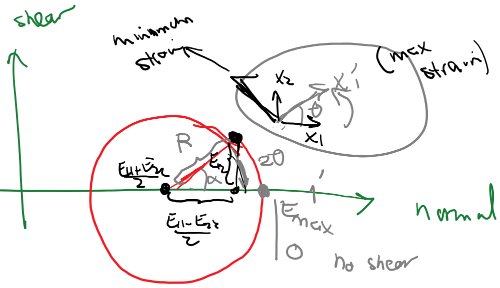
CM2021/10/28

Thursday, October 28, 2021 4:32 PM

How to get principal directions and values

$$tg\alpha = \frac{\varepsilon_{12}}{\frac{\varepsilon_{11}-\varepsilon_{22}}{2}}$$

atan2



$$\varepsilon_{max} = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} + R$$

$$R = \sqrt{\varepsilon_{12}^2 + \left(\frac{\varepsilon_{11}-\varepsilon_{22}}{2}\right)^2}$$

$$\varepsilon_{min} = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} - R$$

Recall rotation from $1 \to 1'$ being θ

in Mohr circle we had -2θ rotation

$$\alpha = -2\theta$$

You can always use eigen function to calculate these

$$\begin{bmatrix} \varepsilon_{11}-\lambda & \varepsilon_{12} \\ \varepsilon_{12} & \varepsilon_{22}-\lambda \end{bmatrix} \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

eigen vector in direction θ

$$\lambda^2 - (\varepsilon_{11}+\varepsilon_{22})\lambda + \varepsilon_{11}\varepsilon_{22} - \varepsilon_{12}^2 = 0$$

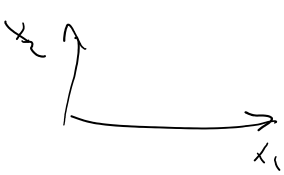
$$\lambda_{1,2} = \frac{\varepsilon_{11}+\varepsilon_{22}}{2} \pm \sqrt{\left(\frac{\varepsilon_{11}+\varepsilon_{22}}{2}\right)^2 - 4(\varepsilon_{11}\varepsilon_{22} - \varepsilon_{12})}$$

they match ε_{max}, ε_{min} above

Please read theorem 139 (Cesaro line integral representation)

idea

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \longrightarrow \varepsilon = \begin{bmatrix} \varepsilon_{11} & \varepsilon_{12} \\ \varepsilon_{12} & \varepsilon_{22} \end{bmatrix}$$

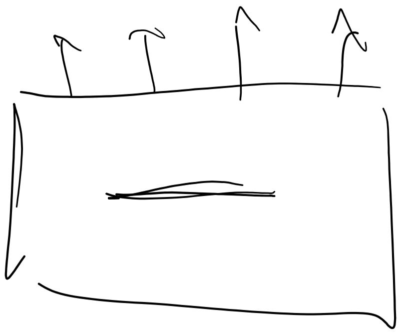


$$\varepsilon_{11} = u_{1,1}$$
$$\varepsilon_{22} = u_{2,2}$$
$$\varepsilon_{12} = .5(u_{1,2} + u_{2,1})$$

we cannot always go from strain to displacement

Why we may even want to go from strain to displacement?

We have these functions called Airy stress functions -> we get stress solutions that are very good for many different problems



σ_{11}
σ_{22} ✓ $\rightarrow \begin{bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ 2\varepsilon_{12} \end{bmatrix} = C^{-1} \begin{bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{bmatrix}$
σ_{12}

next $\varepsilon_{11} = u_{1,1} \rightarrow u_1 = \int \varepsilon_{11} dx_1$

we cannot always Integrate strains to get displacements. We need Additional compatibility eqns.

