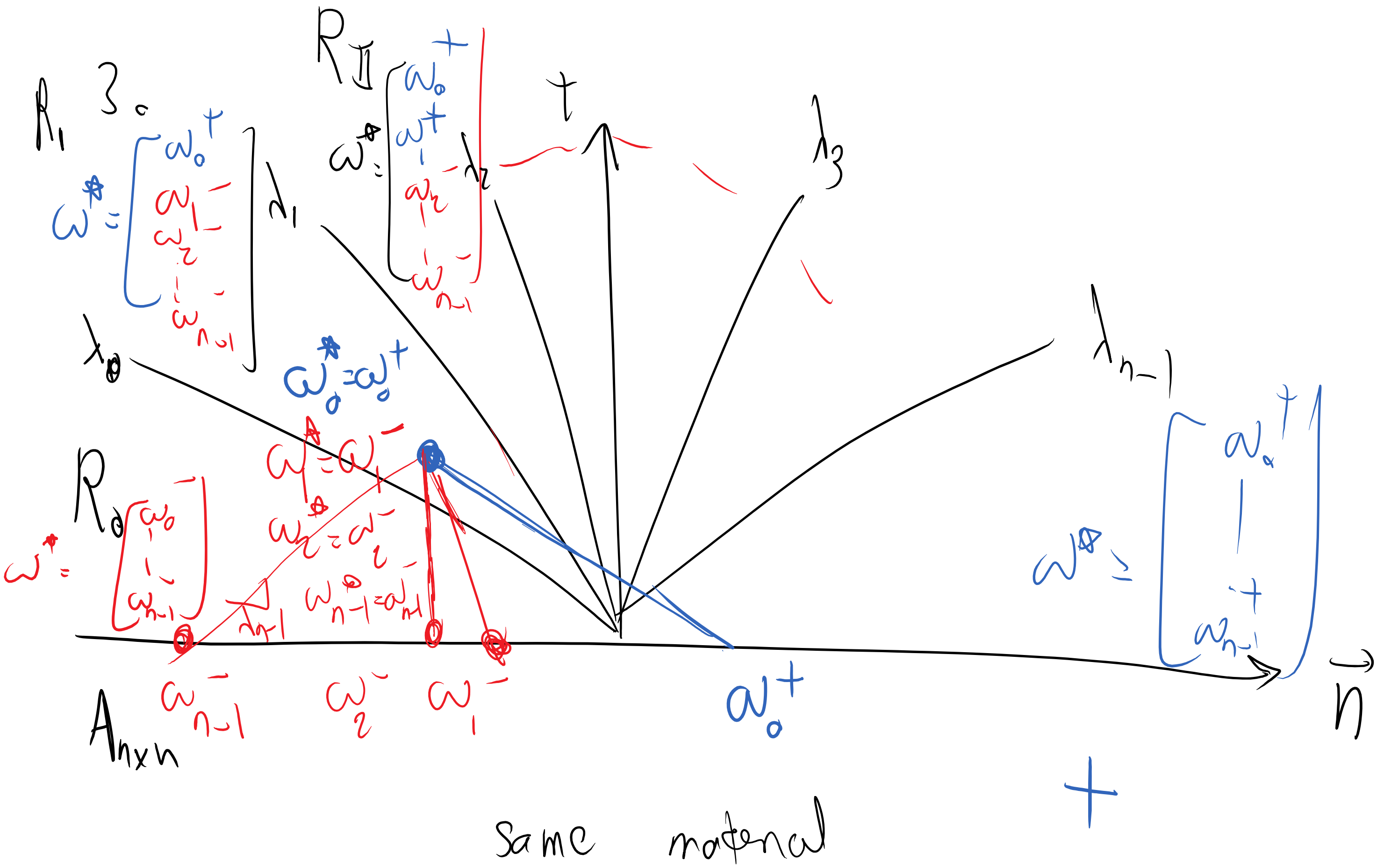
2018/04/04

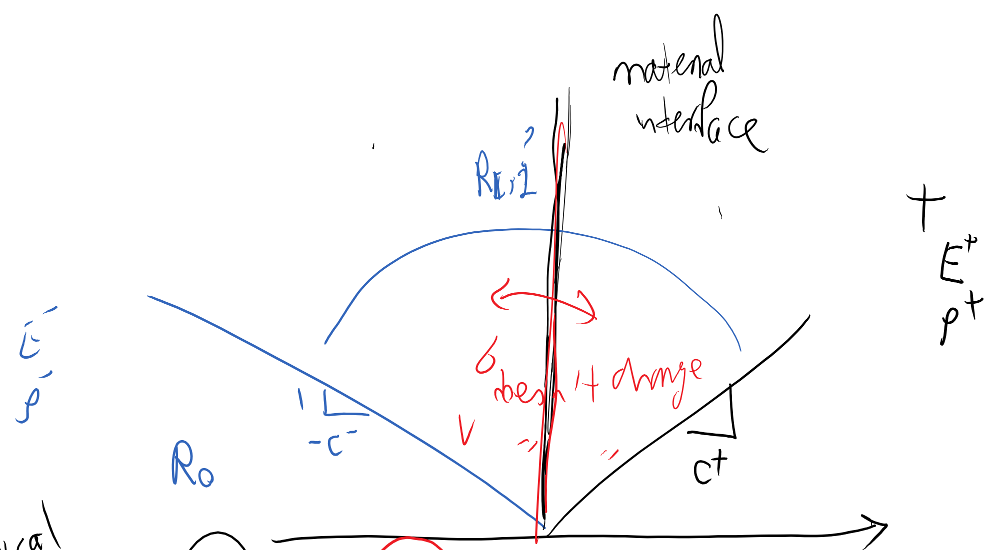
Wednesday, April 04, 2018 11:40 AM



For elastodynamics:

For elastodynamics spatial flux is

$$f_x = \begin{bmatrix} \sigma \\ v \end{bmatrix}$$



typical way $\leftarrow$ $\begin{matrix} p \\ \varepsilon \\ \hline f_t \end{matrix}$ OR $\begin{matrix} \sigma_- \\ v \\ f_x \end{matrix}$ much nicer to use these as IC

correct way of writing the balance law

$$\Gamma \begin{cases} \dot{p} - \sigma_{,x} = \rho b \\ \dot{\varepsilon} - v_{,x} = 0 \end{cases}$$

$$\rho^0 \dot{\varepsilon} - v_{,x} = 0 \qquad \text{balance law}$$

all kinetic quantities

$$f_t = \begin{bmatrix} p \\ \varepsilon \end{bmatrix} \qquad f_x = -\begin{bmatrix} \sigma \\ v \end{bmatrix}$$

If we express the solution for spatial flux quantities and IC is also specified in term of spatial flux ->

1. the mid-regions on the sides of the material interface will have the same solution in terms of the spatial flux
2. BTW, we only need spatial flux on vertical faces

