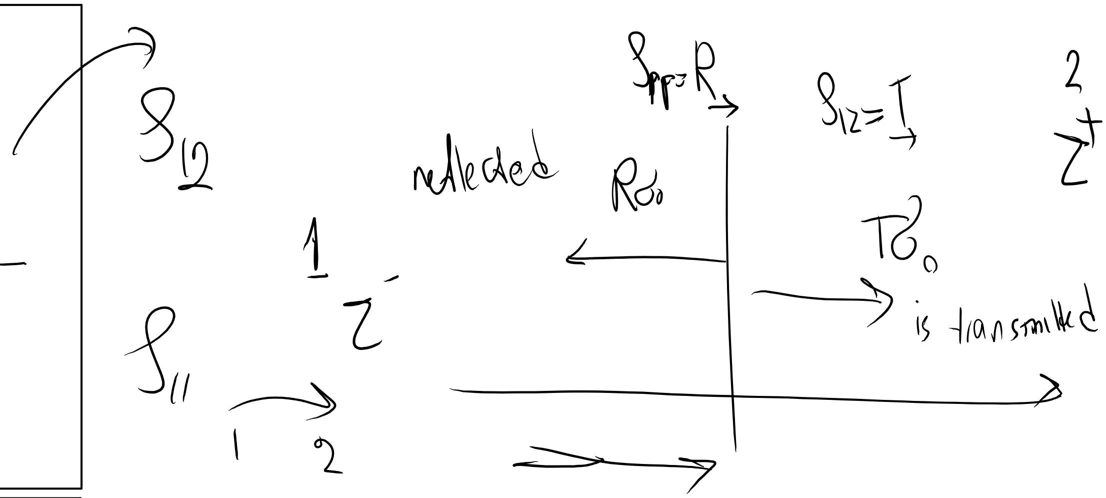
2018/04/09

Monday, April 09, 2018 11:39 AM

From last time we derived T, R coefficients for stress

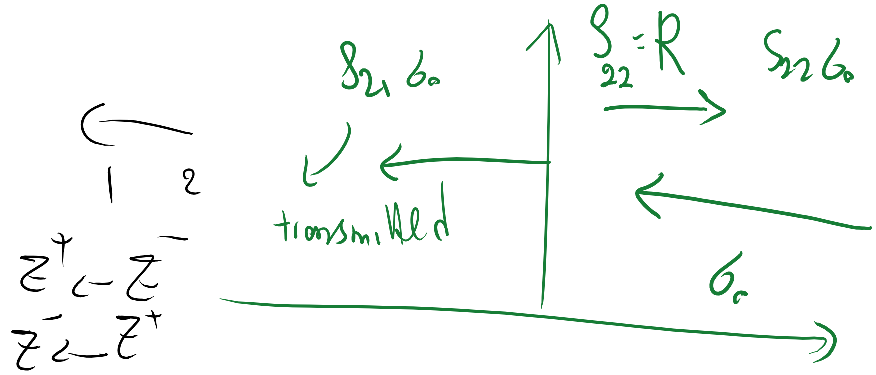
$$S_{12} = T = \frac{2Z^+}{Z^- + Z^+}$$

$$S_{11} = R = \frac{Z^+ - Z^-}{Z^- + Z^+}$$



$$S_{21} = \frac{2Z^-}{Z^- + Z^+}$$

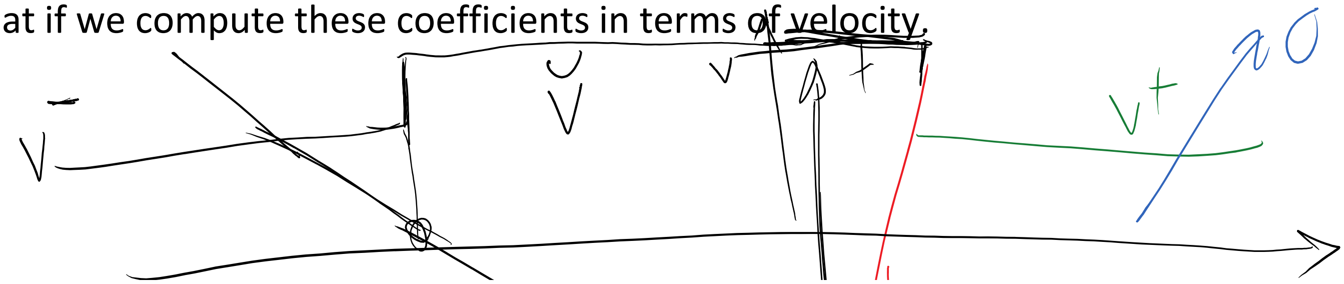
$$S_{22} = \frac{Z^- - Z^+}{Z^- + Z^+}$$

