Hyperbolic PDEs:

Homework 5: Part 1:

- Complete ComputeDG_KM and ComputeDG_F functions in PDE1DFEM class for hyperbolic PDE formulation. Also, add / complete implementation for computing $\mathbf{M}$ in ComputeDG_KM for parabolic PDE (if any changes relative to parabolic formulation is needed).
- Complete the time marching scheme in `function [objout, F, A, slnDGXs, slnDGYs, slnXs, slnYs] = ComputeDG_Sln_Hyperbolic(obj)` by using forward Euler method.
- Then run "config_DG_PeriodicSine.txt" (make sure PDEtype is changed to 2) and send me your PDE1DFEM.m file along with two generated files config_DG_PeriodicSine_u.png and config_DG _PeriodicSine _DuDx.png and the output file config_DG_PeriodicSine.out after scriptSolvePDE.m is ran with `configNameWOExt = 'config_DG_PeriodicSine';` is chosen as the active config file (last entry). Note that no other file must be changed.

Homework 4: Part 2 (Be **brief** in your answers. Try to answer all questions in less than two pages (but be specific and to the point answering the question asked).

1. Stability limit, we need to find a stability limit factor f, so that

$$\Delta t = f \min_e \frac{h_e}{\sqrt{\kappa_e / c_e}} \tag{1}$$

so that forward Euler scheme is stable for `hyperStartOption` = 1 (Riemann flux option), and `hyper1FLambdaScalingOn` = 1 (factor for dimensional consistency included), and element polynomial $p = 1$. Other cases can be studies similarly (different parameters for star value, different element order and time integration scheme). Use `scriptSolvePDE.m` file with `config_DG_LeftEssential0_RightNatural1` as the activ entry to numerically obtain f for these choices of flux, p, spatial dimension, and time integration. You can follow a process similar to that performed for parabolic PDEs.

2. Run `MAIN_PDEComparison.m` with configName taking the values `config_DG_LeftEssential0 _Right Natural1.txt`, `config_DG _PeriodicSine.txt`, and `config_DG _Hyperbolic2regions.txt`. The description of initial boundary value problems can be found in the assignment for elliptic PDEs. Based on the results, answer the following:

   (a) What can be said for the `choiceshyperStartOption = 1` and `hyper1FLambdaScalingOn = 0` (Riemann flux, without dimensional factor fix)? Is the scheme stable?

   (b) What can be said for the `choiceshyperStartOption = 0` and `hyper1FLambdaScalingOn = 0` (Average flux, without dimensional factor fix)? For the step function solution for $u_{,x}$ (2region and flux on the left config files), is the solution u this choice monotonically increasing (or decreasing) and how oscillatory are u and $u_{,x}$?

   (c) Between Riemann flux and average flux (both with `hyper1FLambdaScalingOn = 1`, which one provides a more dissipative solution (waves being more dissipated / smoothened out). In particular, refer to $u_{,x}$ plots for different resolutions for 2 region config results [1].

   (d) The damping matrix $\mathbf{C}$ in $\mathbf{M}\ddot{\mathbf{A}} + \mathbf{C}\dot{\mathbf{A}} + \mathbf{K}\mathbf{A} = \mathbf{F}$ for semi-discrete solution of this problem corresponding to all damping term contributions. One source is if d in $c\ddot{u} + d\dot{u} - \nabla.\kappa\nabla u = Q$ is nonzero. However, in the DG formulation of this problem with one field u interpolated with Riemann or average fluxes $\mathbf{C}$ is still nonzero. What contributes to damping of the solution? Which flux choice provides more damping (Riemann or average)?

[1] More on this topic, specifically for this problem, can be found at "'R. Abedi and S. Mudaliar, Error analysis and comparison of Riemann and average fluxes for a spacetime discontinuous Galerkin electromagnetic formulation In: Proceeding XXXIInd International Union of Radio Science General Assembly & Scientific Symposium, URSI 2017 GASS, Palais des congres, Montreal, Canada August 19-26th, 2017, paper no. 2480 (4 pages)" at www.rezaabedi.com under publications.