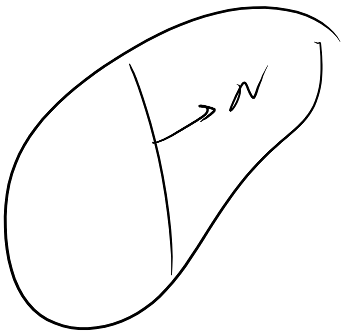
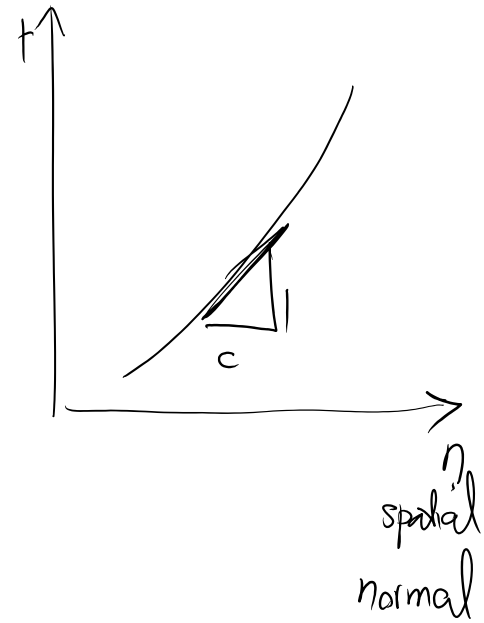
2018/04/06

Friday, April 06, 2018 11:36 AM

The other approach for solving Riemann solutions (using jump conditions):



$[F] \cdot N = 0$ $\xrightarrow{\text{1D setting}}$

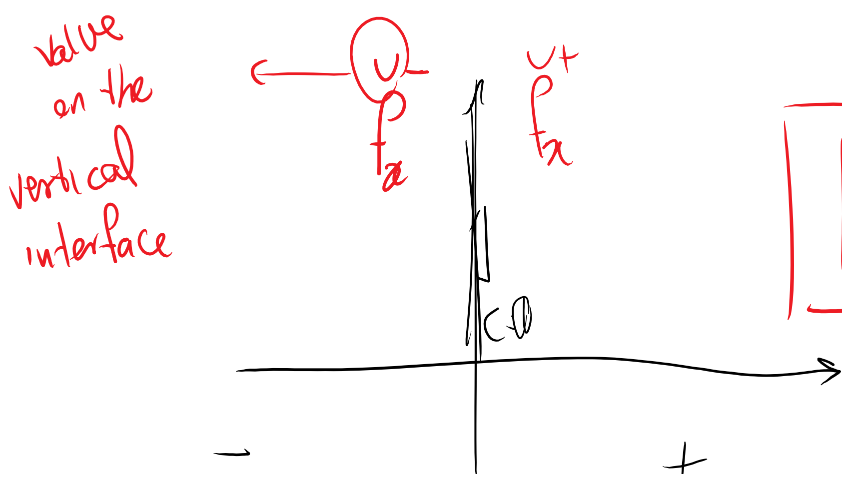


$$\boxed{[f_x] = c[f_t]} \quad (1)$$

So basically jumps on $[f_x]$ & $[f_t]$ are not arbitrary

Spatial cases:

(1) $c=0$ Vertical interface such as a material interface



$$\boxed{[f_x] = 0}$$

$-$ | $+$

(2) $c = 0$ not interesting, because the interface is cancel anyway

$[f_t] = 0$ $f_t^+ = f_t^-$ in continuum setting

So what happens to (1) for a linear problem

$$\left.\begin{aligned} f_t &= q \\ f_x &= Aq \\ [f_x] &= c[f_t] \end{aligned}\right\} \longrightarrow \boxed{[Aq] = c[q]} \quad (2)$$

what about (2) for a material interface problem:

