Advanced Finite Element Methods

This course is intended to serve as a sequel to an introductory finite element course. It is designed to deepen the student's understanding of the mathematical background of the method. An introduction to the mathematical theory that supports the finite element method, error estimates, adaptive analysis, and newer finite element methods for parabolic and hyperbolic problems are among the topics covered. In particular, a great emphasis would be on hyperbolic systems where conventional finite element methods perform poorly. After the mathematical description of systems of conservation laws, characteristics, and Riemann solutions, we describe a few advanced numerical techniques for solving these systems including finite volume method and the Spacetime Discontinuous Galerkin Finite Element Method.

Course Objectives

1. Provide sufficient mathematical background to read the current literature and understand new developments in the field.
2. Familiarize the students with approximation theory, error estimates and error indicators and their application in adaptive analysis.
3. Introduce finite element methods for parabolic and hyperbolic boundary-value problems, where the standard Galerkin finite element method fails.
4. Relate theory to practical applications in computational science and engineering.
5. Develop the student's capabilities for technical communication and independent research in computational science and engineering.

Note: This course does not include detailed coverage of basic topic such as shape functions, isoparametric elements, numerical quadrature, etc.

Course Objectives

Your course grade will be based on your performance on homework, midterm exams, and term projects. The course grade will be determined as follows:

Homework	15%
Midterm exams	15%
Term projects	70%

The term projects consist of computer-based exercises that provide practical experiences in one of more of the topics introduced in the lecture. The students have the choice to use the SDGFEM, and advanced finite element code developed based on Spacetime Discontinuous Galerkin Finite Element Method, for their term projects. The

term projects are in lieu of a final examination and are due at the course's scheduled final exam.

Recommended Text

- S. C. Brenner and L. R. Scott, The Mathematical Theory of Finite Element Methods, Springer-Verlag, 1994.
- Claes Johnson, Numerical Solution of Partial Differential Equations by the Finite Element Method, 2nd Edition, Cambridge University Press, 1987.
- R. J. Leveque, Finite Volume Methods for Hyperbolic Problems, Cambridge University Press, 2003.