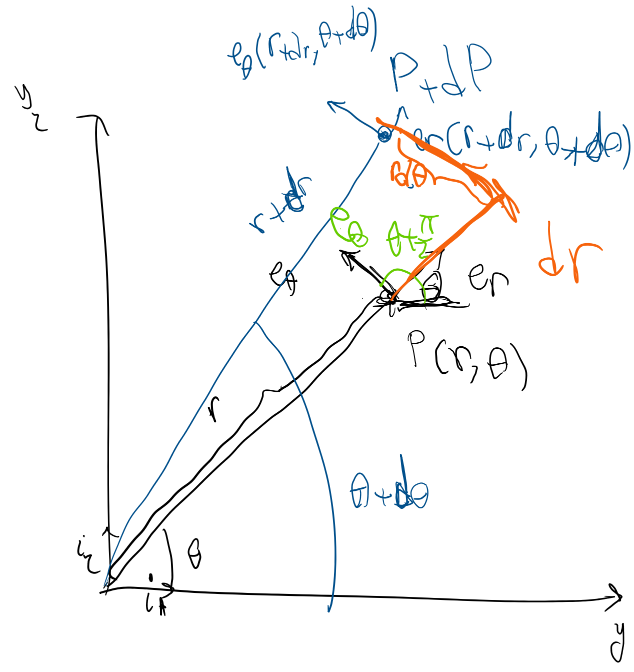
CM2023/10/18

Wednesday, October 18, 2023 9:40 AM

$$d\vec{P} = \underbrace{(1)}_{h_{y_1}=1} i_1\, dy_1 + \underbrace{(1)}_{h_{y_2}=1} i_2\, dy_2$$

$$= \underbrace{(1)}_{h_r} e_r\, dr + \underbrace{(r)}_{h_\theta} e_\theta\, d\theta$$



Martinec_Zdenek_Charles_U_Prague_Martinec-ContinuumMechanics.pdf
Appendix C

$$d\vec{p} = \sum_{k=1}^{3} \frac{\partial \vec{p}}{\partial x_k} dx_k = \sum_{k=1}^{3} h_k \vec{e}_k dx_k, \qquad \text{(C.11)}$$

The function h_k are called the *scale factors*, or the *Lamé coefficients*. They are defined by relation

$$\boxed{h_k := \sqrt{\frac{\partial \vec{p}}{\partial x_k} \cdot \frac{\partial \vec{p}}{\partial x_k}}.} \qquad \text{(C.9)}$$

A Perfectly Matched Layer for the Absorption of Electromagnetic Waves

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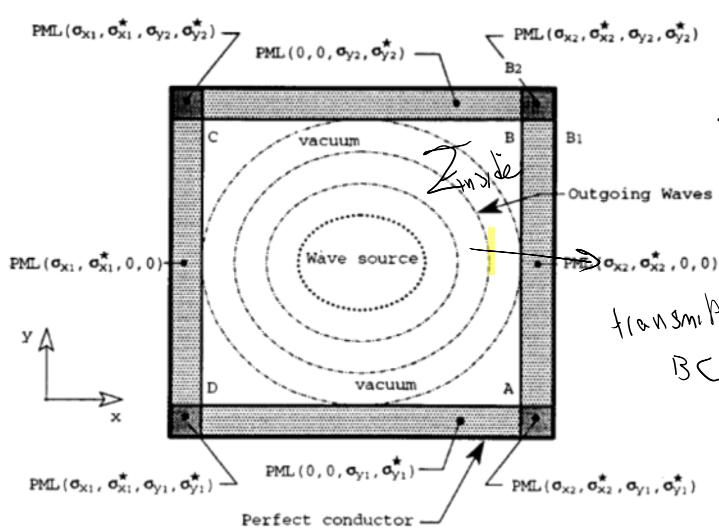
Received July 2, 1993



Z outside

transmitting BC

Z = impedance

Elasto dynamics (1D)

$Z = \sqrt{E\rho}$

($Z_{inside} = Z_{outside}$ no reflection

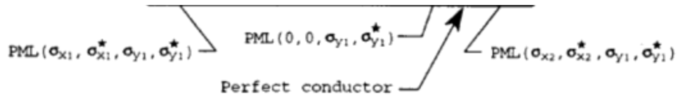


FIG. 3. The PML technique.

