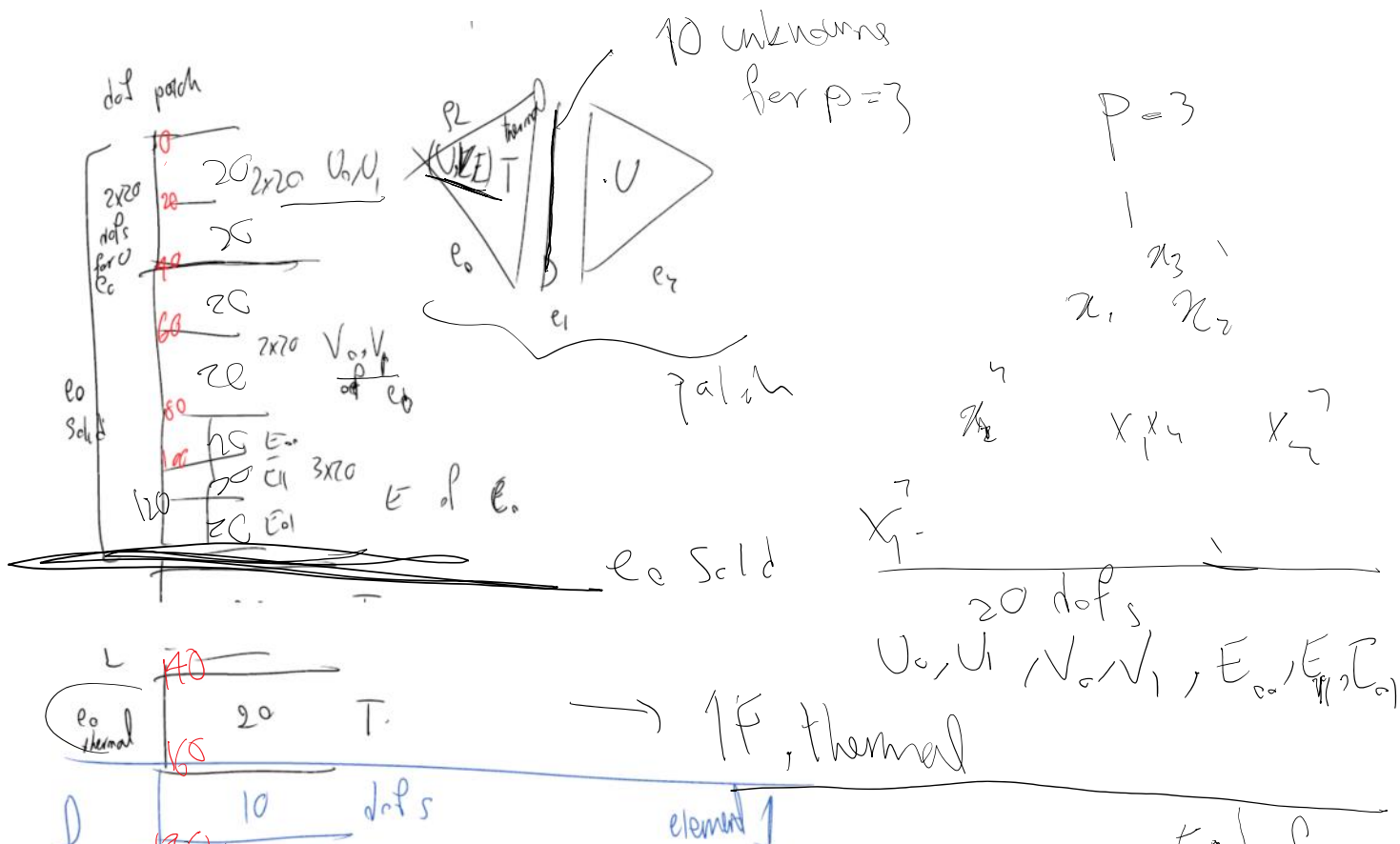
2  $\longrightarrow$   $H(x)$   
 $\frac{dH}{dx_i}$   
 $\frac{d^2H}{dx_i dx_j}$

