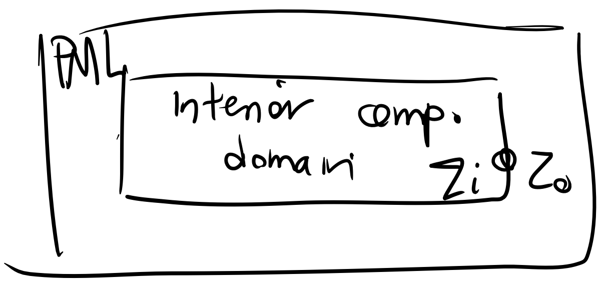
FYI information (beyond our class)

Fun fact: related topic
JOURNAL OF COMPUTATIONAL PHYSICS 114, 185-200 (1994)
A Perfectly Matched Layer for the Absorption of Electromagnetic Waves
JEAN-PIERRE BERENGER
Cited ~ 10,000 times!
http://www.ate.uni-due.de/data/coft1/JOCP_1994_Berenger.pdf

Chew’s work using coordinate stretching in complex plane:
http://onlinelibrary.wiley.com/doi/10.1002/mop.4650071304/abstract
A 3D perfectly matched medium from modified maxwell's equations with stretched coordinates
Weng Cho Chew, William H. Weedon
September 1994
If you cannot find the paper, let me know so that I can send it to you.
Explanation: The first paper (by Berenger) revolutionized many computational fields (electromagnetics, elastodynamics, acoustic, aerodynamics) with applications in modeling unbounded domains in air, outer space, and earth. While the concept was very novel, the formulation was quite complex and proofs and demonstrations were quite involved. Also, the first approach by Chew had some stability problems.

Chew’s idea was genius! He said why not stretch the material in complex space, this ensures that impedances remain matched (between the interior and exterior domain):

$Z_i = Z_0$




So there is no reflection, at the same time by stretching material (and its constitutive equation in complex plain) make it dissipative, so that waves die down inside PML.
In case you look at this paper, some equations would be familiar to you:

where

$$\nabla_e = \hat{x}\frac{1}{e_x}\frac{\partial}{\partial x} + \hat{y}\frac{1}{e_y}\frac{\partial}{\partial y} + \hat{z}\frac{1}{e_z}\frac{\partial}{\partial z} \qquad (5)$$

….
And this is when the coordinate transformation is done in complex plane:

## 5. MODIFIED EQUATIONS IN THE TIME DOMAIN

For the general case of a matched medium, we let $e_x = h_x = s_x$, $e_y = h_y = s_y$ and $e_z = h_z = s_z$. Then, $\nabla_e = \nabla_h = \hat{x}\frac{1}{s_x}\frac{\partial}{\partial x} + \hat{y}\frac{1}{s_y}\frac{\partial}{\partial y} + \hat{z}\frac{1}{s_z}\frac{\partial}{\partial z}$. In Eq. (1), we write the curl as

…

Interestingly, with the use of Christoffel symbols and general grad, div, … expressions in curvilinear systems Chew and collaborators expressed PML formulations for many different geometries: cylinder-> cylindrical coordinate, sphere-> spherical coordinate, etc)

While the details of these two works are clearly outside the scope of this class, it's a demonstration on how simple things such as coordinate transformation (and curvilinear coordinate system of geometries mentioned above) can produce outstanding results!