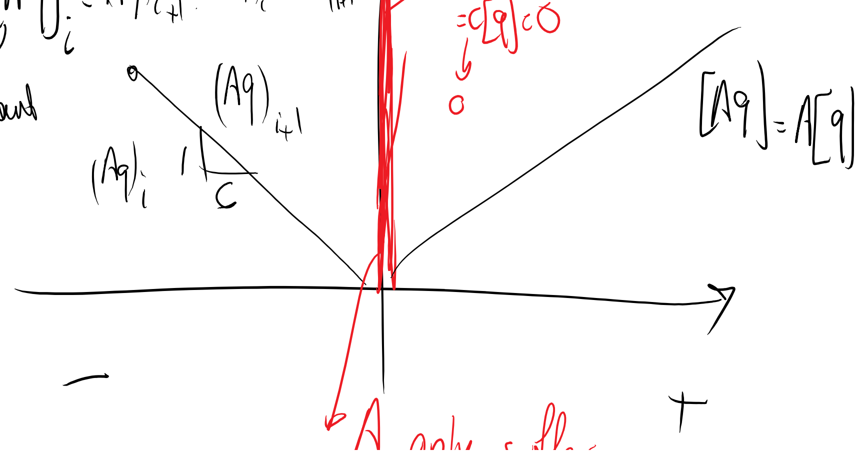
i. $c \neq 0$ $[Aq]_i = (Aq)_{i+1} - (Aq)_i = A(q_{i+1} - q_i)$

material-dependent

$[Aq] = A^+q^+ - A^-q^- = 0$

$= c[q] = 0$

0



Same for $c \neq 0$ on the $+$ side

A only s...

*A only suffers jump here

T

spatial flux is continuous

$[f_x] =$

$A^+q^+ - A^-q^- = 0$ (?)

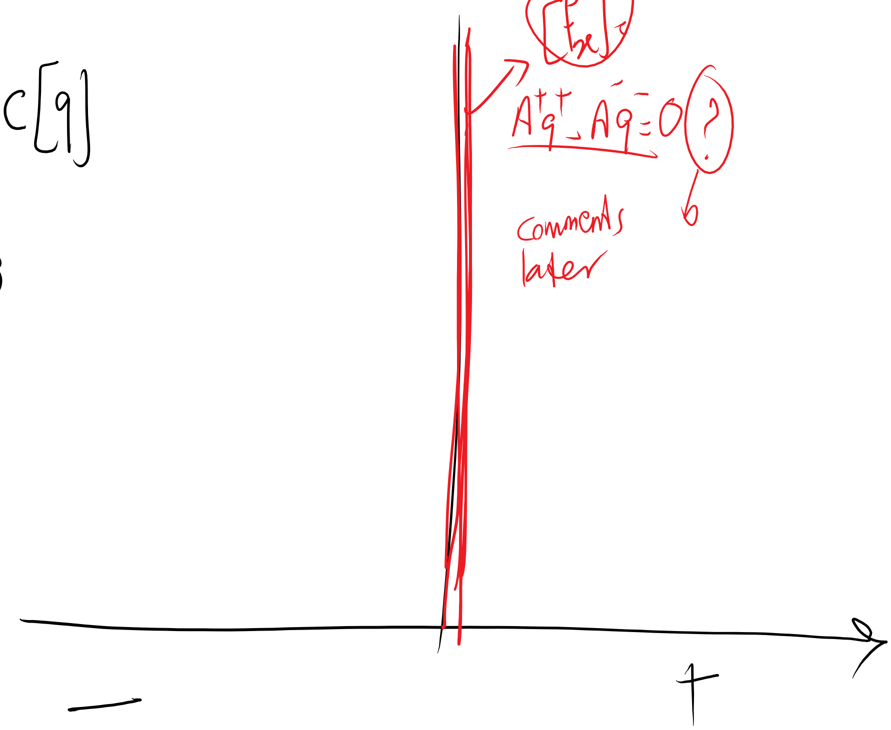
comments later

$A^-[q] = c[q]$

$c < 0$

$A^+[q] = c[q]$

$c > 0$



$A^{\mp}[q]_i = c[q]_i \iff c$ is an eigenvalue

between region (+) & (−)

$[q]_i$ is an eigenvector of A.