$$\dot{v} \leftarrow \dot{p} - \sigma_{,x} = \rho b \qquad / \rho$$

$$\dot{\sigma} \leftarrow \dot{\varepsilon} - v_{,x} = 0 \qquad \times E$$

$$\boxed{\begin{aligned} \dot{v} - \frac{1}{\rho}\sigma_{,x} &= b \\ \dot{\sigma} - E v_{,x} &= 0 \end{aligned}}$$

☆ This is NOT the right form of the balance laws but for 1 material this is fine

$$q = \begin{bmatrix} v \\ \sigma \end{bmatrix} \qquad A = \begin{bmatrix} 0 & -\frac{1}{\rho} \\ -E & 0 \end{bmatrix} \qquad q_{,t} + A q_{,x} = S \qquad S = \begin{bmatrix} b \\ 0 \end{bmatrix} \rightarrow 0$$

for this Riemann sln

Left eigenvalues & vectors are

$$\det \begin{bmatrix} -\lambda & -\frac{1}{\rho} \\ -E & -\lambda \end{bmatrix} = 0 \quad \longrightarrow \quad \lambda^2 - c^2 = 0 \qquad c = \sqrt{\frac{E}{\rho}}$$

$$\lambda = \mp c$$

Eigen values

$\lambda = -c$

$$\begin{bmatrix} v_1^1 & v_2^1 \end{bmatrix} \begin{bmatrix} c & -\frac{1}{\rho} \\ -E & c \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$v_1^1 c - v_2^1 \underbrace{E}_{\rho c^2} = 0 \longrightarrow v^{1} = [\underbrace{\rho c}_{} \quad 1] \qquad \lambda^1 = -c$$

