2019/10/21

Monday, October 21, 2019 11:44 AM

Continue from last time (U was obtained), now we want to find the rotation part of deformation:

$F = RU$

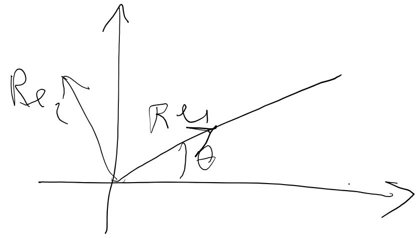
last time

$\Longrightarrow$

$U = \begin{bmatrix} .9478 & 0.0642 \\ & -.0020 \end{bmatrix}$

$F = \begin{bmatrix} 0.95 & .125 \\ 0.063 & 0.992 \end{bmatrix}$

$$\boxed{R = \begin{bmatrix} 0.9981 & 0.061 \\ -0.0610 & 0.9981 \end{bmatrix}}$$



$R = \left[\, Re_1 \mid Re_2 \,\right] = \begin{bmatrix} c & -s \\ s & c \end{bmatrix}$

$c = \cos\theta$
$s = \sin\theta$

Angle of rotation?

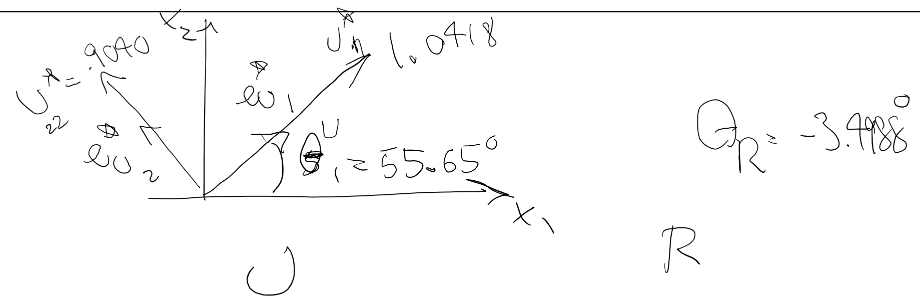
$-.0610 = \sin\theta_R \Longrightarrow$

$.9981 = \cos\theta_R$

$\theta_R = -0.061 \Longrightarrow$

$\theta_R = -0.061 \times \frac{180}{\pi}$ in degrees

$$\boxed{\theta_R = -3.4988^\circ}$$



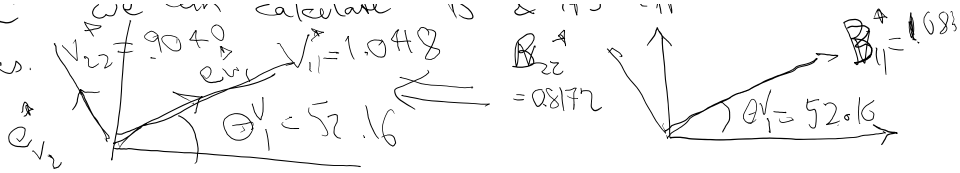
Eulerian view point

$F = VR$ $\qquad dy \to dx$ is where these are used

$V = \sqrt{B}$ $\qquad B = FF^t$

Similar to C we can calculate B & its expression in its principal axes.

Similar to C we can calculate ... principal axes.

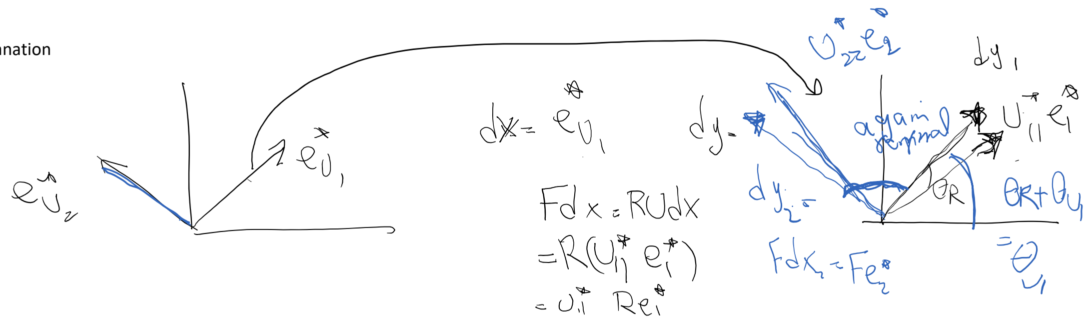


1) Eigenvalues of U and V are equal! (same can be said about C and B)

