2018/01/22

Monday, January 22, 2018 11:08 AM

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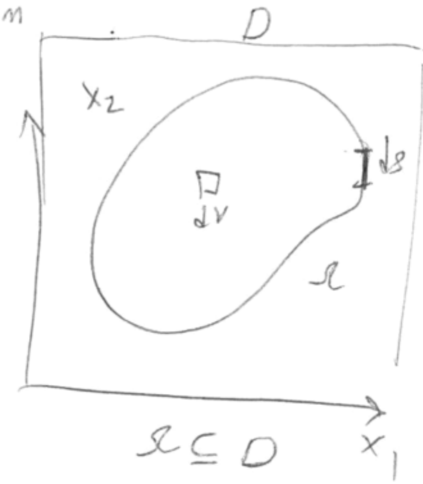
(1) ~~Thermal~~ heat conduction:

Balance law

$$\frac{D}{Dt} \underbrace{E}_{\text{total energy}} = \int_{\Omega} \overbrace{Q}^{\text{heat source term}} \, dV - \int_{\partial\Omega} q \cdot n \, dS$$



$$E = \int_{\Omega} \underbrace{e}_{\text{energy density}} \, dV$$

$$e = \underbrace{C_V}_{\text{Volumetric heat capacity}} T$$

for thermal problem
no other physics ...

$$\frac{D}{Dt} \int_{\Omega} CT = \int_{\Omega} Q \, dV \underbrace{- \int_{\partial\Omega} q \cdot n \, dS}_{\text{divergence theorem}}$$

$$\int_{\Omega} \frac{D}{Dt}(CT) - Q = - \int_{\Omega} \nabla \cdot q \, dV$$

$$\rightarrow \int_{\Omega} \underbrace{\left[\frac{D}{Dt}(CT) + \nabla \cdot q - Q\right]}_{\text{continuous}} dV = 0$$

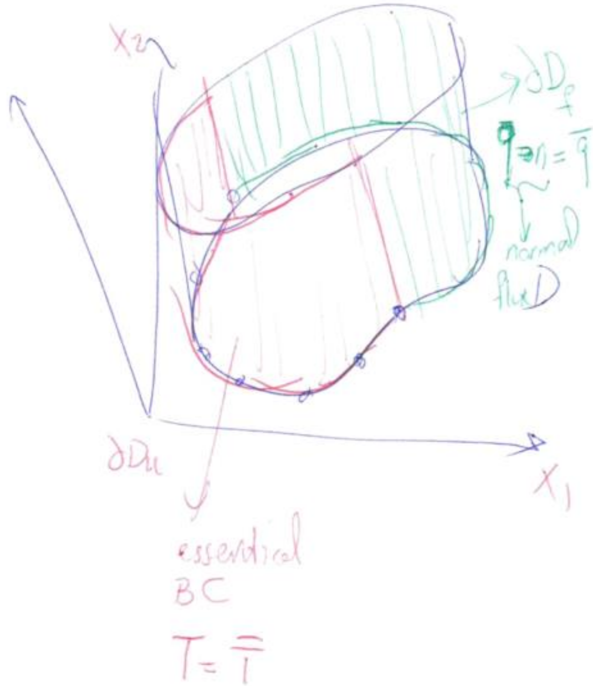
Ω is arbitrary

$$\boxed{\frac{D}{Dt}(CT) + \nabla \cdot q - Q = 0}$$ Differential equation

(2) Residuals

$$R_i = \frac{D}{Dt}(CT) + \nabla \cdot q - Q$$

on D



$$R_u = \bar{T} - T \quad \text{on } \partial D_u$$

essential BC residual

$$R_f = \bar{q} - q \cdot n \quad \text{on } \partial D_f$$

natural BC residual

Types of FE formulation

(A) Continuous finite element method (CFEM)

$$T^h(\vec{x},t) = \phi_p(\vec{x},t) + \sum_{i=1}^{n} \underbrace{N_i(x)}_{\text{shape functions}} a_i(t)$$