$\left\{ \begin{array}{l} Z_{inside} = Z_{outside} \text{ no reflecti} \\ \text{material outside lossy} \end{array} \right.$

A 3D PERFECTLY MATCHED MEDIUM FROM MODIFIED MAXWELL'S EQUATIONS WITH STRETCHED COORDINATES

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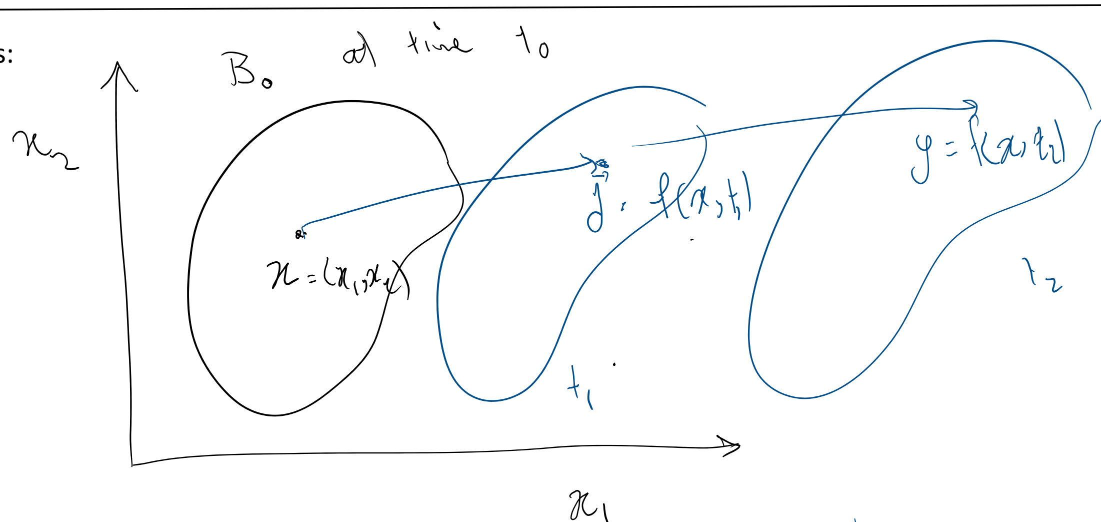
$$\nabla_e = \hat{x}\frac{1}{e_x}\frac{\partial}{\partial x} + \hat{y}\frac{1}{e_y}\frac{\partial}{\partial y} + \hat{z}\frac{1}{e_z}\frac{\partial}{\partial z} \tag{5}$$

$$\nabla_h = \hat{x}\frac{1}{h_x}\frac{\partial}{\partial x} + \hat{y}\frac{1}{h_y}\frac{\partial}{\partial y} + \hat{z}\frac{1}{h_z}\frac{\partial}{\partial z}. \tag{6}$$

$h = 1 + i(\alpha)$

Same h's as above

Kinematics:

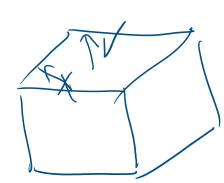


Definition 72 *Let* $\overset{0}{\mathcal{B}}$ *be an open, bounded, regular region of a Euclidean point space* $\mathcal{E}$*. A deformation* f *is a mapping (function) of points in* $\overset{0}{\mathcal{B}}$ *onto another open region of* $\mathcal{E}$ *with the properties*

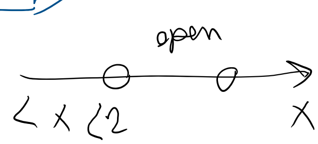
1. f *is one-to-one; i.e.,* $f(x) = f(y) \Rightarrow x = y \;\forall\; x, y \in \overset{0}{\mathcal{B}}$,
2. $f \in C^2(\overset{0}{\mathcal{B}})$, $f^{-1} \in C^2(f(\overset{0}{\mathcal{B}}))$,
3. $\det \nabla f(x) > 0 \;\forall\; x \in \overset{0}{\mathcal{B}}$.