$$\Theta_{V_i} = \Theta_{U_i} + \Theta_R$$

principal axes of V are principal axes of U rotated

Physical explanation



$dx = e^*_{U_1}$ $\quad dy_1 =$

$Fdx = RUdx$
$= R(U^*_1 e^*_1)$
$= U^*_1 Re^*_1$

$Fdx_2 = Fe^*_2$

Mathematical explanation:

$$RU = VR \Rightarrow \boxed{RUR^t = V}$$

$\vec{v}_i, \lambda_i$ are eigen vector & eigen value # i of U

$$Uv_i = \lambda_i v_i$$

$$U\underbrace{(R^t R)}_{I}v_i = \lambda_i (R^t R) v_i$$

$$Uv_i = \lambda_i v_i \longrightarrow$$

$$(RUR^t)(Rv_i) = \lambda_i (Rv_i)$$

$$\boxed{V \underbrace{(Rv_i)}_{w_i} = \lambda_i (Rv_i)}$$

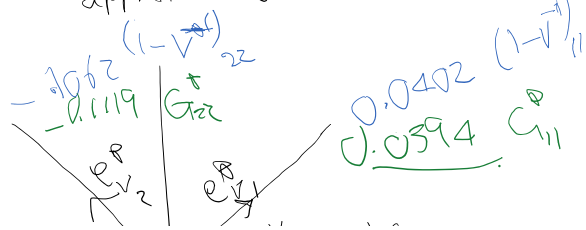
$$Vw_i = \lambda_i w_i$$

HW6: we'll see

$$\underbrace{I - V^{-1}}_{\text{exact}} \quad \& \quad G^* = \underbrace{\tfrac{1}{2}(1 - B^{-1})}_{\text{approximate}} = \tfrac{1}{2}(I - V^{-2})$$