$$\boxed{Z = \rho c}$$

similarly for $\lambda = c$

$$v^{2} = [-Z \quad 1] \qquad \lambda^2 = c$$

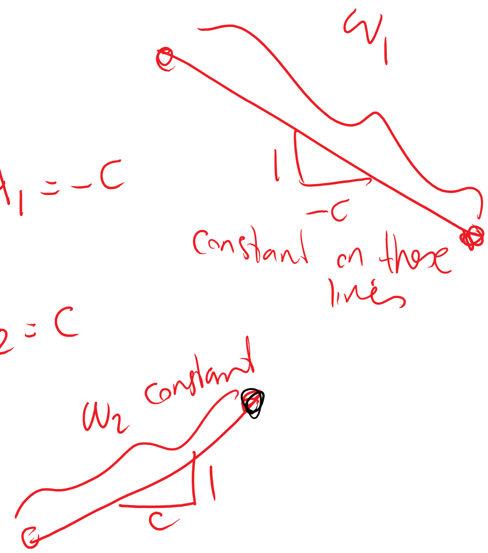
$$U = \begin{bmatrix} v^{1} \\ \hline v^{2} \end{bmatrix} = \begin{bmatrix} Z & 1 \\ -Z & 1 \end{bmatrix} \qquad D = \begin{bmatrix} \lambda^1 & \\ & \lambda^2 \end{bmatrix} = \begin{bmatrix} -c & 0 \\ 0 & c \end{bmatrix}$$

$$\omega = Uq \rightarrow \quad \omega = \begin{bmatrix} Z & 1 \\ -Z & 1 \end{bmatrix}\begin{bmatrix} v \\ \sigma \end{bmatrix}$$

$\rightarrow$

$$\omega_1 = \sigma + Zv \qquad \lambda_1 = -c$$

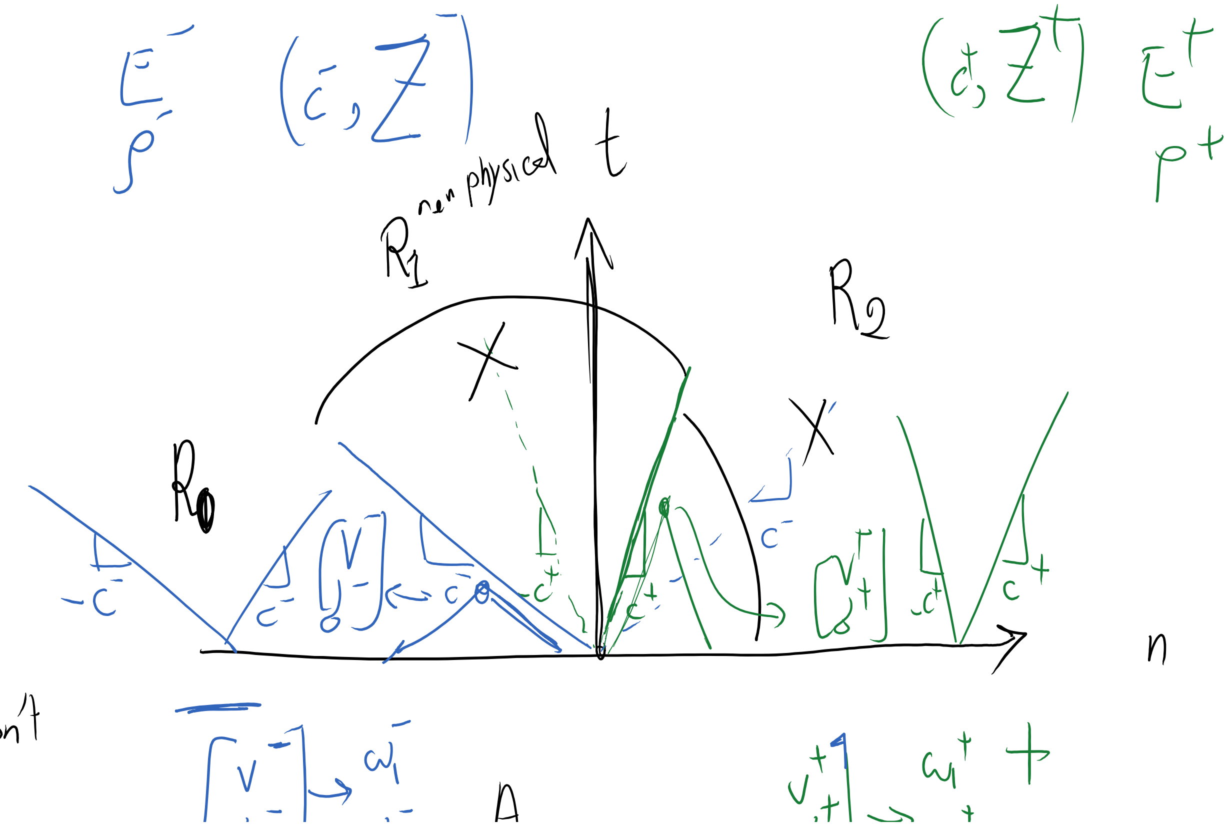
$$\omega_2 = \sigma - Zv \qquad \lambda_2 = c$$