The notation $f(\overset{0}{\mathcal{B}})$ *refers to the mapped region, which is called the image of the set* $\overset{0}{\mathcal{B}}$ *under* f.

regular : we can define normal vectors almost everywhere

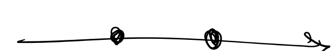


open : $x \in (1, 2)$



$1 < x < 2$

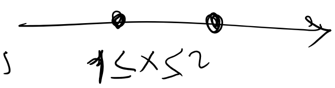
closed $x \in [1, 2]$

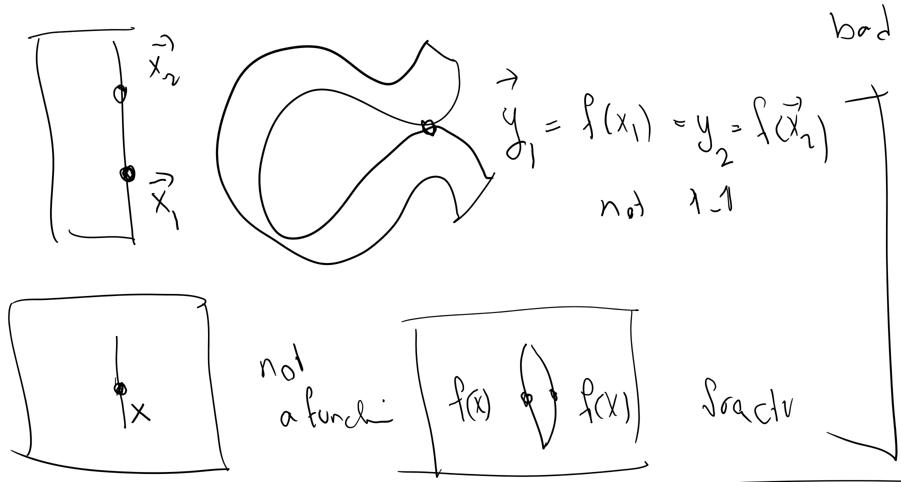


of the set $\mathcal{B}$ under f.

closed $x \in [1,2]$

bad for definitions





we don't like working with closed sets

why $f \in C^2(B_0)$ it means $f, \frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial t}, \frac{\partial^2 f}{\partial x_1^2}, \frac{\partial^2 f}{\partial x_1 \partial x_2}, \ldots \frac{\partial^2 f}{\partial t^2}$ exist & are continuous

