1. **Exercise 25**(**20 Points**)
2. Are there any nonzero isotropic vectors (refer to definition 48).(**10 Points**)
3. **Exercise 46**(**30 Points**)
4. **Exercise 55 (and 56)** (no need to return exercise 56)(**20 Points**)
5. If $\mathbf{P}$ is positive definite, show that the definition $< \mathbf{u}, \mathbf{v} >:= \mathbf{u}.\mathbf{P}\mathbf{v}$ is an inner product.(**20 Points**)
6. Find polar decomposition for $\mathbf{F}$ ($\mathbf{R}, \mathbf{U}, \mathbf{V}$ as in the course notes),(**30 Points**)

$$\mathbf{F} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -3 \\ 0 & 2 & 0 \end{bmatrix}$$

7. Consider polar curvilinear coordinate $x_1 = r, x_2 = \theta, x_3 = z$, where (r, θ, z) are polar coordinates. They are related to Cartesian coordinates (y_1, y_2, y_3) through,(**70 Points**)

$$\begin{aligned} y_1 &= x_1 \cos x_2 = r\cos(\theta) \\ y_2 &= x_1 \sin x_2 = r\sin(\theta) \\ y_3 &= z \end{aligned}$$

For grad, curl, div, and Laplacian calculations directly use formulas in terms of scale moduli and Christoffel symbols. Express your results in component form (That is summarize them in vectors and matrices when applicable).

   (a) Obtain expressions for scale moduli $h_r := h_1, h_\theta := h_2, h_z := h_3$.
   (b) Obtain the only two non-zero Christoffel symbols.
   (c) For a scalar field ϕ obtain $\nabla\phi = \text{grad}\phi$, $\nabla^2\phi$ (Laplacian).
   (d) For a vector field $\mathbf{v} := v_r\mathbf{e}_r + v_\theta\mathbf{e}_\theta + v_z\mathbf{e}_z$ obtain $\nabla.\mathbf{v} = \text{div}\mathbf{v}$, $\nabla\mathbf{v} = \text{grad}\mathbf{v}$ and $\text{curl}\mathbf{v} = \nabla \times \mathbf{v}$.
   (e) For a second order tensor field $\mathbf{T} = T_{rr}\mathbf{e}_r \otimes \mathbf{e}_r + T_{r\theta}\mathbf{e}_r \otimes \mathbf{e}_\theta + \cdots + T_{zz}\mathbf{e}_z \otimes \mathbf{e}_z$ express $\text{div}\mathbf{T} = \nabla.\mathbf{T}$.