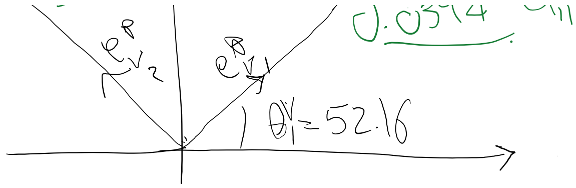
are aslo strains $dy \to dx$

$$[I - V^{-1}]^{*V} = \begin{bmatrix} 0.0402 & \\ & -0.1062 \end{bmatrix}$$

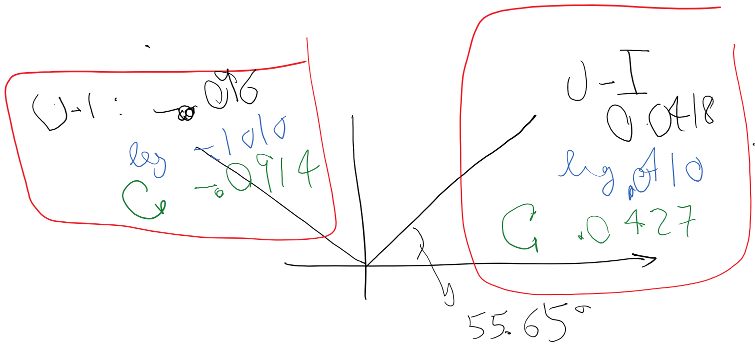


$$[I - V] = \begin{bmatrix} & \\ & -0.1062 \end{bmatrix}$$

$$[G_B]^{*V} = \begin{bmatrix} 0.0394 & \\ & -0.1119 \end{bmatrix}$$



very close to U-based strain values



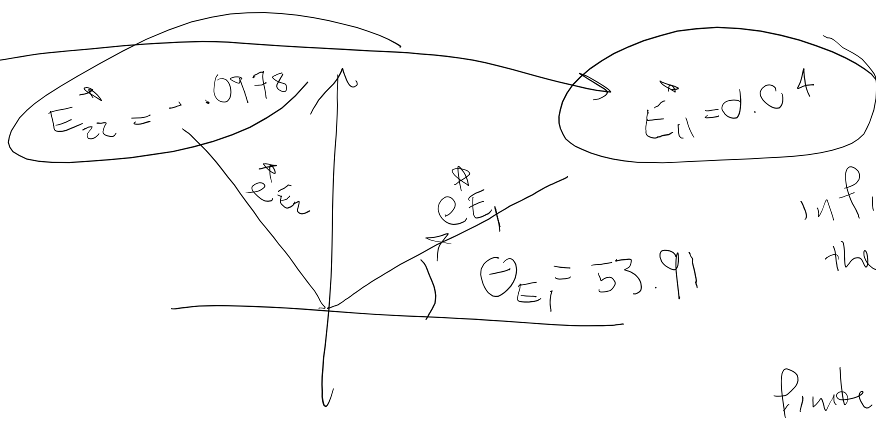
Infinitesimal theory for strain

$$H = F - I = \begin{bmatrix} -0.05 & .125 \\ -.0063 & -.0078 \end{bmatrix} \rightarrow$$

$$E = \text{Sym}\, H = \frac{H + H^t}{2} = \begin{bmatrix} -.005 & 0.0656 \\ \text{sym} & -.0078 \end{bmatrix} \quad W = \text{asym}\, H = \frac{H - H^t}{2} = \begin{bmatrix} 0 & 0.0594 \\ -0.0594 & 0 \end{bmatrix}$$

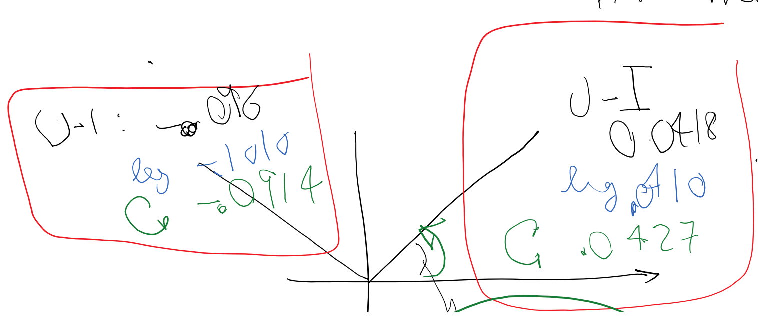
Principal strains and axes:
[eigv, eigd, eigvalues, theta1, theta2, theta1D] = eigSym2([-0.05 0.0656; 0.0656 -.0078])

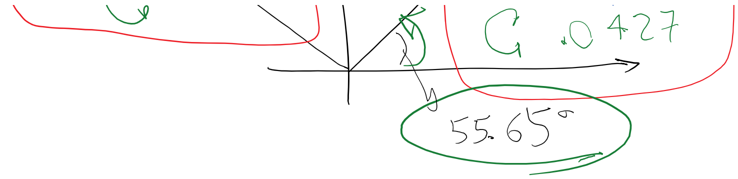
```
eigv =

  -0.5890   -0.8081
  -0.8081    0.5890


eigd =

  0.0400        0
       0  -0.0978


eigvalues =

  0.0400   -0.0978


theta1 =

  0.9410


theta2 =

  2.5118


theta1D =

 53.9151
```