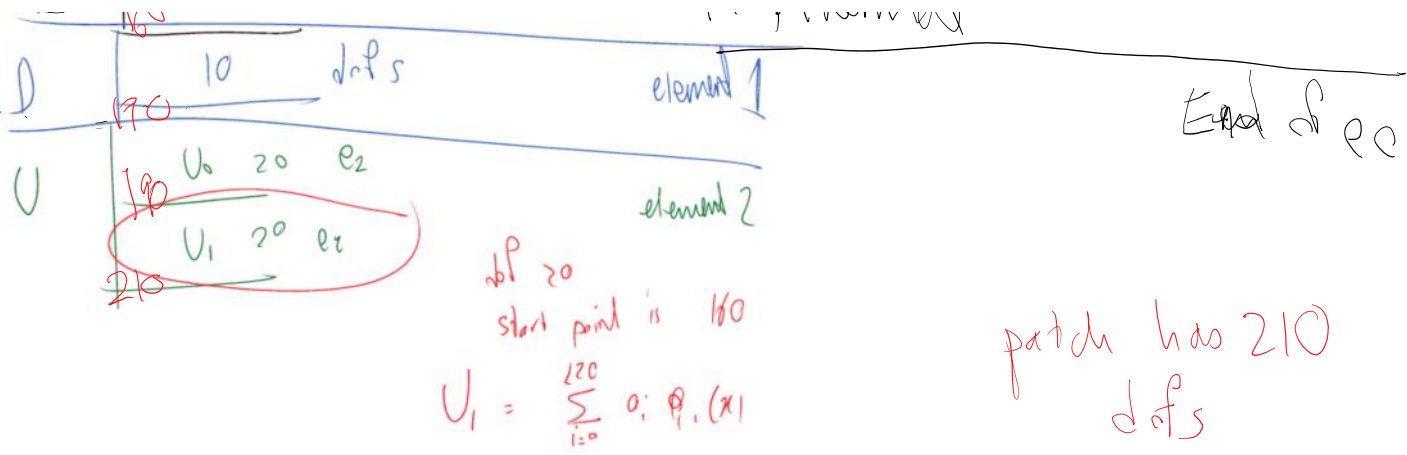
```
class phyField
{
  PhyBasisElement pBasis; // Basis for the field
}
```

```
VECTOR pCoef; // Coefficients for the given physics interpolant
```

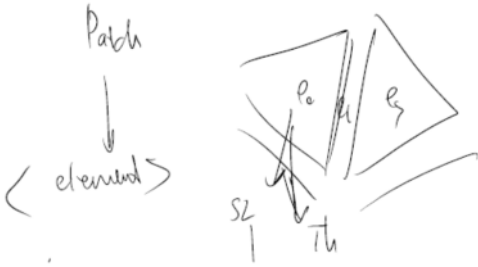
vector of unknowns for this coefficient

```
posDof pDof; // for storing the position and dof info of the given physics interpolant
```

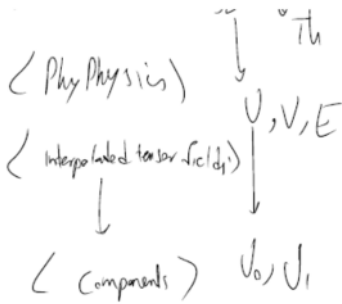




posDof is a class that stores dof of a thing (component of a tensor field, tensor field, physics, element) and its position w.r.t. to all objects owning it (tensor field, physics, element, patch)