Solving elastodynmic problem with the new approach:

Balance of linear momentum

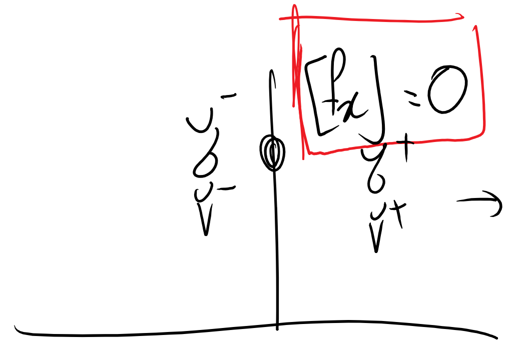
compatibility

$$\boxed{\begin{aligned} \dot{p} - \sigma_{,x} &= \rho b \\ \dot{\epsilon} - v_{,x} &= 0 \end{aligned}}$$

2

$$f_t = q = \begin{bmatrix} p \\ \varepsilon \end{bmatrix} \qquad \boxed{f_x = \begin{bmatrix} -\sigma \\ -v \end{bmatrix}}$$

correct spatial flux for this problem



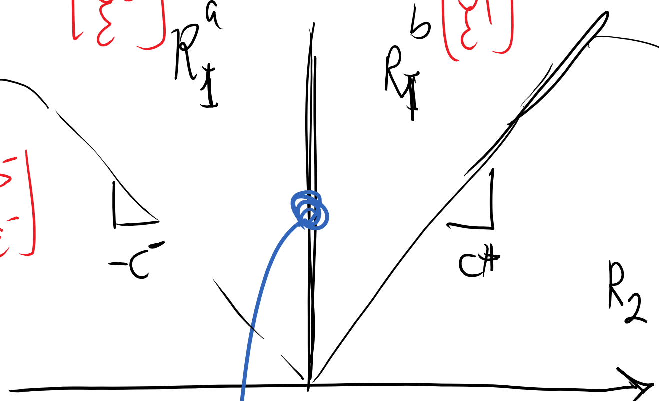
$$\rightarrow \begin{bmatrix} \breve{\sigma} \\ \breve{v} \end{bmatrix} = 0$$

$\breve{\sigma}$, $\breve{v}$ are unique values

$$A = \begin{bmatrix} 0 & -E \\ -\frac{1}{\rho} & 0 \end{bmatrix}$$

$$q_t + A q_x = 0$$

eigenvalues are $\mp c$



$$\begin{bmatrix} \breve{p}^- \\ \breve{\varepsilon}^- \end{bmatrix} - \begin{bmatrix} p^- \\ \varepsilon^- \end{bmatrix} = \alpha_1 V^1$$

first eigenvalue of A^- corresponding to $-c^-$

p^-, ε^- $\qquad$ p^+, ε^+

$$\begin{bmatrix} \breve{p}^+ \\ \breve{\varepsilon}^+ \end{bmatrix} - \begin{bmatrix} p^+ \\ \varepsilon^+ \end{bmatrix} = \alpha_2 V^2$$

2nd eigenvector of A^+ corresponding to c^+

$\boxed{[f_x] = 0}$ always true (except some parts of compatibility — not the balance law parts — are relaxed)

$f_x = A f_t \rightarrow \quad A^- \begin{bmatrix} \breve{p}^- \\ \breve{\varepsilon}^- \end{bmatrix} = A^+ \begin{bmatrix} \breve{p}^+ \\ \breve{\varepsilon}^+ \end{bmatrix}$

$[f_x] = 0 \equiv \text{Jump} \begin{bmatrix} -\breve{\sigma} \\ -\breve{v} \end{bmatrix} = 0$

6 unknowns $\begin{pmatrix} \breve{p}^{\mp} \\ \breve{\varepsilon}^{\mp} \end{pmatrix}$ & α_1, α_2 , 6 eqn 2x2 jumps

& 2 for $[f_x] = 0$)

and we solve it.