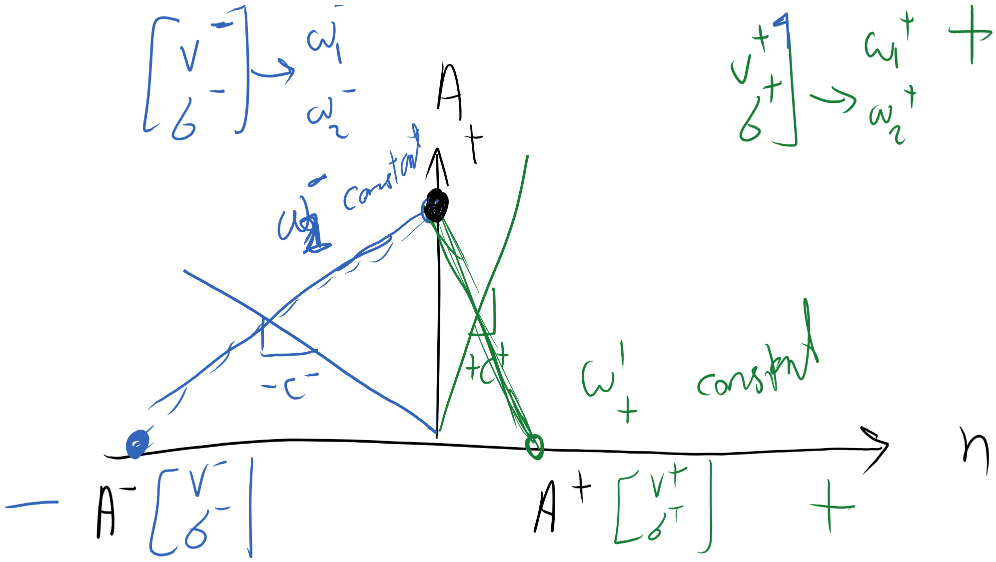
How do we use these for the solution of Riemann values at an interface:

For $\begin{bmatrix} v \\ \sigma \end{bmatrix}$ solution inside R_1 we have different ε & ρ slns

[but we don't care | |



Lbut we don't care !!]



$A^-, A \to \omega_2^A = \omega_2^{A^-}$ all in $-$ material

$A^+, A \to \omega_1^A = \omega_1^{A^+}$ all in $+$ material

$\to$ $\omega_2 = \sigma - Zv$, $\omega_1 = \sigma + Zv$

$$\sigma^A - Z^- v^A = \underset{\sigma^-}{\sigma^{A^-}} - Z^- \underset{v^-}{v^{A^-}}$$

$$\sigma^A + Z^+ v^A = \underset{\sigma^+}{\sigma^{A^+}} + Z^+ \underset{v^+}{v^{A^+}}$$

$$\begin{cases} (1) & \breve{\sigma} - Z^- \breve{v} = \sigma^- - Z^- v^- \\ (2) & \breve{\sigma} + Z^+ \breve{v} = \sigma^+ + Z^+ v^+ \end{cases}$$

breve $\breve{}$ Riemann slns on vertical direction

2 unknowns $\breve{\sigma}, \breve{v}$

2 eqns

(2) $-$ (1) $(Z^- + Z^+)\breve{v} = \ldots \; Z^+v^+ + Z^-v^-$

(2) $-$ (1) $\quad (Z^- + Z^+)\,\check{v} = [\sigma] + Z^+ v^+ + Z^- v^-$

(1) Z^+ + (2) Z^- $\quad (Z^+ + Z^-)\,\check{\sigma} = Z^+\sigma^- + Z^-\sigma^+ + Z^- Z^+ [v]$

