I already mentioned in the class that in **gradient** operator the ek from nabla operator should appear as the **last** unit vector, in polyadic products. If you need to read more about this, there are a couple of good references below, otherwise the material discussed in class is sufficient.

Side note (FYI): Care should be taken in working with general nabla operator

$$\nabla = \sum_k \frac{\vec{e}_k}{h_k}\frac{\partial}{\partial x_k}. \qquad (C.37)$$

To calculate grad, div, etc. operations that we encounter in continuum mechanics. Below is a short discussion of potential problem if one is not careful.

C.5.4 Gradient of a vector

The gradient of vector $\vec{v}$ is a non-symmetric, second-order tensor, defined by the left-dyadic product of the nabla operator with a vector,

$$\operatorname{grad}\vec{v} := \nabla \otimes \vec{v}. \qquad (C.49)$$

from (ref1). One should be very careful in putting unit vector

$$\nabla = \sum_k \frac{\vec{e}_k}{h_k}\frac{\partial}{\partial x_k}. \qquad (C.37)$$

e_k at the end of v NOT beginning

This is more discussed on this link (for div operator there)
https://en.wikipedia.org/wiki/Tensor_derivative_(continuum_mechanics)

Cartesian coordinates [edit]

Note: the Einstein summation convention of summing on repeated indices is used below.

In a Cartesian coordinate system we have the following relations for a vector field **v** and a second-order tensor field $\boldsymbol{S}$.

$$\boldsymbol{\nabla}\cdot\mathbf{v} = \frac{\partial v_i}{\partial x_i}$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{S} = \frac{\partial S_{ki}}{\partial x_k}\,\mathbf{e}_i$$

Note that last relation can be found in reference [4] under relation (1.14.13). Note also that according to the same paper in the case of the second-order tensor field, we have:

$$\boldsymbol{\nabla}\cdot\boldsymbol{S} \neq \operatorname{div}\boldsymbol{S} = \boldsymbol{\nabla}\cdot\boldsymbol{S}^{\mathrm{T}}.$$

and grad operator in ref (2)

Although for a scalar field $\operatorname{grad}\phi$ is equivalent to $\nabla\phi$, note that the gradient defined in 1.14.3 is *not* the same as $\nabla\otimes\mathbf{a}$. In fact,

$$(\nabla\otimes\mathbf{a})^{\mathrm{T}} = \operatorname{grad}\mathbf{a} \qquad (1.14.7)$$

since

$$\nabla \otimes \mathbf{a} = \mathbf{e}_i \frac{\partial}{\partial x_i} \otimes a_j \mathbf{e}_j = \frac{\partial a_j}{\partial x_i} \mathbf{e}_i \otimes \mathbf{e}_j \tag{1.14.8}$$

These two different definitions of the gradient of a vector, $\partial a_i / \partial x_j \mathbf{e}_i \otimes \mathbf{e}_j$ and $\partial a_j / \partial x_i \mathbf{e}_i \otimes \mathbf{e}_j$, are both commonly used. In what follows, they will be distinguished by labeling the former as $\text{grad}\mathbf{a}$ (which will be called the gradient of **a**) and the latter as $\nabla \otimes \mathbf{a}$.

Note the following:

- in much of the literature, $\nabla \otimes \mathbf{a}$ is written in the contracted form $\nabla \mathbf{a}$, but the more explicit version is used here.
- some authors define the operation of $\nabla \otimes$ on a vector or tensor $(\bullet)$ not as in 1.14.8, but through $\nabla \otimes (\bullet) \equiv (\partial(\bullet)/\partial x_i) \otimes \mathbf{e}_i$ so that $\nabla \otimes \mathbf{a} = \text{grad}\mathbf{a} = (\partial a_i / \partial x_j)\mathbf{e}_i \otimes \mathbf{e}_j$.