Tricky place CORRECT

$[f_x] = 0$ must be satisfied at the interface

If want to be adventurous and solve the problem with fewer unknowns we can choose the quantities that form spatial flux as unknowns

$$\dot{p} - \sigma_{,x} = \rho b \longrightarrow \times \frac{1}{\rho} \qquad \dot{v} - \frac{1}{\rho}\sigma_{,x} = \cancel{b}\ 0$$

$$\dot{\varepsilon} - v_{,x} = 0 \rightarrow \times E \qquad \dot{\sigma} - E\, v_{,x} = 0$$

$$q = \begin{bmatrix} v \\ \sigma \end{bmatrix} \qquad q_{,t} + A\, q_{,x} = 0$$

$$A = \begin{bmatrix} 0 & -\frac{1}{\rho} \\ -E & 0 \end{bmatrix} \qquad [Aq] = 0$$

is it correct $\begin{bmatrix} -\frac{1}{\rho}\sigma \\ -Ev \end{bmatrix} = 0$

No it's not

but we know for this problem in conservation law correct form $f_x = \begin{bmatrix} -\sigma \\ -v \end{bmatrix} \Rightarrow \sigma,\ v$ continuous on the material interface

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$$q_{,t} + A\, q_{,x} = 0 \qquad A = \begin{bmatrix} 0 & -\frac{1}{\rho} \\ -E & 0 \end{bmatrix}$$

$q_{,t} + A\, q_{,x} = 0 \qquad A = \begin{bmatrix} & \\ -E & 0 \end{bmatrix}$

to get the jump conditions: $\rightarrow$ solve the RIGHT eigenvalue problem

$$\det(A - \lambda I) = 0 \qquad \det\begin{bmatrix} -\lambda & -\frac{1}{\rho} \\ -E & -\lambda \end{bmatrix} = 0$$

$$\rightarrow \quad \lambda = \mp c \qquad c = \sqrt{\frac{E}{\rho}}$$

eigenvectors

$\lambda = -c$

$$\begin{bmatrix} c & -\frac{1}{\rho} \\ -E & c \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = 0 \qquad c v_1 - \frac{1}{\rho} v_2 = 0 \rightarrow$$