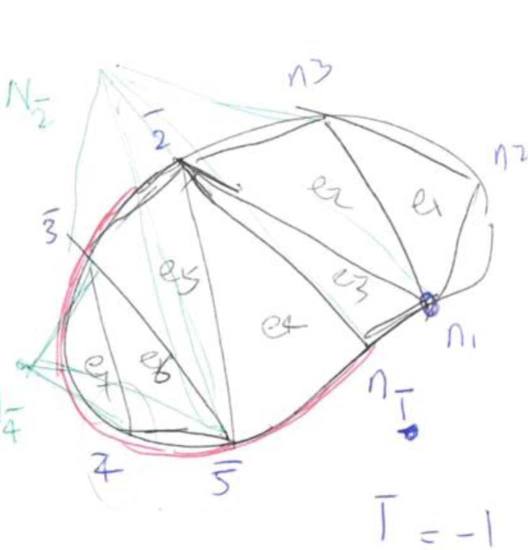
$a_i(t)$: unknown coefficients

$\phi_p(\vec{x},t) = \bar{T}(x,t)$ on ∂D_u $\quad N_i(x) = 0$ on ∂D_u

So we satisfy essential BC a priori

Just need to satisfy PDE (R_i) and natural BC (R_f)

(3)



prescribed temperature at node $n_{\bar i}$ and time time t

$$\phi_p(\vec{x}, t) = \sum_{i=\bar 1}^{\bar 5} N_{\bar i}(x)\, \bar T(n_{\bar i}, t)$$

spatial location

# of prescribed dof

# of prescribed dof

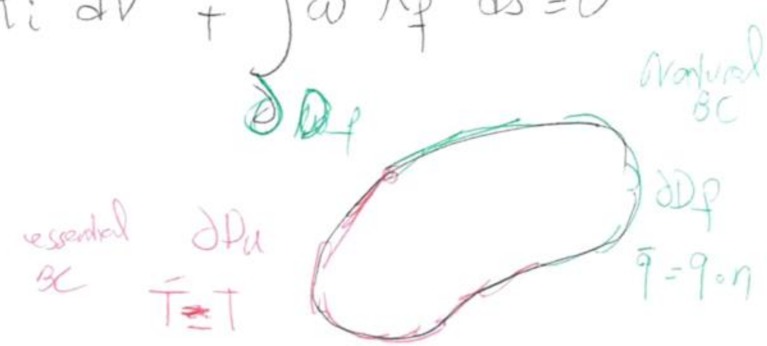
$$\phi_p(n_{\bar j}, t) = \sum_{\bar i = \bar 1}^{\bar n_p} N_{\bar i}(n_{\bar j})\, \bar T(n_{\bar i}, t)$$

$$= \sum_{\bar i = \bar 1}^{\bar n_p} \delta_{\bar i \bar j}\, \bar T(n_{\bar i}, t) = \bar T(n_{\bar j}, t)$$

ϕ_p satisfies essential BC's at all prescribed nodes ($\bar 1$ to $\bar 5$ here)

Forming Weighted residual statement (WRS)

$$\forall \omega \quad \int_D \omega R_i \, dV + \int_{\partial D_f} \omega R_f \, ds = 0$$



④ 0th order in T

2nd order in T

$$\boxed{\int_{D} \omega \left(\frac{D}{Dt}(CT) + \nabla \cdot q - Q \right) dV + \int_{\partial D_f} \omega \left(\bar{q} - q \cdot n \right) ds = 0} \quad (\mathrm{I})$$

Fourier heat law $q = -k \nabla T$
conductivity matrix

$$\nabla \cdot (\omega q) = \nabla \omega \cdot q + \omega \nabla \cdot q$$

$$\int_D \omega \nabla \cdot q \, dV = \int_D \nabla \cdot (\omega q) \, dV - \int_D \nabla \omega \cdot q \, dV$$

$$\boxed{\int_D \omega \nabla \cdot q \, dV = \int_{\partial D} \omega q \cdot n \, ds - \int_D \nabla \omega \cdot q \, dV} \quad (\mathrm{II})$$

Plug (II) into (I)