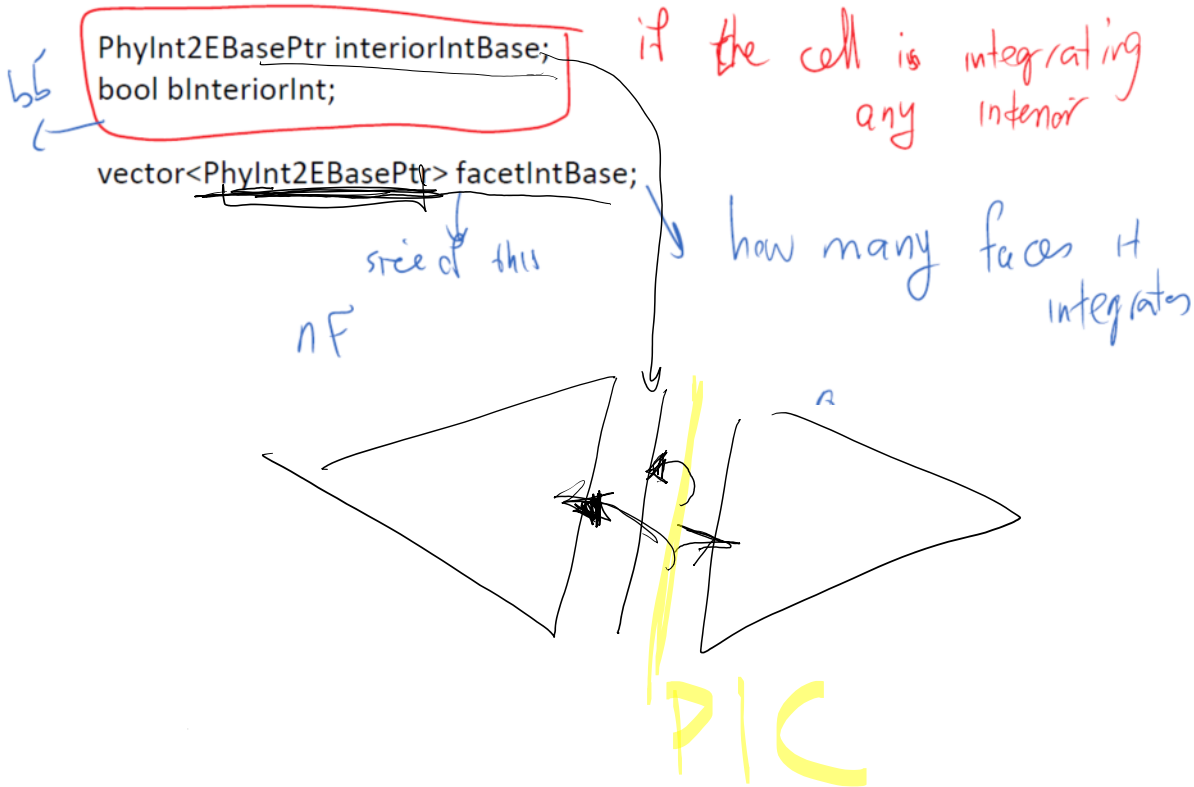


DG Page 6



class PhyIntCellBase

-----



PhyElements and PICs are stored in PhyPatch

Class PhyPatch

....

vector<PhyElementBase\*> phyElementsBase;  
 int num\_elementsBase; } elements

vector<PhyIntCellBase\*> phyIntsBase;  
 int num\_phyIntsBase; } integration cells

Storage members:

$$\int_{\Omega} \psi \left( \frac{1}{2} \mathbf{u} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{p} \right) dv \quad \Gamma_{2D}$$

Solid  $\int_{\partial Q} (\hat{u}_p + \nabla \hat{u}_d + \hat{u}_{pb}) \cdot \nu \, d\Omega$

$$\int_{\partial Q} (-\hat{u}_p \cdot n + \hat{u}_d \cdot n_x + [E] \delta n_t + [\hat{u}] \delta n_x + \hat{u} \cdot [u] n_t) \, dS = 0$$

Riemann

Damage  $\int_Q \hat{D}(\hat{D} - D_{src}) \, dv + \int_{\partial Q} \hat{D}(\hat{D}^\dagger - D) n_t \, dS = 0$