2D

$$\left.\begin{aligned} \varepsilon_{11} &= u_{1,1} \\ \varepsilon_{22} &= u_{2,2} \\ \varepsilon_{12} &= .5(u_{1,2} + u_{2,1}) \end{aligned}\right] \begin{matrix} ,22 \\ ,11 \\ ,12 \end{matrix}$$

3 strains

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

2 displacements

3 - 2 = 1 additional "compatibility" eqns needed to ensure we can integrate strain to get displacement.

$$\boxed{\varepsilon_{11,22} + \varepsilon_{22,11} - 2\varepsilon_{12,12} = 0} \quad (*)$$

In 3D there are 3 compatibility equations (6 strains - 3 displacements = 3)

In 1, there is no compatibility equations. So, every reasonable strain can be integrated.

1D $\frac{du}{dx} = \varepsilon(x) = 0.005x + 1e{-}5\, x^2$

$$u = \int \frac{du}{dx} \cdot dx = 0.01 x^2 + \frac{1e{-}5 x^3}{3} + C$$

Motion:

We have time dependency of deformation

Definition 87 *A* motion *of a body is a family of deformations ordered by a single real parameter called* time, *denoted t. We introduce a reference time t_0 associated with the undeformed state of the body.*[16] *Then a motion is denoted by*

$$\{\mathbf{f}(\cdot, t)\}, t \in [t_0, \infty),$$

where

$$\mathbf{y} = \mathbf{f}(\mathbf{x}, t)$$

is the position vector at time t of the material point identified by the position vector $\mathbf{x}$ in the undeformed state at time t_0. A motion inherits all the required properties of a deformation, except that the numbered properties in Definition 72 are superceded by the requirements

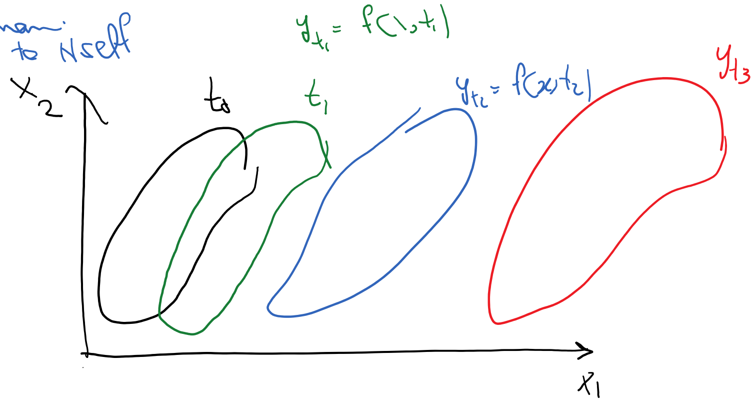
1. $\mathbf{f}(\mathbf{x}, t_0) = \mathbf{x}$;
2. $\mathbf{f} \in C^2(\overset{0}{\mathcal{B}} \times [t_0, \infty), \mathcal{V})$.[17]

@ time zero domain maps to itself

Motion is a time-dependent deformation

$y = x$ @ $t_0 \rightarrow F = I$

$\det F = 1$



Basically for each time we want y_t to be a deformation

Definition 72 *Let $\overset{0}{\mathcal{B}}$ be an open, bounded, regular region of a Euclidean point space $\mathcal{E}$. A* deformation $\mathbf{f}$ *is a mapping (function) of points in $\overset{0}{\mathcal{B}}$ onto another open region of $\mathcal{E}$ with the properties*

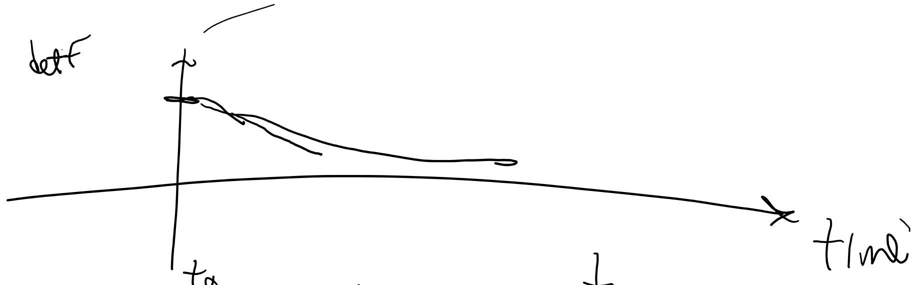
1. $\mathbf{f}$ *is one-to-one; i.e.,* $\mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{y}) \Rightarrow \mathbf{x} = \mathbf{y} \ \forall\, \mathbf{x}, \mathbf{y} \in \overset{0}{\mathcal{B}}$,
2. $\mathbf{f} \in C^2(\overset{0}{\mathcal{B}})$, $\mathbf{f}^{-1} \in C^2(\mathbf{f}(\overset{0}{\mathcal{B}}))$,
3. $\det \nabla \mathbf{f}(\mathbf{x}) > 0 \ \forall\, \mathbf{x} \in \overset{0}{\mathcal{B}}$.