Equation of motion as we'll see is

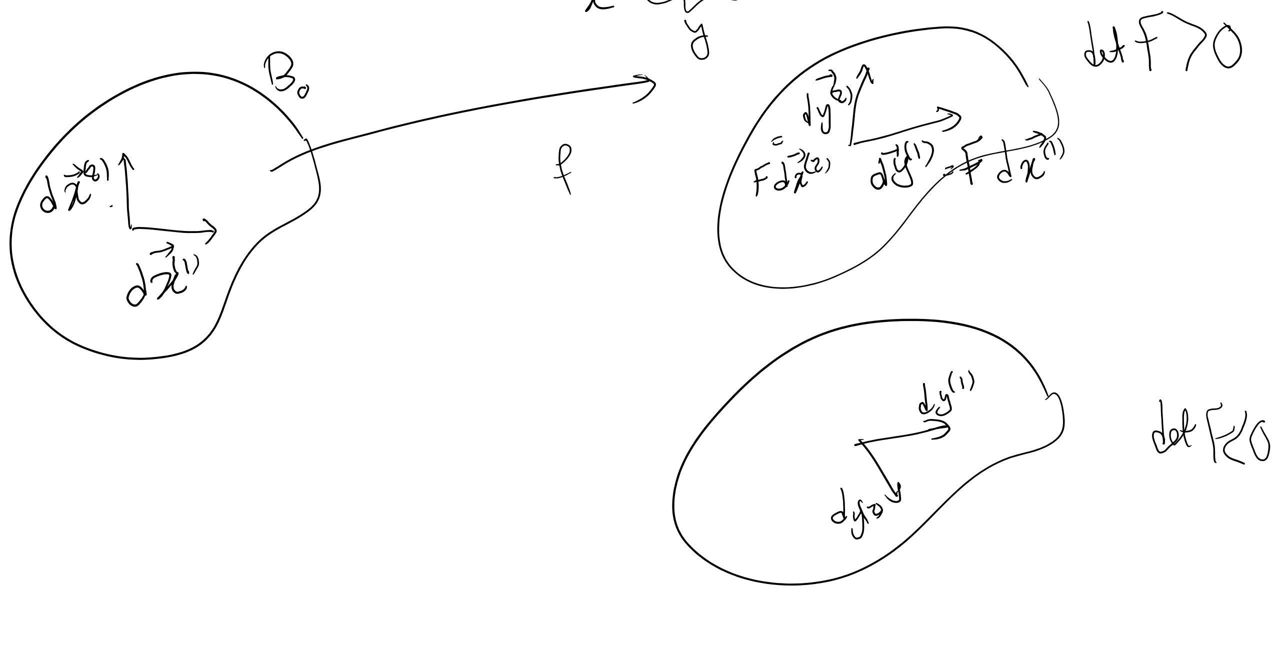
$$\underbrace{\rho \ddot{u}}_{\text{2 derivatives in time}} - \nabla \cdot \sigma = \rho b \qquad \sigma = C \nabla u$$

$$\underbrace{\nabla \cdot C \nabla u}_{\text{2 derivatives in space}}$$

u: displacement

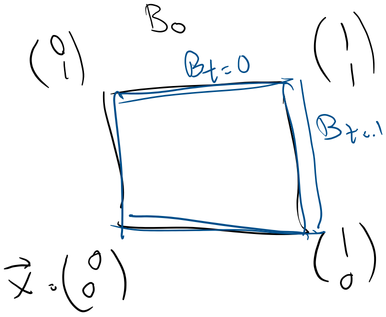
$u = y - x$

$\det F > 0 \qquad F = \nabla_x \underbrace{f(x)}_{y}$



Examples

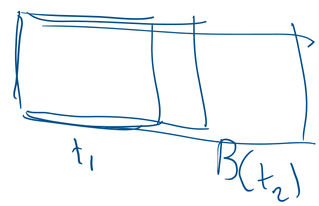
2D



$y_1 = (1+.1t)x_1$

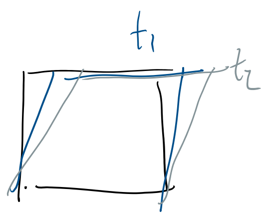
$y_2 = x_2$

$t \nearrow$



shear motion

$$\begin{cases} y_1 = x_1 + .1t\,x_2 \\ y_2 = x_2 \end{cases}$$



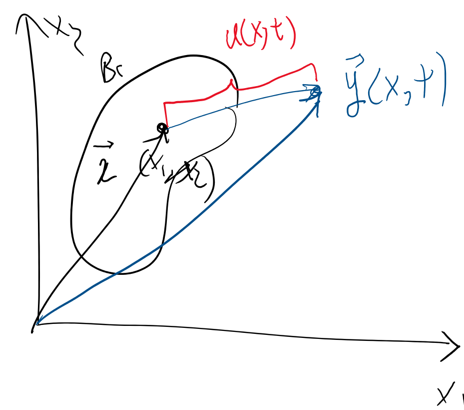
Displacement vector:

$$U(X,t) = \underbrace{y(X,t)}_{\text{new position}} - \underbrace{X}_{\text{old position}}$$

$$\Longleftrightarrow$$

$$y(X,t) = u(X,t) + X$$

(1)



$$u_i(x_1,x_2,x_3,t) = \underbrace{y_i(x_1,x_2,x_3,t)}_{} - X_i \qquad i \in \{1,2,3\}$$

gradient of U w.r.t x

gradient of y w.r.t x

$$\underbrace{\frac{\partial u_i}{\partial x_j}} = \frac{\partial y_i}{\partial x_j} - \frac{\partial x_i}{\partial x_j}$$