$$\underbrace{c\rho}_{Z}\, v_1 - v_2 = 0$$

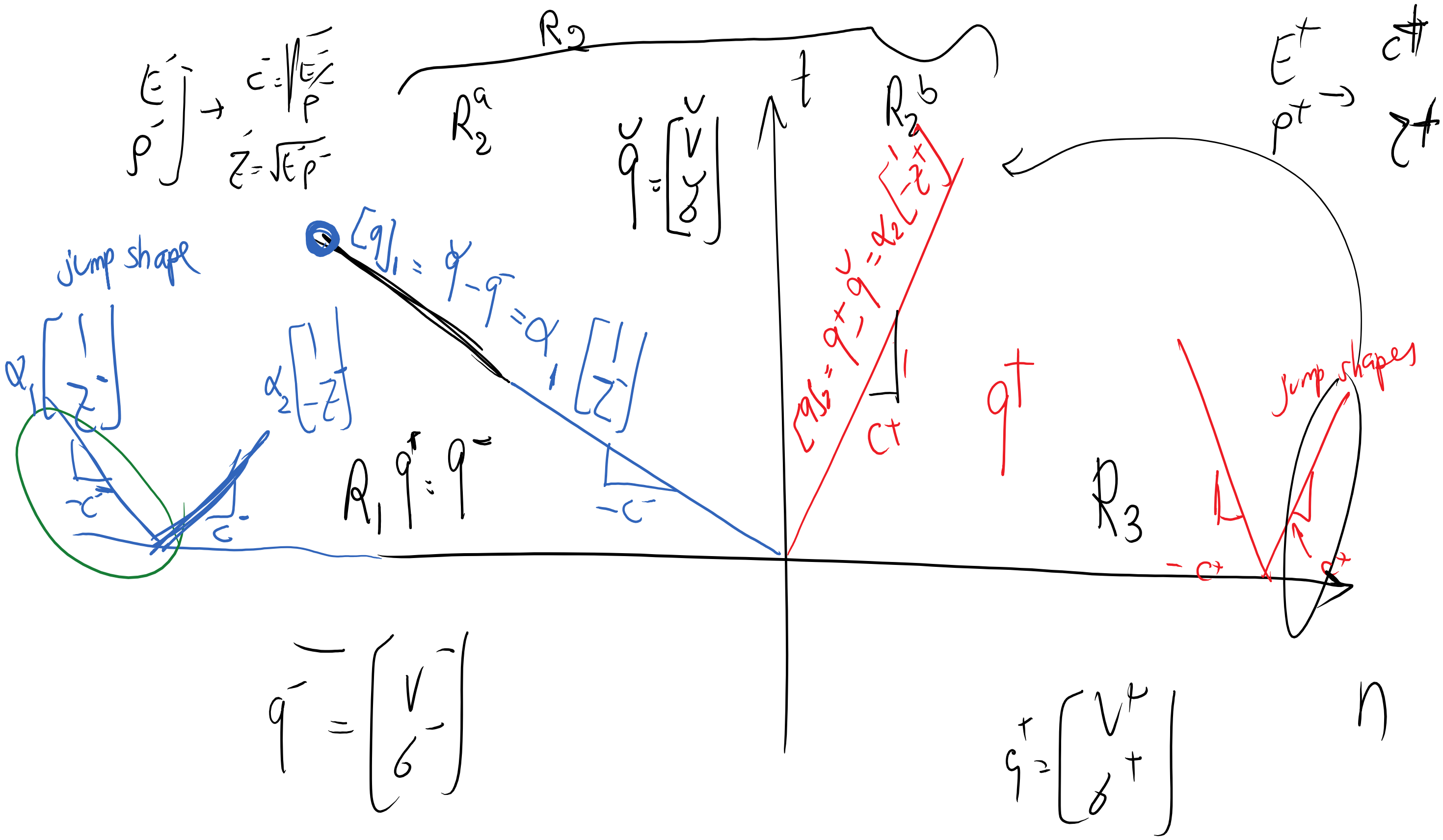
$$v_1 = 1 \quad v_2 = Z$$

$$\lambda^1 = -c, \quad \vec{V}^1 = \begin{bmatrix} 1 \\ Z \end{bmatrix}$$

similarly

$$\lambda^2 = c \qquad \vec{V}^2 = \begin{bmatrix} 1 \\ -Z \end{bmatrix}$$

3



$$\begin{cases} [q]_1 = \check{q} - q^- = \alpha_1 \begin{bmatrix} 1 \\ Z^- \end{bmatrix} \\ [q]_2 = q^+ - \check{q} = \alpha_2 \begin{bmatrix} 1 \\ -Z^+ \end{bmatrix} \end{cases}$$

add all together

$$[q] := q^+ - q^- = \underbrace{\begin{bmatrix} 1 & 1 \\ Z^- & -Z^+ \end{bmatrix}}_{R} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} \quad (4)$$

no subscript

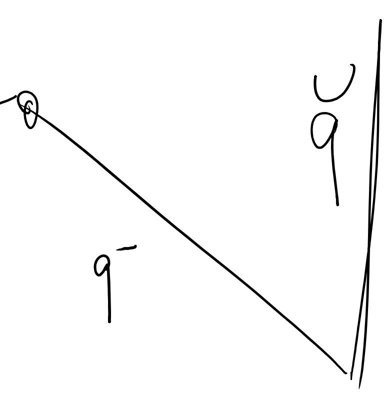
V_1

V $\quad V^1_- \quad V^2_+$

$$\begin{bmatrix}\alpha_1\\ \alpha_2\end{bmatrix} = V^{-1}[q] = \frac{1}{Z^- + Z^+}\begin{bmatrix} Z^+ & 1 \\ Z^- & -1\end{bmatrix}[q]$$

$$\rightarrow \boxed{\begin{aligned}\alpha_1 &= \frac{Z^+[v] + [\sigma]}{Z^- + Z^+}\\ \alpha_2 &= \frac{Z^-[v] - [\sigma]}{Z^- + Z^+}\end{aligned}} \qquad (6)$$