$[\sigma] = \sigma^+ - \sigma^-$

$[v] = v^+ - v^-$

$$\check{v} = \frac{Z^- v^- + Z^+ v^+}{Z^- + Z^+} + \frac{1}{Z^- + Z^+}[\sigma]$$

$$\check{\sigma} = \frac{Z^- Z^+}{Z^- + Z^+}[v] + \frac{Z^-\sigma^+ + Z^+\sigma^-}{Z^- + Z^+}$$

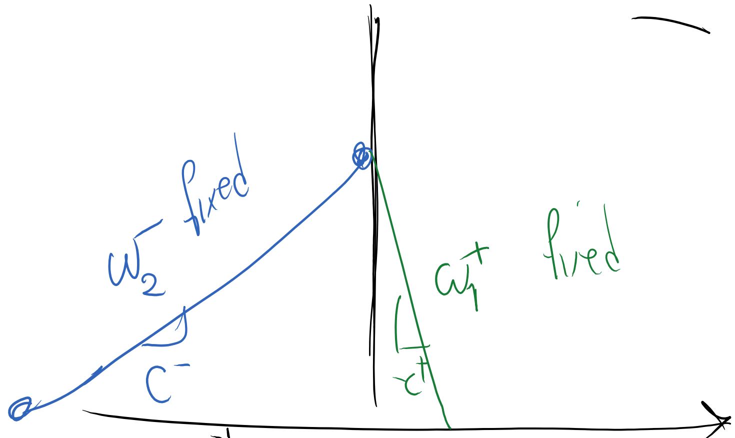
What happens if $Z^- = Z^+ \rightarrow c^-\rho^- = c^+\rho^+ \rightarrow \sqrt{E^-\rho^-} = \sqrt{E^+\rho^+}$

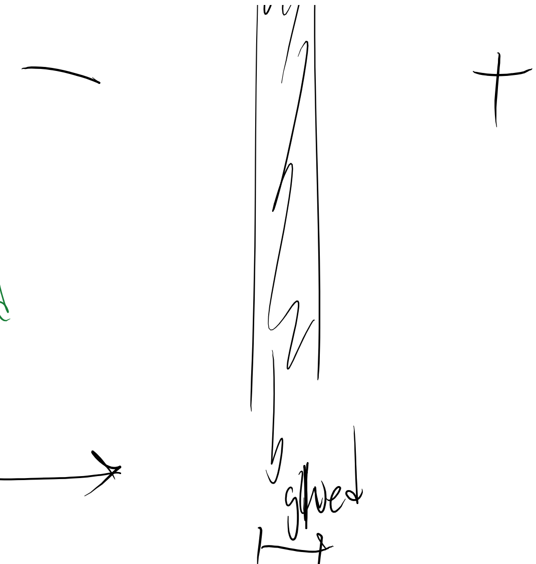
$$\check{v} = \overbrace{\left(\frac{v^- + v^+}{2}\right)}^{\{v\}} + \frac{1}{2Z}[\sigma]$$

$$\check{\sigma} = \frac{Z}{2}[v] + \underbrace{\left(\frac{\sigma^- + \sigma^+}{2}\right)}_{\{\sigma\}}$$

1

How to use Riemann solutions to come up with interesting interface matching conditions:





almost always satisfied

$\sigma = k \Delta u$

+

$\sim \quad \check{\sigma}^- \neq \check{\sigma}^+$ $\quad$ f_x for $\quad \dot{p} - \check{\sigma}_{,x} = \rho b$ $\quad$ BO Lin Momentum

? $\quad \check{v}^- \neq \check{v}^+$ $\quad$ f_x for $\quad \dot{\varepsilon} - \check{v}_{,x} = 0$ $\quad$ Compatibility

$\check{\sigma}$ unique

$\check{v}^- \neq \check{v}^+$

3 unknowns

$\check{v}$ from the left side

− side ω_2 preserved $\quad \check{\sigma} - Z^- \check{v}^- = \sigma^- - Z^- v^-$