infinitesimal theory

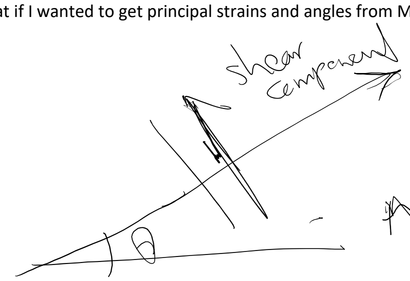
finite theory

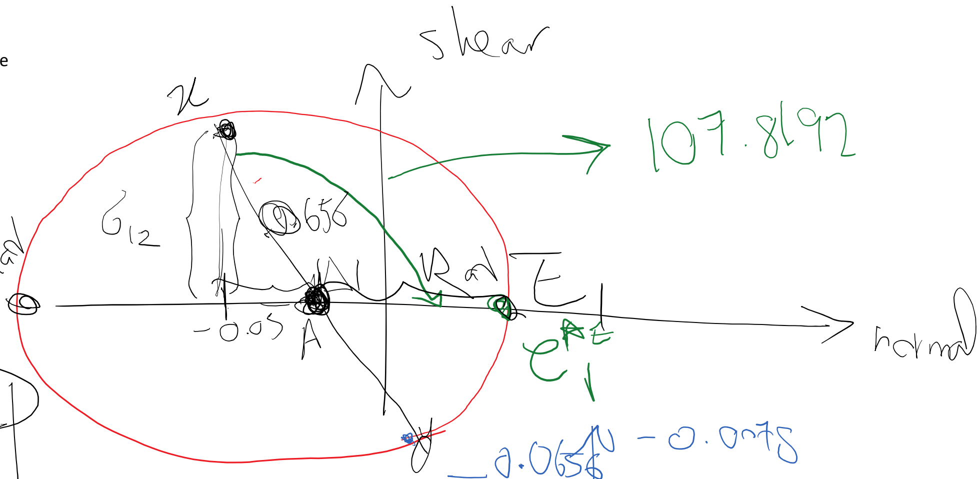




The angles (53.91 and 55.65) and values of infinitesimal and finite deformation theories are pretty close!

What if I wanted to get principal strains and angles from Mohr circle





$$E = \begin{bmatrix} -0.05 & 0.0656 \\ sym & -.0078 \end{bmatrix}$$

x axis normal = -0.05 shear 0.0656

y axis normal = -.0078 shear

$$A = \frac{E_{11}+E_{22}}{2} = -.0289$$

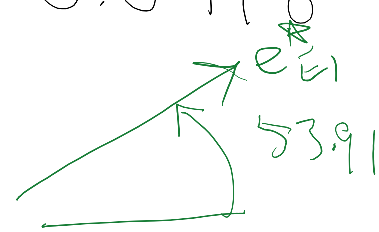
$$\Delta = \frac{E_{11}-E_{22}}{2} = -.0211$$

$$Radius = \sqrt{\Delta^2+\sigma_{12}^2} = 0.0689$$

$$E_1^* = A + Radius = 0.04$$

$$E_2^* = A - Radius = -0.0978$$

$107.81 = -2\theta_E$ →



matches figure above

Infinitesimal rotation

Infinitesimal rotation

$$W = \begin{bmatrix} 0 & .594 & 0 \\ -.594 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

write it o 3D vector

what is Θ_W (infinitesimal rotation)?

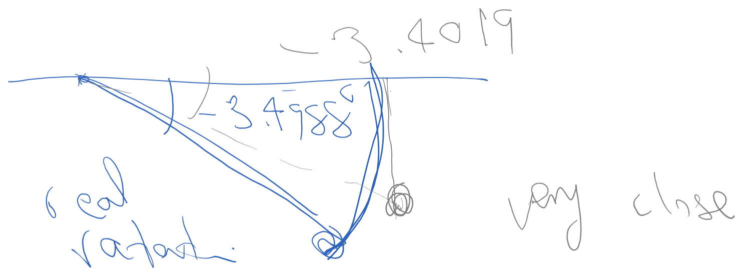
$$W \rightarrow \underbrace{\text{ax}\,W}_{\text{rotation vector}} \qquad W\,du = (\text{ax}\,W) \times du$$

$$\text{ax}\,W = -\begin{bmatrix} W_{23} \\ W_{31} \\ W_{12} \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \\ .594 \end{bmatrix} = \underbrace{-.594}_{\text{rotation angle}} \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{\text{axes of rotation}}$$

$$\text{ax}\,W = \underbrace{\frac{\text{ax}\,W}{|\text{ax}\,W|}}_{\substack{\text{unit vector} \\ \text{axis of rotation}}} \cdot \underbrace{|\text{ax}\,W|}_{\text{rotation angle}}$$