Now we have α's we can obtain q's one region after the other:



$$\breve{q} - q^- = \alpha_1 V_1$$

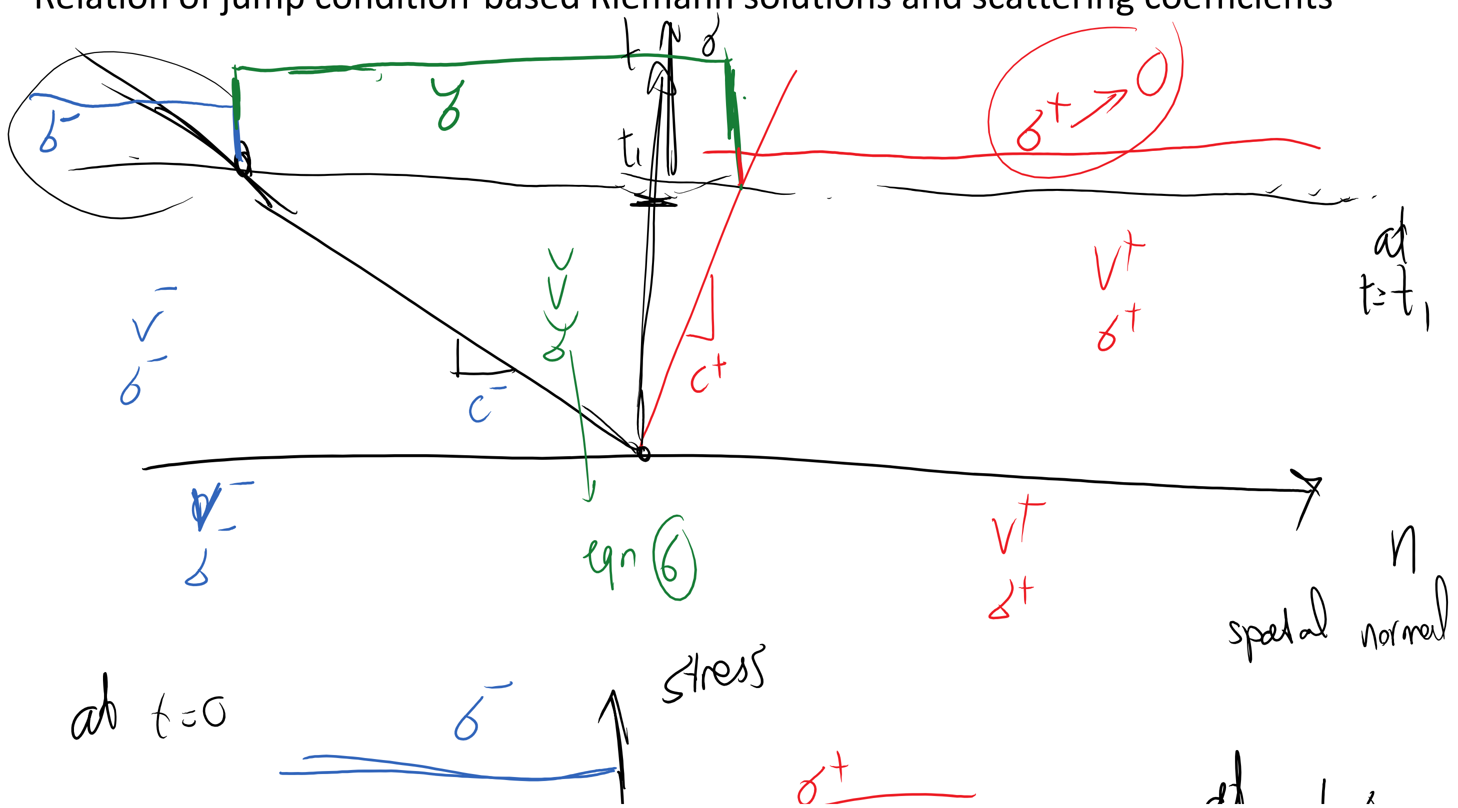
$$\breve{q} = q^- + \alpha_1\begin{bmatrix}1\\ Z^-\end{bmatrix} = \begin{bmatrix}v^-\\ \sigma^-\end{bmatrix} + \frac{Z^+[v] + [\sigma]}{Z^- + Z^+}\begin{bmatrix}1\\ Z^-\end{bmatrix}$$

$$\rightarrow \quad \check{q} = \begin{bmatrix} \check{v} \\ \check{\sigma} \end{bmatrix} = \begin{bmatrix} v^- + \dfrac{Z^+[v] + [\sigma]}{Z^- + Z^+} \\ \sigma^- + \dfrac{Z^+[v] + [\sigma]}{Z^- + Z^+} Z^- \end{bmatrix}$$

