1. Exercise 11. (**15 Points**)
2. Exercise 12. (**15 Points**)
3. Exercise 15. (**20 Points**)
4. Exercise 19. (**20 Points**)
5. Exercise 20. (**30 Points**)
   Hint: You need to first do Exercise 17 and Exercise 18 (parts 3 and 4).
6. For a vector $\mathbf{v} = v_1\mathbf{e}_1 + v_2\mathbf{e}_2$ we define

$$l_p(\mathbf{v}) = |\mathbf{v}|_p := \sqrt[p]{|v_1|^p + |v_2|^p}$$

   and for the limiting case $p \to \infty$,

$$l_\infty(\mathbf{v}) = |\mathbf{v}|_\infty := \max\{|v_1|, |v_2|\}$$

   These are often called l_p and l_∞ "norms" of vectors. For $p = 2$ it matches vector magnitude $|\mathbf{v}| = \sqrt{\mathbf{v}.\mathbf{v}}$. Are any of these norms except $p = 2$ actually a scalar? Does it make sense to call them norms in this context? (**extra-credit 20 Points**)
   Hint: A scalar must keep the same value regardless of what coordinate system is used. Consider a simple vector such as $\mathbf{e}_1$ and check if any of these values change by using another coordinate system.

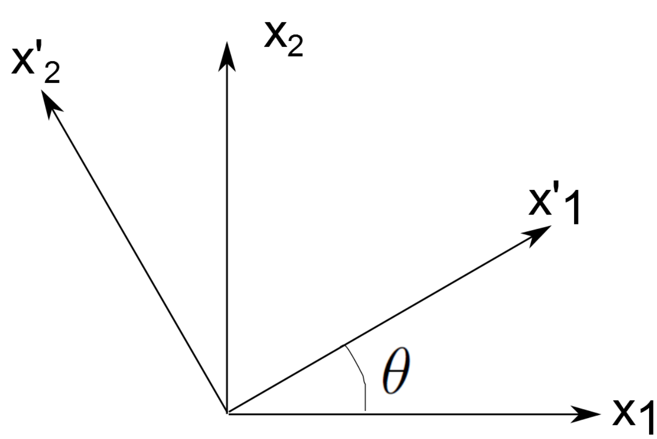

Figure 1: Two coordinate systems $\{e_1, e_2\}$ and $\{e'_1, e'_2\}$ at relative angle θ.

7. Figure 1 shows two coordinate systems $\{e_1, e_2\}$ and $\{e'_1, e'_2\}$ at angle $\theta = 30°$ (degrees) (**45 Points**).
   - Compute matrix Q (λ in TAM551.pdf) to transform tensor components from coordinate system from $\{e_1, e_2\}$ to $\{e'_1, e'_2\}$ (with the convention used in course notes).
   - Vector,

$$\mathbf{v} = \begin{bmatrix} 4 \\ -1 \end{bmatrix}$$

     is expressed in $\{e_1, e_2\}$ coordinate system. Find its components in $\{e'_1, e'_2\}$ coordinate system.

- Second order tensor,

$$\mathbf{T} = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$$

  is expressed in $\{e_1, e_2\}$ coordinate system. Find its components in $\{e'_1, e'_2\}$ coordinate system.
- Scalar $\phi = 12$ is expressed in $\{e_1, e_2\}$ coordinate system. Express it in $\{e'_1, e'_2\}$ coordinate system.