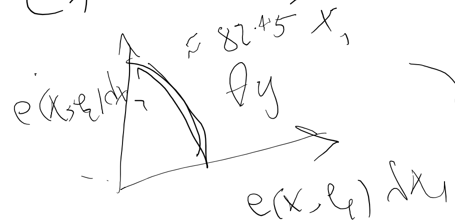
$$\boxed{\begin{aligned} &\Theta_W = -.0594^{\text{rad}} = 0.0594 \times \frac{180}{\pi} = -3.4019^{\circ} \\ &\text{infinitesimal} \\ &\Theta_R = -3.4988^{\circ} \end{aligned}}$$

The rotations are also very close



Final question: how do strains in x,y axes compare?
We have already compared strains in each theory's corresponding principal axes.
- Normal strains were very close
- The rotation parts of deformation were very close
- Shear strains are zero in principal axes for both theories and match

VS



$$\gamma_{12} = \frac{\pi}{2} - \theta_y$$

$$e(x, e_1) = \sqrt{C_{11}} - 1 = \underline{-0.05}$$

$$e(x, e_2) = \sqrt{C_{22}} - 1 = \underline{3.0517e{-5} \approx 0}$$

finite theory

$$\sin \gamma_{12} = \frac{C_{12}}{\sqrt{C_{11}}\sqrt{C_{22}}} \Rightarrow \gamma_{12} = 0.1319 \rightarrow \text{degrees}$$

$$.1319 \times \frac{180^\circ}{\pi} = 7.55^\circ$$

finite theory

infinitesimal theory

$$[E]_{x_1x_2} = \begin{bmatrix} -0.05 & 0.0656 \\ & -.0078 \end{bmatrix}$$

$$e_E(e_1) = E_{11} = \underline{-0.05}$$

$$e_E(e_2) = E_{22} = \underline{-.0078}$$

$$\underbrace{(\gamma_{12})_E}_{\text{eng strain}} = 2\underbrace{E_{12}}_{\text{tensorial strain}} = 0.133$$

very close

$$\underline{7.52^\circ}$$

FYI: Read theorem 139 (Cesaro Line Integral representation)

OK

Idea

$$\begin{matrix} u_1 \\ u_2 \\ u_3 \end{matrix} \longrightarrow \varepsilon = \begin{bmatrix} u_{1,1} & \frac{u_{1,2}+u_{2,1}}{2} \\ & \end{bmatrix}$$

u_2
u_3 $\longrightarrow$

$\underbrace{\qquad}_{3}$ $\underbrace{E_{11}, E_{12}, E_{13}, E_{22}, E_{23}, E_{33}}_{6}$

$u = ?$ $\longleftrightarrow$ given

$3 = 6 - 3$ comp eq needed — not always possible

2D $\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \longrightarrow E = \begin{bmatrix} u_{1,1} & \frac{u_{1,2}+u_{2,1}}{2} \\ & u_{2,2} \end{bmatrix}$

$2 \xleftarrow{?} 3$

1 compatibility eqn needed

$$\left(\begin{aligned} E_{1,1} &= u_{1,1} \\ E_{2,2} &= u_{2,2} \\ E_{1,2} &= \tfrac{1}{2}(u_{1,2} + u_{2,1}) \end{aligned} \right) \quad \begin{aligned} &,22 \\ &,11 \\ &,12 \end{aligned}$$

$$\boxed{E_{1,122} + E_{2,211} - 2E_{12,12} = 0} \star$$

compatible (good ☺) strain field in 2D must satisfy (★) $\Longrightarrow$

refer to notes to obtain U from E

Can be used to propose E fields for complicated problems (e.g crack tip fields). All need to check is satisfaction of ★ because we start from E.