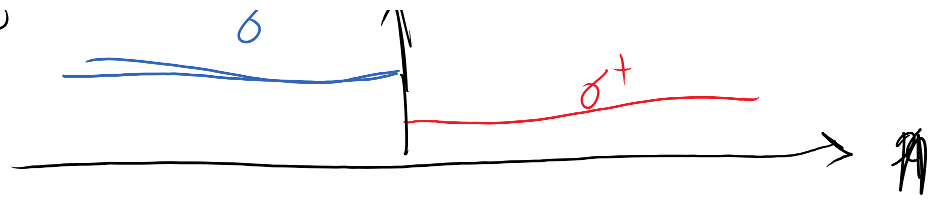
$$\rightarrow \quad \boxed{\begin{aligned} \check{v} &= \frac{v^- Z^- + v^+ Z^+}{Z^- + Z^+} + \frac{1}{Z^- + Z^+}[\sigma] \\ \check{\sigma} &= \frac{\sigma^- Z^+ + \sigma^+ Z^-}{Z^- + Z^+} + \frac{Z^- Z^+}{Z^- + Z^+}[v] \end{aligned}} \quad (6)$$

As expected, this matches our solution from characteristic approach.

Relation of jump condition-based Riemann solutions and scattering coefficients



at $t=0$



at $t=0$

To compute tranmission and reflection coefficients we assume that there is a stress wave moving from the left side (-) and has reached the material interface at time t = 0. On the right side (+) solution is zero at time zero ($\sigma^+, v^+ = 0$) (1)

As will be seen later "a stress wave moving to the right on the left side" -> imposes a relation between σ^-, v^- (2)

$$T^\sigma_{12} = S^\sigma_{12} = \frac{\check{\sigma}}{\sigma^-}$$

often not shown

scattering coefficient

side it goes to

side wave moves from

stress going in this region

makes it non-dimensional

$$R^\sigma_{11} = S^\sigma_{11} = \frac{\check{\sigma} - \sigma^-}{\sigma^-}$$

reflected magnitude

side the wave goes to

$$\boxed{T^\sigma = S^\sigma_{12} = \frac{\check{\sigma}}{\sigma^-} \qquad T, R, I}$$

$$T = S_{12} = \frac{\check{\sigma}}{\sigma^-} \qquad T = R + 1$$

$$R^\sigma = S_{11}^\sigma = \frac{\check{\sigma} - \sigma^-}{\sigma^-} = \frac{\check{\sigma}}{\sigma^-} - 1 \qquad S_{12}^\sigma = S_{11}^\sigma + 1 \qquad (7)$$

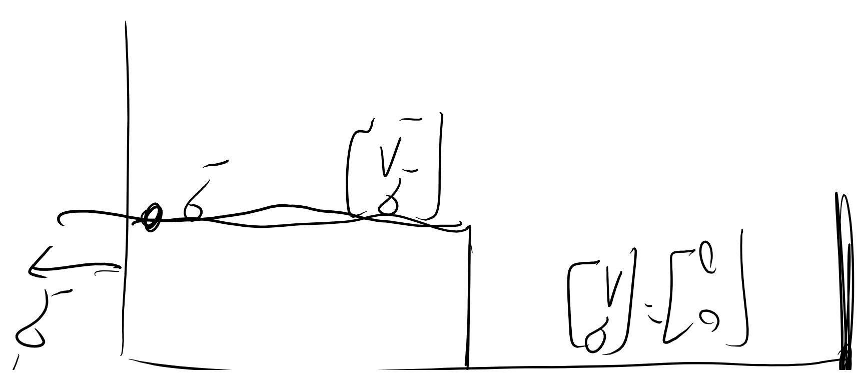
All we need to do is to plug $\check{\sigma}$ in (7)