naming tensor fields



class **PhyFldC**

...  
**phyFld** phyF;  $\rightarrow U, V, P, E, \dots$

compT ct;  $\rightarrow$  Val,  $\frac{DT}{\partial t}, \frac{DX}{\partial x}, \dots$   
 $\nabla(\cdot)$  star value, ...

```

typedef enum {
    pfNONE = -1,
    pfVoid = 0, pfDisp, pfVel, pfStrnL, pfplm, pfStrsL, pfStrsElaSL, pfRho,
    pfID,
    pfT, pfHeatFlx, pfUEnergy, pfKEnergy, pfTEnergy, pfDampingF, pfConc, pfConcFlx, pfMaxWaveSpeed, pfBDM, pfBDKappa, pfRTEI,
    /// EM fields, FEM is the field solved (e.g. scatter), inc is incident, and tot is total = inc + FEM
    pfEM_EFEM = 40, pfEM_Einc = 41, pfEM_Etot = 42, // E field
} phyFld;

typedef enum {
    ctVal, //Value
    // // crDT = ctDangle0 for RTE, ctDS2 = ctDangle1 for RTE
    ctDT, /* ctDS2,*/ ctDX, //First order derivatives BASE ones
    ctDT2, ctDXDT, ctDXY, //Second Order Derivatives BASE ones
    ctDXSym,
    ctSource, // source term
    ctTFlux, ctTFluxStar, ctTFluxRiemann, ctTFluxExact, // conservation law Time flux
    ctXFlux, ctXFluxStar, ctXFluxRiemann, ctXFluxExact, // conservation law Space flux
    ctStar, ctStarDT, ctRiemann, ctRiemannSlip, ctExact, // general value Riemann, Star values
    ctTFluxDT, ctXFluxDiv,
    ctStarMVal, ctExactMVal, ctStarMExact,
    // Derived values from others
    ctVisual,
    ctProj, ctFilter01,
    ctDiv, ctCurl, //First order derivatives derived ones
    ctDXSymDT, ctDivDT, ctCurlDT, ctLaplacian, ctdS, // ctdS directional derivative for solid angle RTE //Second Order Derivatives BASE ones
    ctDXDiv,
    // computation types (mostly related to the J term in EM problem)
    // eps = E -> D, H -> B factors; (not including total part) sigma = conductivity (not including PML part)
    // wl = dispersive linear, (not including total part)
    // pml = PML layer, (including all PML related)
    // inc related to incident wave

```



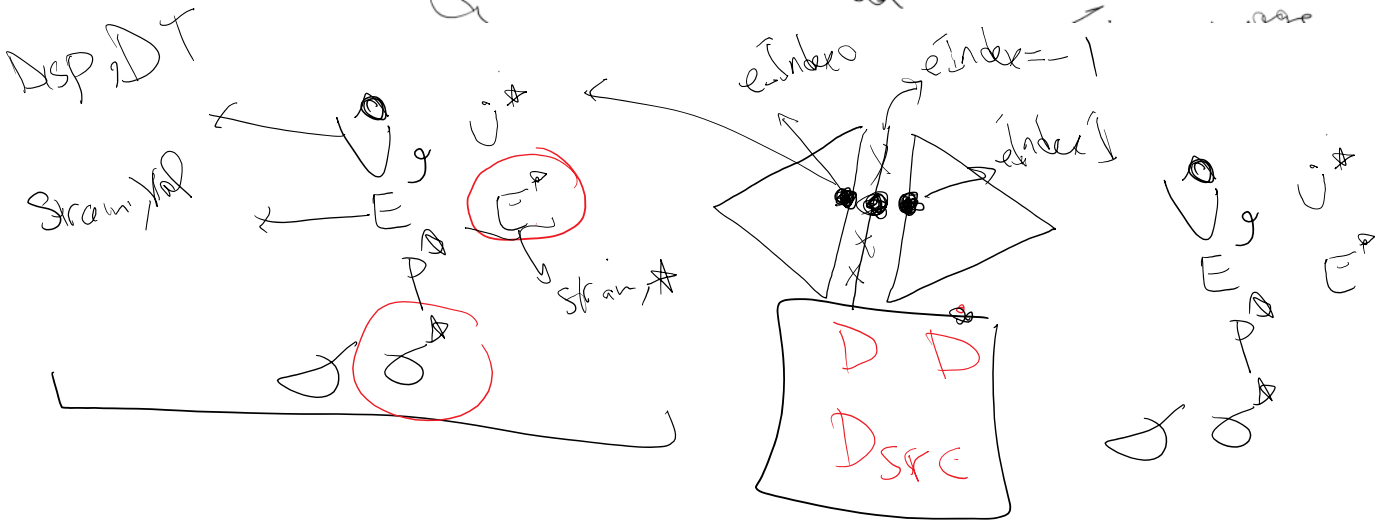
```
// tot = total = inc + sig + wl + pml
ctEMTot = 40, ctEMInc = 41, ctEMEps = 42, ctEMSig = 43, ctEMwl = 44, ctEMPML = 45
// ctInc is used to store incident value in new incident / scatter formulation
, ctInc = 50, ctIncDT = 51,
} compT;
```