Motion:

$y = f(X)$ deformation

$y = f(X, t)$ time

Definition 87 *A motion of a body is a family of deformations ordered by a single real parameter called* time, *denoted* t. *We introduce a reference time* t_0 *associated with the undeformed state of the body.*[16] *Then a motion is denoted by*

$$\{\mathbf{f}(\cdot, t)\}, t \in [t_0, \infty),$$

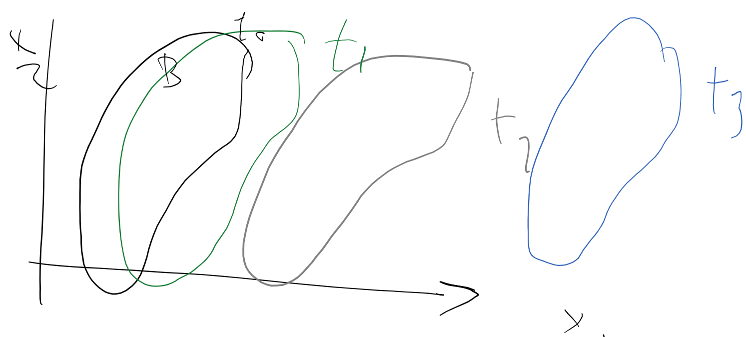
where

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, t)$$

is the position vector at time t *of the material point identified by the position vector* $\mathbf{x}$ *in the undeformed state at time* t_0. *A motion inherits all the required properties of a deformation, except that the numbered properties in Definition 72 are superceded by the requirements*

1. $\mathbf{f}(\mathbf{x}, t_0) = \mathbf{x}$;
2. $\mathbf{f} \in C^2(\overset{0}{\mathcal{B}} \times [t_0, \infty), \mathcal{V})$.[17]

$f(X, t_0) = X$



why we don't need this

$\det F = \det \nabla f > 0$

at $t_0 = F = \nabla_X y = I$

$\det F(t = t_0) = 1$

Proof on why $\det F > 0$?

Theorem 143 *Let* $\{\mathbf{f}(\cdot, t)\}$ *be a motion. Then*

$$J(\mathbf{x}, t) > 0 \text{ on } \overset{0}{\mathcal{B}} \times [t_0, \infty).$$

Proof. Our definition of a motion requires that for any fixed value of t, the mapping $\mathbf{f}(\cdot, t)$ is a deformation on $\overset{0}{\mathcal{B}}$. Therefore, $\mathbf{f}(\cdot, t)$ is invertible $\Rightarrow$ its Jacobian determinant $J(\mathbf{x}, t) \neq 0$ on $\overset{0}{\mathcal{B}} \times [t_0, \infty)$. Since $J(\mathbf{x}, t)$ must be a continuous function of both position and time, the requirement $J(\mathbf{x}, t) \neq 0$ on $\overset{0}{\mathcal{B}} \times [t_0, \infty) \Rightarrow$ either $J(\mathbf{x}, t) > 0$ or $J(\mathbf{x}, t) < 0$ everywhere on $\overset{0}{\mathcal{B}} \times [t_0, \infty)$. At $t = t_0$ we have

$$\mathbf{f}(\mathbf{x}, t_0) = \mathbf{x} \Rightarrow \; f_{i,j}(\mathbf{x}, t_0) = \delta_{ij}.$$

Thus,

$$\begin{aligned} J(\mathbf{x}, t_0) &= \det \mathbf{F}(\mathbf{x}, t_0) \\ &= \det [f_{i,j}(\mathbf{x}, t_0)] \\ &= \det [\delta_{ij}] \\ &= 1. \end{aligned}$$

$\therefore J(\mathbf{x}, t) > 0$ everywhere on $\overset{0}{\mathcal{B}} \times [t_0, \infty)$. ■