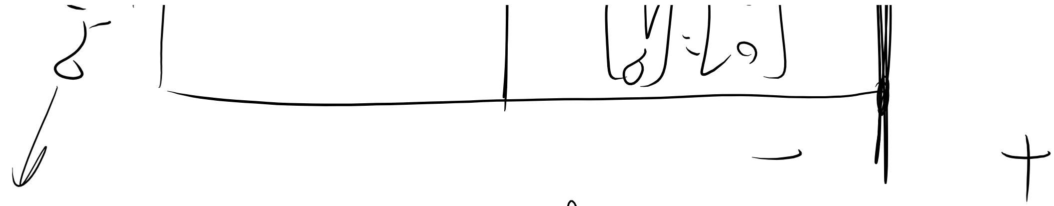
$$\check{\sigma} = \frac{Z^- \cancel{\sigma^+}^{\,\to 0} + Z^+ \sigma^-}{Z^- + Z^+} + \frac{Z^- Z^+}{Z^- + Z^+}[v]$$

$$[v] = \overset{\to 0}{v^+} - v^- = -v^-$$

$$\boxed{\check{\sigma} = \frac{Z^+ \sigma^-}{Z^- + Z^+} - \frac{Z^- Z^+}{Z^- + Z^+} v^-} \qquad (8)$$

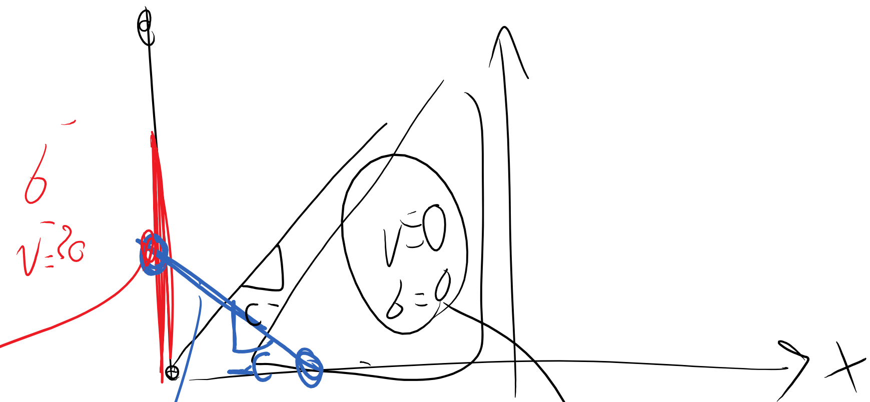
For a "right-going" wave is there a condition between σ^-, v^-





if $\bar{\delta}$ is applied   natural BC $\bar{V}$ is not specified

where $\bar{v}$ comes from?



$\omega' = \text{constant}$ $\omega' = \delta + Z$

$$\bar{\delta} + \bar{Z}\bar{v} = 0 + \bar{Z}\cdot 0 = 0$$

$$\boxed{\bar{v} = -\frac{\bar{\delta}}{\bar{Z}}} \qquad (9)$$

$$|V| = \frac{\delta}{Z}$$

$$|\delta| = |V| Z$$

plug (9) into (8) to get:

$$\gamma = \frac{Z^+ \sigma^-}{Z^- + Z^+} - \frac{Z^- Z^+}{Z^- + Z^+}\left(\frac{-\sigma^-}{Z^-}\right)$$

$$\gamma = \frac{Z^+ \sigma^- + Z^+ \sigma^-}{Z^- + Z^+} \rightarrow \boxed{\gamma = \frac{2Z^+}{Z^- + Z^+}\sigma^-} \quad 10$$

$$\rightarrow T = \frac{\gamma}{\sigma^-} = \frac{2Z^+}{Z^- + Z^+}$$

$$R = \frac{\gamma - \sigma^-}{\sigma^-} = T - 1 = \frac{2Z^+ - Z^- - Z^+}{Z^- + Z^+} =$$

$$\frac{Z^+ - Z^-}{Z^- + Z^+}$$

$$\boxed{\begin{aligned} T &= \frac{2Z^+}{Z^- + Z^+} \\ R &= \frac{Z^+ - Z^-}{Z + Z^+} \end{aligned}} \quad \begin{aligned} &S_{12} \\ &S_{11} \end{aligned}$$