Solid  $\int_Q (\hat{u}_p + \nabla \hat{u}_d + \hat{u}_{pb}) dv \int_{\partial Q}$

$$\int_{\partial Q} (\hat{u}_p n + \hat{u}_d \delta^* n_x + [E] \hat{\delta} n_T + [\hat{u}] \hat{\delta} n_x + \hat{u} [u] n_T) ds = 0$$

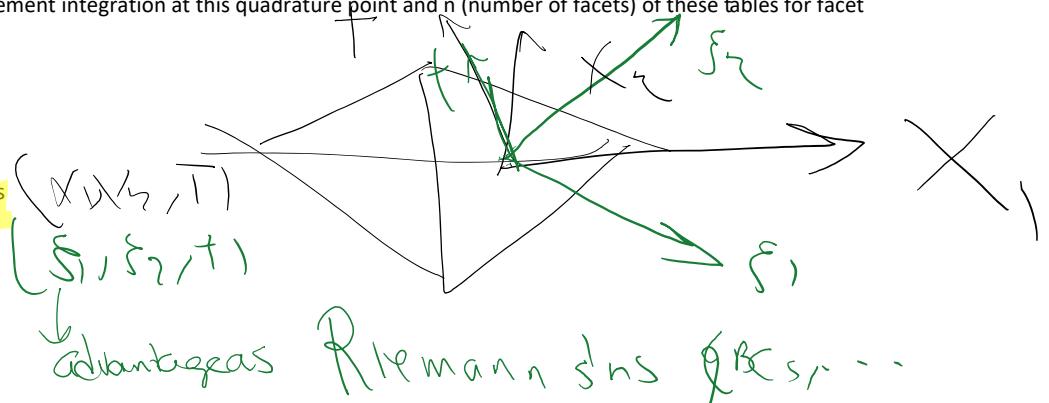
Random

Damage  $\int_Q \tilde{D}(\tilde{D} - D_{src}) dv + \int_{\partial Q} \tilde{D}(\tilde{D} - D) n_T ds = 0$



We need 0 or 1 table for the interior of the element integration at this quadrature point and n (number of facets) of these tables for facet integrations.

```
class PhyFieldVals
{
  PhyElementFields cVal; // Cartesian values
  PhyElementFields rVal; // rotated values
}
```



advantageous Riemann sums  $\int_{\partial Q} \dots$

```
class PhyElementFields
{
  PhyFieldElement eI;
  vector<PhyFieldElement> eF; //size = numFTotal
  bool bInterior; // if there is an interior for field values
}
```

table for storing interior values (eg  $D, \tilde{D}, D_{src}$  above)

values

tables for storing

```

{
PhyFieldElement eI;
vector<PhyFieldElement> eF; //size = numFTotal
bool bInterior; // if there is an interior for field values
mapPfc2Td delValF;
vector< mapPfc2DTd > DdelValF;
mapPfc2Td aveValF;
vector< mapPfc2DTd > DaveValF;
}

```

values → tables for storing  
 facet values (u, v, S, ...  
 for e Index 0  
 above 1  
 above)

shapes  
 (der. w.r.t  
 element unknowns)

Shapes are weights & are needed for stiffness calculation.

$$r = \int \mathcal{D} (D - D_{src}) = 0$$

value

$$(r)_{,j} = \int \mathcal{D}_{,j} (D - D_{src})$$

components

$$(K D)_{Ij} = \int \mathcal{D}_{,I} \left( \frac{\partial D}{\partial a_j} - \frac{\partial D_{src}}{\partial a_j} \right)$$

Shapes

class PhyFieldElement -> class for storing Values (in green above) and shapes (in yellow above) for interior cell (eI above) of facet cells (eF above) at a quadrature points.