+ side ω_1 " $\quad \check{\sigma} + Z^+ \check{v}^+ = \sigma^+ + Z^+ v^+$

$\check{\sigma} = k \Delta u$

$$\gamma = k\Delta u$$

$$\breve{V}^- = \frac{\breve{\gamma} - \sigma^-}{Z^-} + V^-$$

$$\breve{V}^+ = -\frac{\breve{\gamma} - \sigma^+}{Z^+} + V^+$$

$$[\breve{V}] = [V] - \breve{\gamma}\left(\frac{1}{Z^-} + \frac{1}{Z^+}\right) + \left(\frac{\sigma^+}{Z^+} + \frac{\sigma^-}{Z^-}\right)$$

$$\breve{\gamma} = \underbrace{k\Delta\breve{u}}_{\int[\breve{V}]} + \underbrace{d\Delta\dot{\breve{u}}}_{[\breve{V}]}$$

if the glue had damping

$$k\int[\breve{V}] + d[\breve{V}]$$

take time derivative

ODE for the interface

$$\boxed{[\dot{\breve{u}}] = [V] - \left(k[\breve{u}] + d[\dot{\breve{u}}]\right)\left(\frac{1}{Z^-} + \frac{1}{Z^+}\right) + \left(\frac{\sigma^-}{Z^-} + \frac{\sigma^+}{Z^+}\right)}$$

IC e.g. $[\breve{u}] = 0$

2nd approach for Riemann solutions:

This one provides the jump conditions from region to region:

$q^2 \quad + q^3$

Motivation

$q^1 - q^0 = \alpha_1 V_1$, $q^0 = q^-$

$q^2 - q^1 = \alpha_2 V_2$

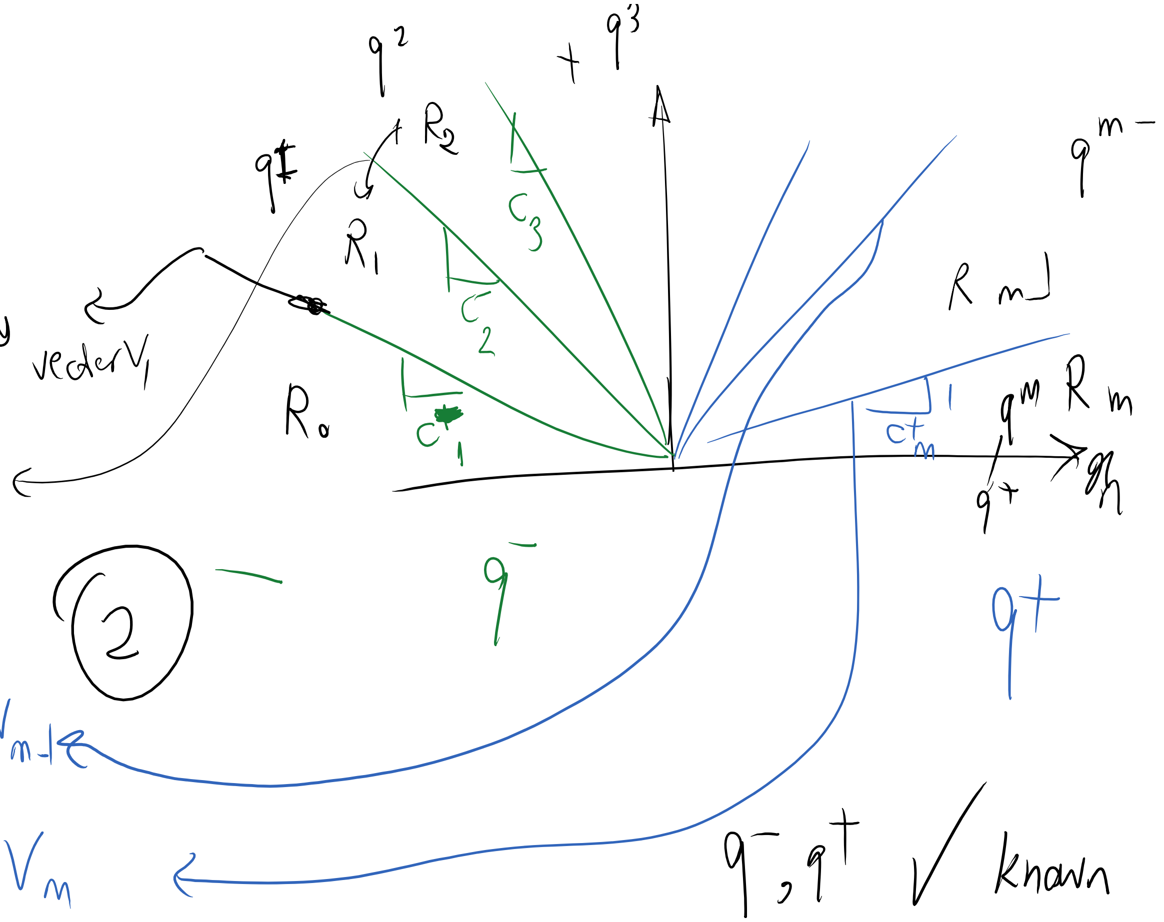
$q^3 - q^2 = \alpha_3 V_3$

$\vdots$

$q^{m-1} - q^{m-2} = \alpha_{m-1} V_{m-1}$

$q^m - q^{m-1} = \alpha_m V_m$

$q^m = q^+$



q^-, q^+ ✓ known

$q^1, q^2, \dots, q^m$ Unknown

eventually $V_1, \dots, V_m$ ✓

$\alpha_1, \dots, \alpha_m$ unknowns

add all the equations

$$q^m - q^0 = V^1\alpha_1 + V^2\alpha_2 + \dots + V^n\alpha_m$$

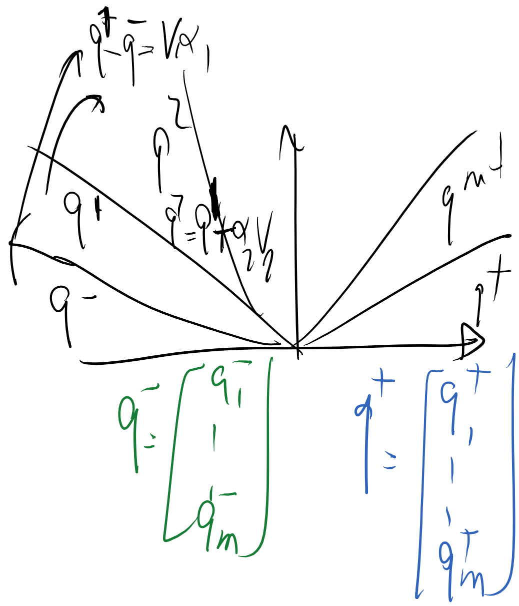
$$q^+ - q^- = \begin{bmatrix} V^1 & V^2 & \cdots & V^m \end{bmatrix}\begin{bmatrix} \alpha_1 \\ \vdots \end{bmatrix}$$

$$= \begin{bmatrix} V \,|\, V \,|\, \cdots \,|\, V \end{bmatrix} \begin{bmatrix} \vdots \\ \alpha_m \end{bmatrix}$$

③ $[q] := q^+ - q^- = V\alpha$

$V := \begin{bmatrix} V_1 \,|\, \cdots \,|\, V^m \end{bmatrix} \quad \alpha = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_m \end{bmatrix}$

$q_{m\times 1}$

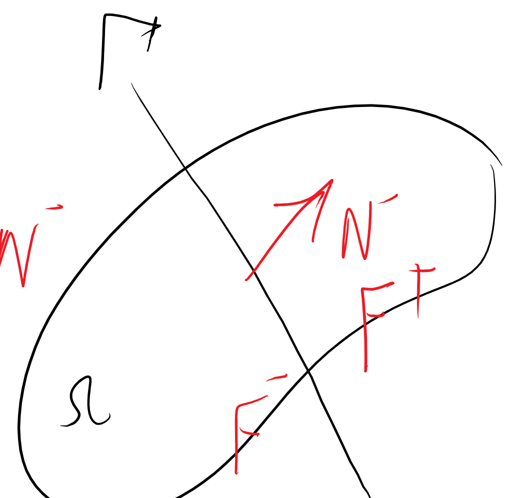


$\alpha = V^{-1}[q] \longrightarrow$ go back to ②

$q^1 = q^- + \alpha^1 V^1, \; q^2 = q^1 + \alpha^2 V^2, \ldots$ we'll get all q's

This method does not require characteristics but uses the jump conditions to derive solution one sector after the other.

balance law $\displaystyle\int_{\partial\Omega} F\, ds = 0 - \int S\, dV \rightarrow \begin{cases} \nabla\cdot F - S = 0 \\ [\![F]\!] = (F^+ - F^-)N^- \\ = -(F^+N^+ + F^-N^-) \end{cases}$