The notation $\mathbf{f}(\overset{0}{\mathcal{B}})$ refers to the mapped region, which is called the image *of the set $\overset{0}{\mathcal{B}}$ under* $\mathbf{f}$.

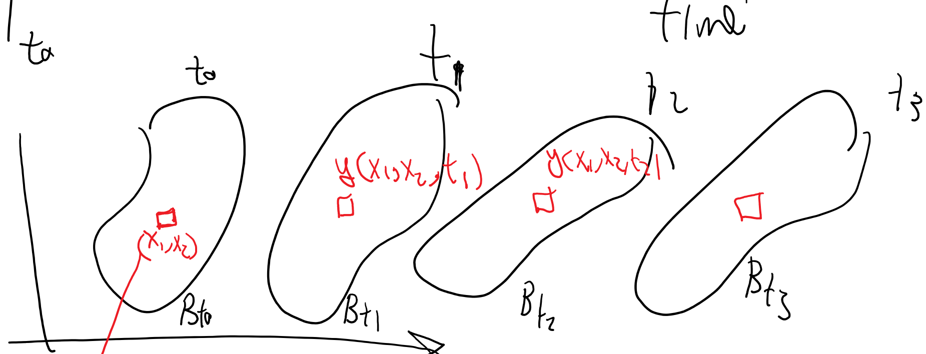
$F = I$

see theorem 143 if $\det F \neq 0 \longrightarrow \det F > 0$

$\det F = 1$ @ t_0 & $\det F \neq 0$ continuous $\longrightarrow$ it always stays > 0