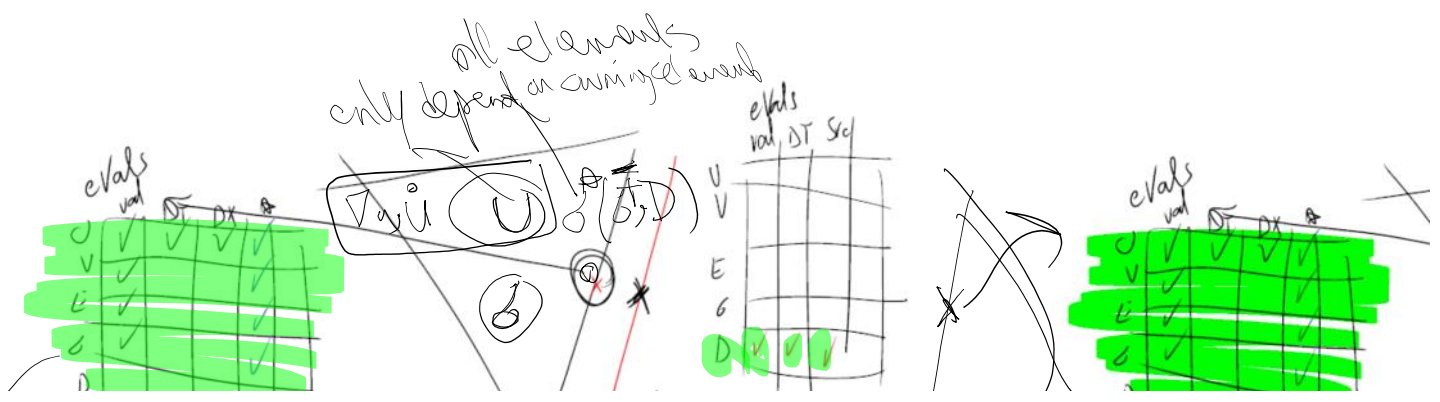
```

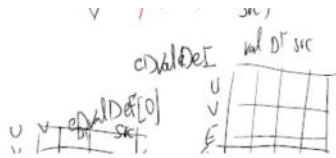
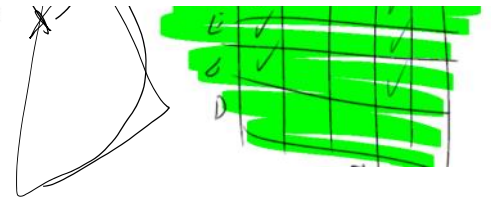
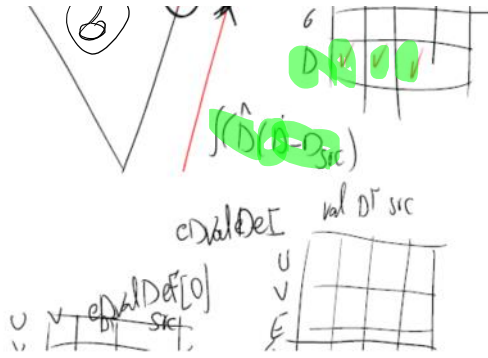
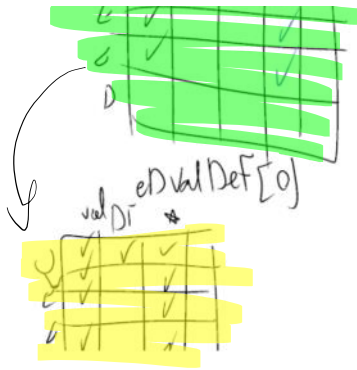
{
mapPfc2Td eVals;
mapPfc2DTd eDValDel; // derivatives wof the vals with respect to the element coefficients on the interior part
vector< mapPfc2DTd > eDValDeF; // derivatives wof the vals with respect to the element coefficients of the elements that share a facet on the intCell
}

```

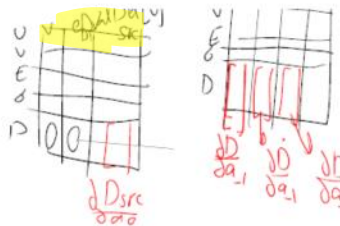
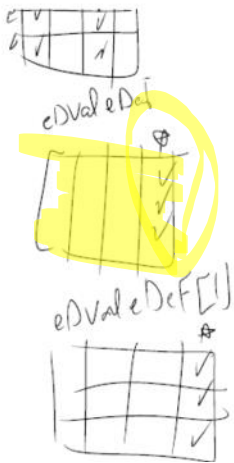
Automatic differentiation: automatically calculates all shapes as calculations go (product rule ....

$$\frac{\partial(UV)}{\partial a} = U \frac{\partial V}{\partial a} + \frac{\partial U}{\partial a} V$$





DG Page 2



similarly  
 $D_{src} \neq 0$