R... problem

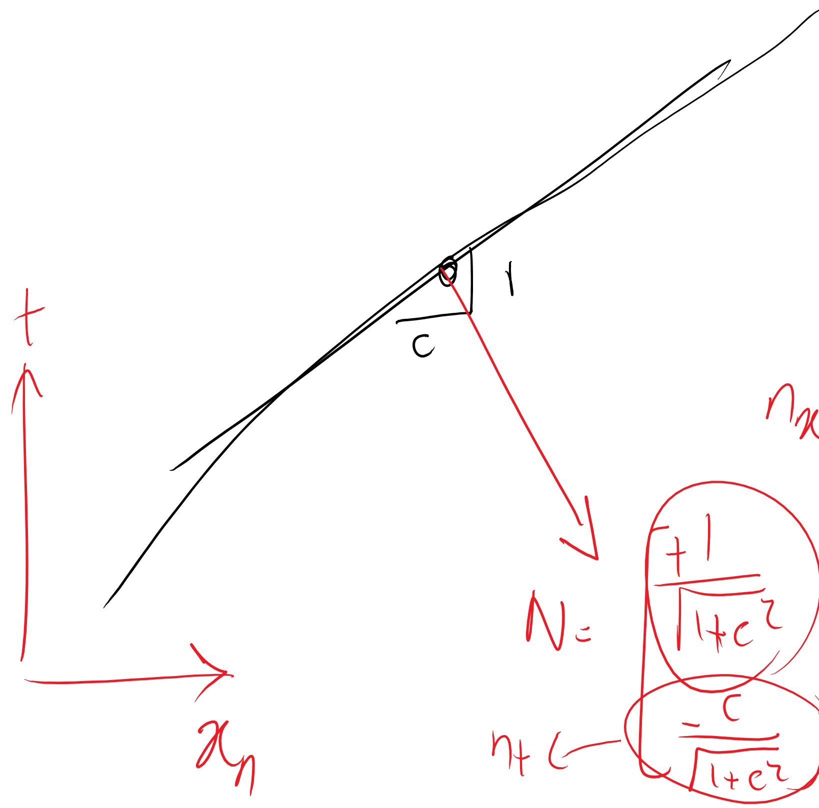
for dynamic problems

$$F = \begin{bmatrix} \vec{f_x} \\ f_t \end{bmatrix}$$

$\vec{f_x}$ → 1 tensor order higher than f_t

$$[F] \cdot N = 0$$

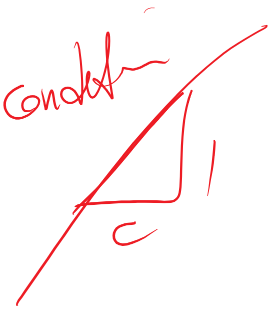
$$\begin{bmatrix} [f_x] \\ [f_t] \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{1+c^2}} \\ \frac{-c}{\sqrt{1+c^2}} \end{bmatrix} = 0$$



$$N = \begin{bmatrix} \frac{+1}{\sqrt{1+c^2}} \\ \frac{-c}{\sqrt{1+c^2}} \end{bmatrix} \begin{matrix} n_x \\ n_t \end{matrix}$$

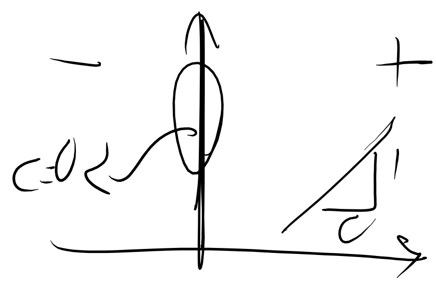
$$[f_x] \frac{1}{\sqrt{1+c^2}} + [f_t] \frac{-c}{\sqrt{1+c^2}} = 0$$

$$\boxed{(5) \quad [f_x] = c\,[f_t] \qquad \text{Jump condition} \qquad F = \begin{bmatrix} f_x \\ f_t \end{bmatrix}}$$



What about a material interface

$$[f_x] = 0\,[f_t] \rightarrow [f_x] = 0$$



— What if the problem is linear

$$\underline{f_x = A q \qquad f_t = q} \longrightarrow \text{put in } (5)$$

and see what we get