Velocity



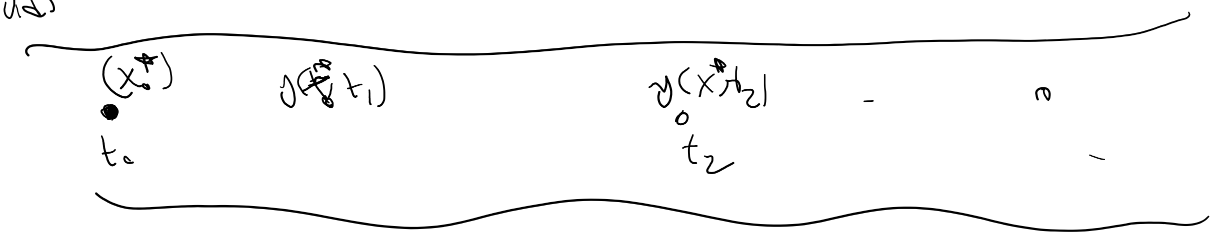
we also use f

reference coordinate

reference coordinate

$$V(x,t) = \frac{Dy(x,t)}{Dt}\Big|_{x \text{ is fixed}}$$

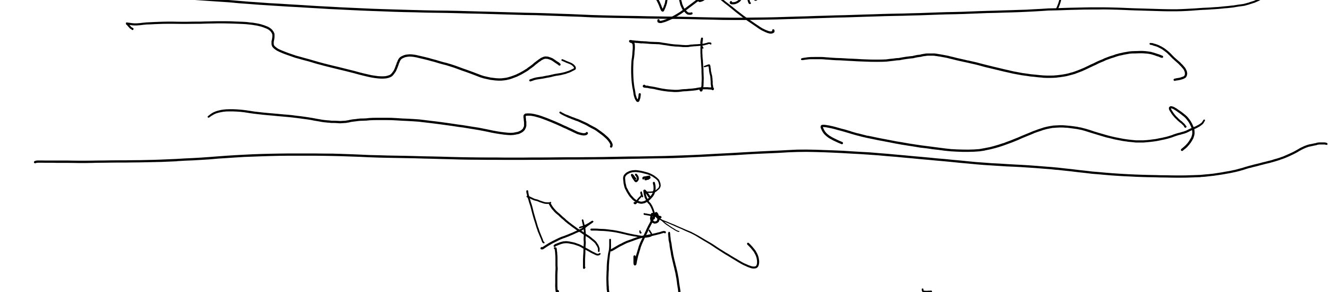
fluids



$$V(x^*,t) = \frac{dy(x^*,t)}{dt}\Big|_{x^* = \text{const}}$$

particle is fixed

$V(\cancel{x,t})$ or $V(y,t)$



looking @ a fixed location

we measure things in y (current configuration)

appropriate for fluid Mechanics

Eulerian $\hat{V}(y,t)$

Lagrangian, we track particles from reference coordinate X

$V(X,t)$ mostly used for solid materials

Many physical laws & equations are

Many physical laws & equations are for a fixed body in reference coordinate or fixed particle

$$V(x,t) = \frac{\partial y(x,t)}{\partial t}\Big|_{\text{fixed } x}$$

$$\cancel{V(y,t) = \frac{\partial y(y,t)}{\partial t}\Big|_{y \text{ fixed}}}$$

for fluids

$$y = f(x,t) \longrightarrow x = f^{-1}(y,t)$$

$$V(x,t)$$

$$\hat{V}(y,t) = V(f^{-1}(y,t),t)$$

Summary

$$V(x,t) = \frac{D\,y(x,t)}{Dt}\Big|_{x\text{-fixed}} = \frac{D\,u(x,t)}{Dt} \quad \text{velocity}$$

$$a(x,t) = \frac{D\,V(x,t)}{Dt}\Big|_{x\text{-fixed}} = \frac{D^2 y(x,t)}{Dt^2}\Big|_{x\text{-fixed}} = \frac{D^2 u(x,t)}{Dt^2}$$