$(\nabla_x u)_{ij}$

(or $\nabla u_{/x}$)

$(u = y - x)$

y a function of x,t

$y = f(x,t)$

"careless" writing

$y = y(x,t)$

(2)

$$\nabla_x u = \nabla_x y - I \qquad (H = F - I)$$

$$H := \nabla_x u \quad \text{gradient of displacement } (u)$$

$$H_{ij} = \frac{\partial u_i}{\partial x_j}$$

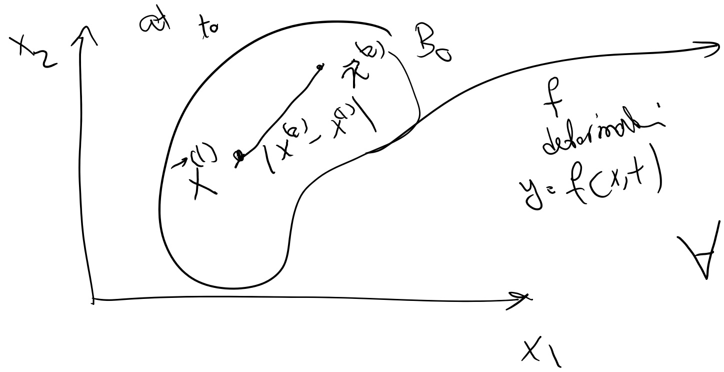
(H is often used for infinitesimal deformation theory)

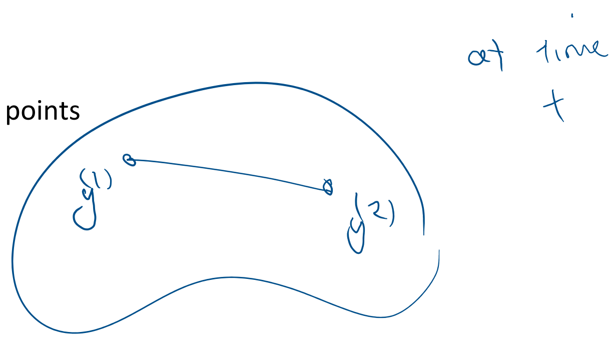
$$F := \nabla_x y \quad \text{gradient of deformation } (y)$$

$$F_{ij} = \frac{\partial y_i}{\partial x_j}$$

(F is often used for finite deformation theory)

Rigid body motion:
Deformation is rigid iff it preserves the distance between all pairs of points



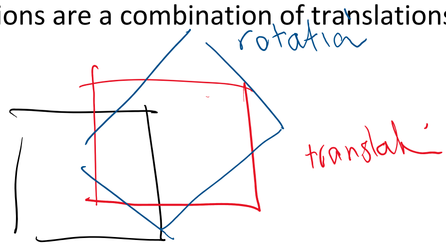


$$\forall x^{(1)}, x^{(2)} \in B_0 : \; |y^{(2)} - y^{(1)}| = |x^{(2)} - x^{(1)}|$$

