Consider the deformation,

$$y_1 = x_1 + \alpha(1 - x_1)x_2 \tag{1a}$$

$$y_2 = \left[1 - \frac{\alpha^2}{2}(1 - x_1)^2\right] x_2 \tag{1b}$$

for parameter $\alpha = 0.45$.

For computing eigenvalues and vectors, with eigenvalues ordered from large to small value (as customary with strains and stresses), and the angle of eigenvectors, you can use the following Matlab function:

`https://rezaabedi.com/wp-content/uploads/Courses/ContinuumMechanics/A/eigSym2.m`

1. (**40 Points**) Consider reference location $x_1 = 0.5$ and $x_2 = 0.2$. Compute $\mathbf{F}, \mathbf{C} = \mathbf{F}^t\mathbf{F}$, $\mathbf{B} = \mathbf{F}\mathbf{F}^t$, and $\mathbf{H} = \mathbf{F} - \mathbf{I}$.

2. (**20 Points**) Compute principal values, orthonormalized vectors, and directions (in degrees) of $\mathbf{C}$. The principal unit vectors corresponding to $\mathbf{C}$ (and $\mathbf{U} = \sqrt{\mathbf{C}}$) are denoted by $\{e^*_{C1}, e^*_{C2}\}$. Write the values and orientations of these unit vectors (note that they are aligned with eigenvectors of $\mathbf{C}$ already computed.).

3. (**30 Points**) Compute $\mathbf{U} = \sqrt{C}$ and express it in both $\{e_1, e_2\}$ and $\{e^*_{C1}, e^*_{C2}\}$ coordinate systems. What are the principal stretches? Specifically write the angle of first principal direction e^*_{C1} in degrees (θ^*_{C1}).

4. (**30 Points**) Compute the following three strains $\mathbf{U} - \mathbf{I}, \mathbf{G} = 0.5(\mathbf{U}^2 - \mathbf{I}) = 0.5(\mathbf{C} - \mathbf{I})$, and $\mathrm{Ln}\mathbf{U}$ in $\{e^*_{C1}, e^*_{C2}\}$ coordinate system. Graphically show these strains in order along $\{e^*_{C1}, e^*_{C2}\}$ axes in a figure. That is along e^*_{C1} write the principal value number one of these three tensors in order and do a similar thing for principal values 2.

5. (**20 Points**) Compute the rotation tensor $\mathbf{R}$ (use $\mathbf{F} = \mathbf{R}\mathbf{U}$). Provide the angle of rotation along x_3 corresponding to $\mathbf{R}$ in degrees (denoted by θ_R).

6. (**20 Points**) Compute principal values and vectors of $\mathbf{B}$ and subsequently $\mathbf{V} = \sqrt{\mathbf{B}}$. Geometrically draw principal axes and values of $\mathbf{V}$. Principal unit vectors of $\mathbf{V}$ and $\mathbf{B}$ are shown as $\{e^*_{B1}, e^*_{B2}\}$. Right the angle of the first principal direction e^*_{B1} in degrees (denoted by θ^*_{B1}).

7. (**10 Points**) What is the relation between principal values of $\mathbf{U}$ and principal values of $\mathbf{V}$? (hint: use $\mathbf{R}\mathbf{U} = \mathbf{V}\mathbf{R}$) and use the fact that $\mathbf{R}$ is a rotation)

8. (**10 Points**) What is the relation between the angle of first principal axes of $\mathbf{U}$ and $\mathbf{V}$ and rotation angle (That is relate θ^*_{C1}, θ^*_{B1}, and θ_R)? (Hint: Again use $\mathbf{R}\mathbf{U} = \mathbf{V}\mathbf{R}$).

9. (**70 Points**) Comparison of infinitesimal and finite strains in their corresponding principal axes:
   (a) Compute $\mathbf{E} = 0.5(\mathbf{H} + \mathbf{H}^t)$ in $\{e_1, e_2\}$ coordinate system.
   (b) Find principal axes (directions) and principal values of $\mathbf{E}$ and show the components of $\mathbf{E}$ in its principal coordinate system (denoted by $\{e^*_{E1}, e^*_{E2}\}$) geometrically.
   (c) Write the angle of the first principal direction in degrees. This angle is denoted by θ^*_{E1}.
   (d) Compare the orientations of the infinitesimal theory first principal axis θ^*_{E1} and its corresponding finite theory value θ^*_{C1} in degrees.
   (e) Compare principal strains 1 and 2 of infinitesimal theory ($\mathbf{E}$) and finite theory ($\mathbf{U} - \mathbf{I}$) (Obviously they have different principal axes. These values are already computed in previous steps).

(f) Compute the infinitesimal rotation tensor $\mathbf{W} = 0.5(\mathbf{H} - \mathbf{H}^t)$ and write the infinitesimal rotation angle θ_W in degrees.

(g) Compare infinitesimal theory rotation θ_W and finite theory rotation θ_R angles in degree.

10. (**40 Points**) Strains in $\{e_1, e_2\}$ coordinate system:

(a) Use the $\mathbf{C}$ tensor to find $e(\mathbf{x}, e_1)$, $e(\mathbf{x}, e_2)$, and $\gamma(\mathbf{x}, e_1, e_2)$ (finite theory, normal strain in e_1 and e_2 directions and shear strain between e_1 and e_2 axes).

(b) Find the corresponding values based on infinitesimal theory (hint: use E_{11}, E_{22}, and $2E_{12}$ in $\{e_1, e_2\}$ coordinate system. Comment on the difference between these values and their corresponding finite theory values.