$\frac{D}{Dt}$ called material time derivative

X is fixed

$\frac{\partial}{\partial t}$ often is spatial time derivative

where y is fixed

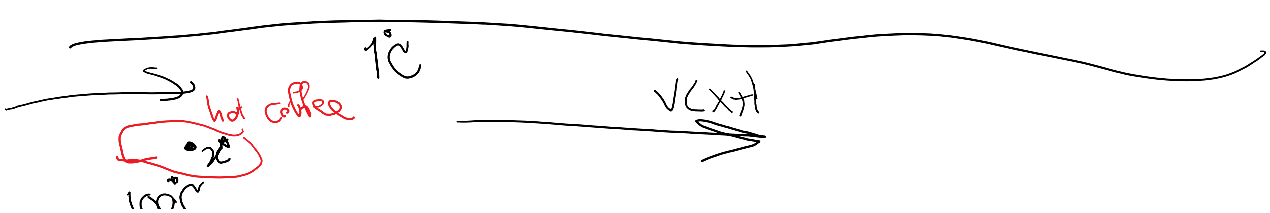
since $y(x,t) = x + u(x,t)$

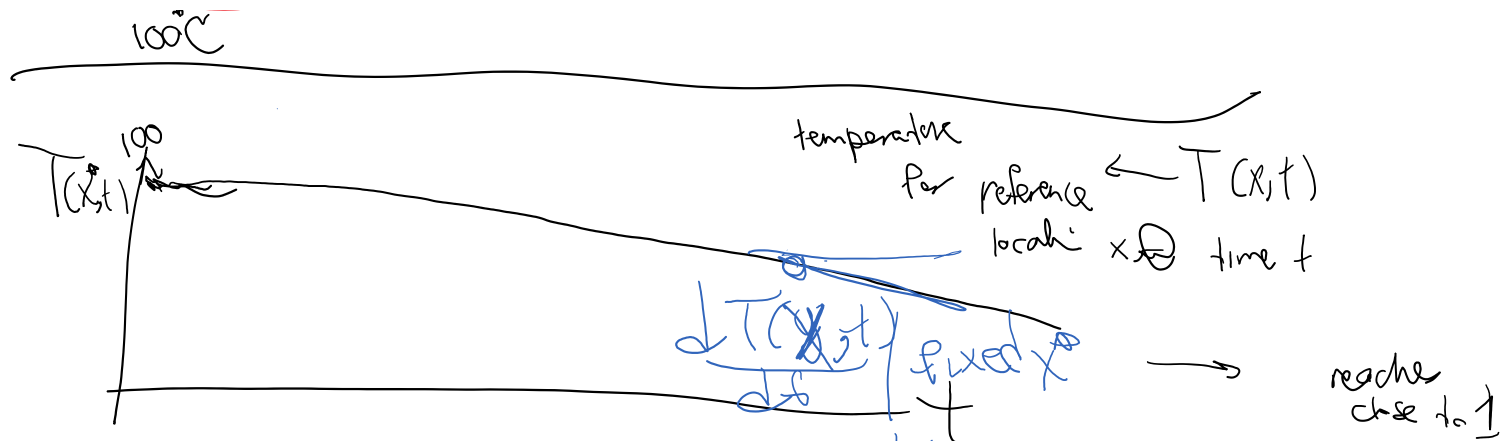
$$V(x,t) = \frac{D\,y(x,t)}{Dt}\Big|_{x\text{-fixed}} = \frac{D\,(x + u(x,t))}{Dt}\Big|_{x\text{-fixed}}$$

$$= \frac{D\,u(x,t)}{Dt}\Big|_{x\text{-fixed}}$$

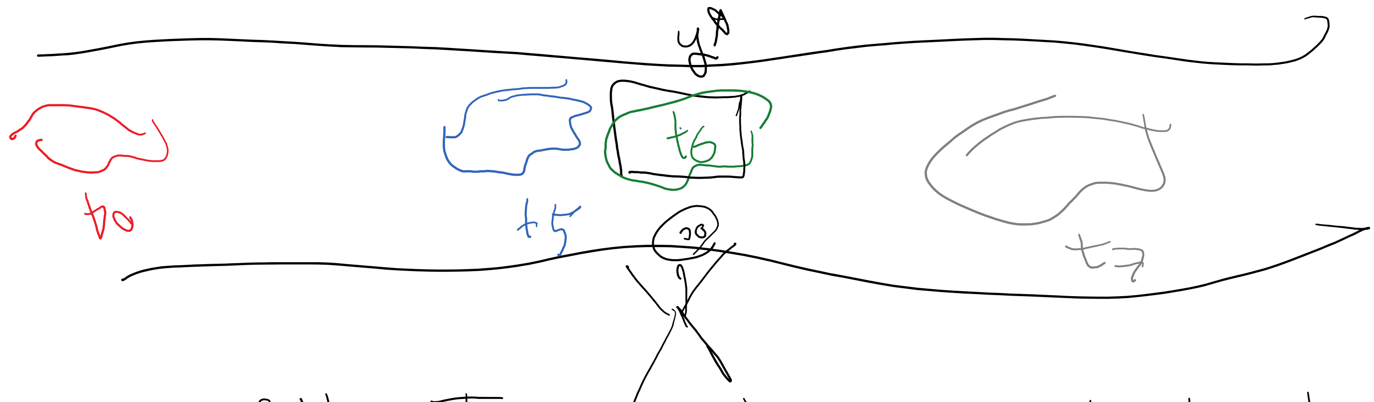
How material and spatial time rates are related?

$$\frac{D}{Dt}\Big|_{x\text{-fixed}} \quad \text{vs} \quad \frac{\partial}{\partial t}\Big|_{y\text{-fixed}}$$





$\frac{DT(x^*,t)}{Dt}$ very small <0 because of diffusion.



measuring T at fixed y (Eulerian)