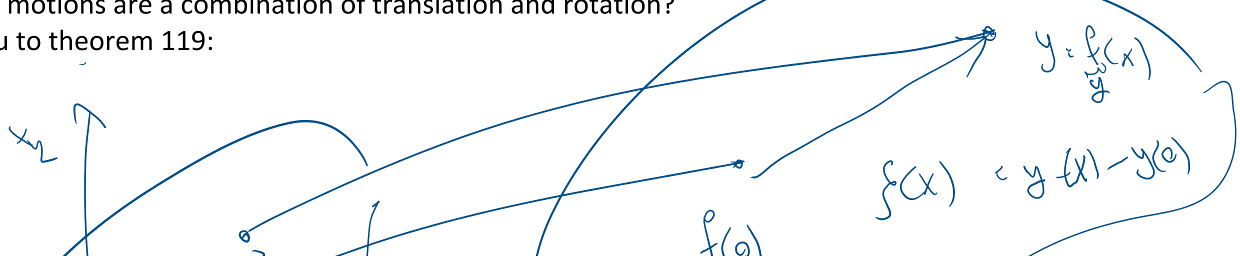
$$y^{(i)} = f(x^{(i)}, t)$$

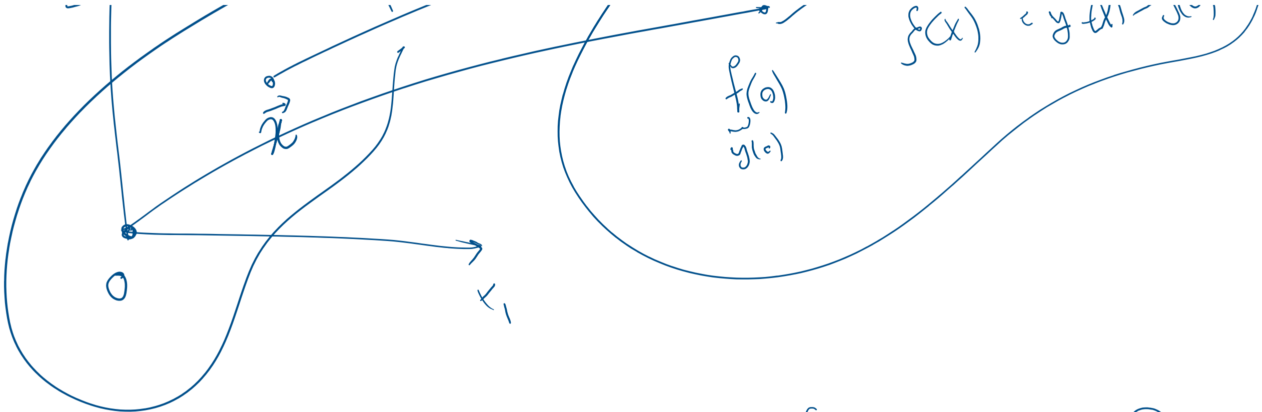
this is called a rigid motion

Rigid motions are a combination of translations and rotations:



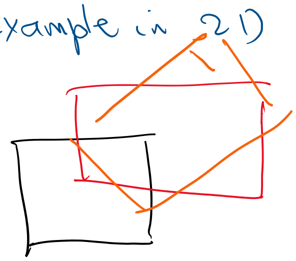
Why rigid motions are a combination of translation and rotation?
I refer you to theorem 119:





this theorem shows $\quad y = f(x) = \underbrace{c}_{\text{translation vector}} + \underbrace{Q}_{\text{rotation tensor}} x$

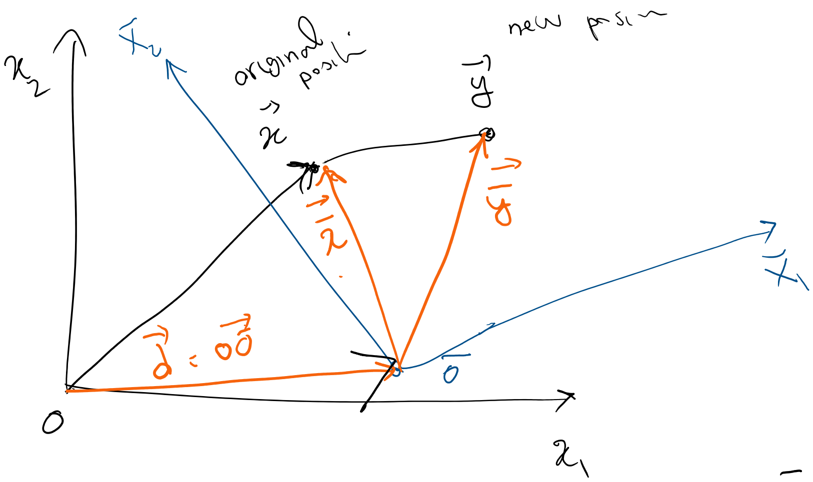
Example in 2D



$$c = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$Q = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix}$$

If a motion (deformation) is rigid from one coordinate system, it's rigid from another coordinate system perspective



y is rigid w.r.t coordinate x_1, x_2

$$\left.\begin{aligned} \vec{y} &= c + Qx \\ x &= d + \bar{x} \\ y &= d + \bar{y} \end{aligned}\right\} \Rightarrow$$

$$d + \bar{y} = c + Q(d + \bar{x})$$

$$\rightarrow \quad \bar{y} = c + Qd - d + Q\bar{x}$$

$$\bar{y} = \bar{c} + Q\bar{x}$$

where $\bar{c} = c + (Q - I)\bar{d}$

the new coordinate system sees the same rotation

but a different translation

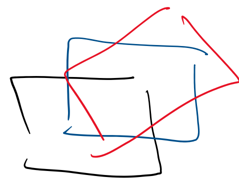
Regardless a rigid motion in 1 coordinate system is rigid in any other one

(3)

rigid motions in 1D

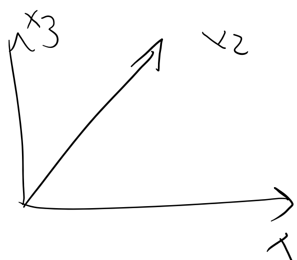
1 rigid motion

2D



$$\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} \quad \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \quad (3 \;\; " \;\; ")$$

3D



$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}$$

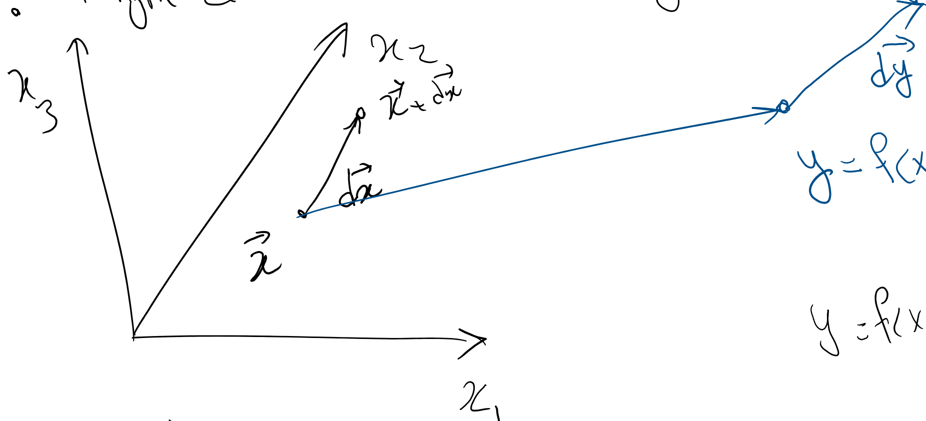
3 rotations

6 " "

Kinematics studies the change in length, angle, area, and volume through deformation
We first study the change in length and orientation of a segment in finite deformation theory, and later approximate them for infinitesimal deformation theory

1. length & orientation of a segment

$\vec{y} + d\vec{y} = f(\vec{x} + d\vec{x})$



$y = f(x)$

$$d\vec{y} = f(\vec{x} + d\vec{x}) - f(\vec{x}) = \underbrace{(\nabla_x f)}\, dx + \underbrace{H.O.T.}_{\text{higher order terms}}$$

$\nabla_x y$

scale ∞

$$\boxed{\vec{dy} = F\vec{dx}} \quad (4)$$

$\nabla_x y$

terms scale as $(dx)^2$ or even smaller

for small $\vec{dx}$