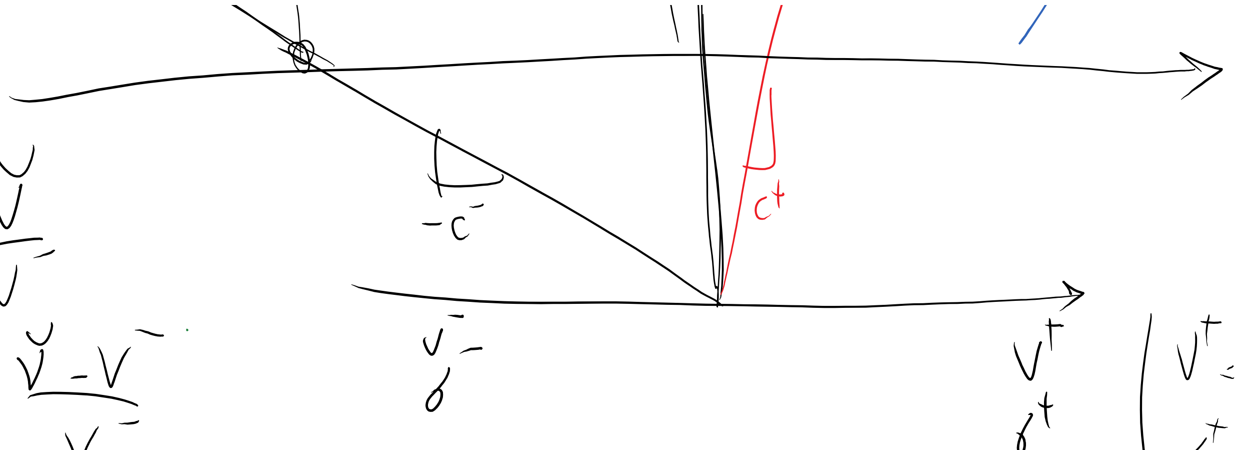


for stress ①

$$S^\sigma = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \frac{Z^+ - Z^-}{Z^+ + Z^-} & \frac{2Z^+}{Z^+ + Z^-} \\ \frac{2Z^-}{Z^- + Z^+} & \frac{Z^+ - Z^-}{Z^- + Z^+} \end{bmatrix}$$

What if we compute these coefficients in terms of velocity.





$$T^v = \frac{\check{v}}{v^-}$$

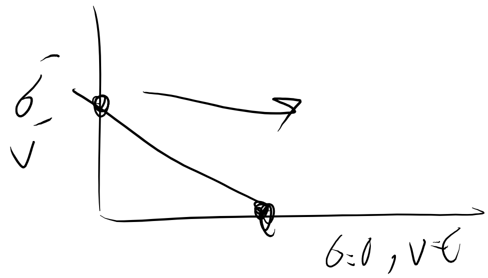
$$R^v = \frac{\check{v} - v^-}{v^-}$$

$$\left(\begin{array}{l} v^+ = 0 \\ \sigma^+ = 0 \end{array} \right)$$

To calculate transmission reflection coefficients we do the following:

1. Everything on the RHS (+ side) is zero.
2. The solution on the LHS (- side) corresponds to a right-going wave.

$$\sigma^- + Z^- v^- = 0 + Z^- 0$$



$$\boxed{\sigma^- = -Z^- v^-}$$

$$T^v = \frac{\check{v}}{v^-}$$

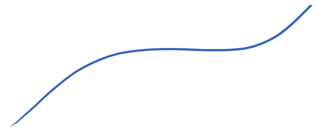
$$R^v = \frac{\check{v} - v^-}{v^-}$$

$$\check{v} = \frac{v^- Z^- + \cancel{v^+}^{\,0} Z^+}{Z^- + Z^+} + \frac{1}{Z^- + Z^+}\left(\cancel{\sigma^+}^{\,0} - \sigma^-\right)$$

(a) $v^+ = 0, \sigma^+ = 0$ (a)

(a) $v^T = 0, \; b^T = 0$ (a)

(b) Right-going wave



$$b^- = -Z^- v^-$$

$$\check{V} = \frac{v^- Z^-}{Z^- + Z^+} + \frac{-(-Z^- v^-)}{Z^- + Z^+}$$

$$\check{V} = \frac{2Z^-}{Z^- + Z^+} \longrightarrow$$

(2)

$$\boxed{\begin{aligned} T^v_{1\to 2} &= \frac{2Z^-}{Z^- + Z^+} \\ T^b_{1\to 2} &= \frac{2Z^+}{Z^- + Z^+} \end{aligned}}$$

$$Y = \frac{1}{Z}$$

$$T^v_{12} = \frac{2\left(\frac{1}{Y^-}\right)}{\frac{1}{Y^-} + \frac{1}{Y^+}} = \frac{2Y^+}{Y^- + Y^+}$$

look the same

Z : impedance $\qquad$ Y : transmittance

$\sigma = Zv \qquad v = Y\sigma$

$$\mathcal{S}^{\sigma} = \begin{bmatrix} \frac{Z^+ - Z^-}{Z^- + Z^+} & \frac{2Z^+}{Z^- + Z^+} \\ \frac{2Z^-}{Z^- + Z^+} & \frac{Z^- - Z^+}{Z^- + Z^+} \end{bmatrix} \xrightarrow{Z \to Y} \mathcal{S}^{v} = \begin{bmatrix} \frac{Y^+ - Y^-}{Y^+ + Y^-} & \frac{2Y^+}{Y^- + Y^+} \\ \frac{2Y^-}{Y^- + Y^+} & \frac{Y^- - Y^+}{Y^- + Y^+} \end{bmatrix}$$

$R_{1\to 2}$ $\quad$ $T_{1\to 2}$

1. What happens when Z- = Z+?

$(c\rho)^- = (c\rho)^+$

$R' s = 0 \qquad T = 1$

The wave is not reflected at all and all of it is transmitted to the other material.

Impedance & transmittance for different problems

Elastodynamic	σ	v	c	Z	E	ρ	$\sigma = E\varepsilon$, $p = \rho v$
3D	σ_{11}	v_1	$c_d \sqrt{\frac{\lambda + 2\mu}{\rho}}$	Z_d	(E, ν)		

wave propagation parameters

constitutive

primary parameters

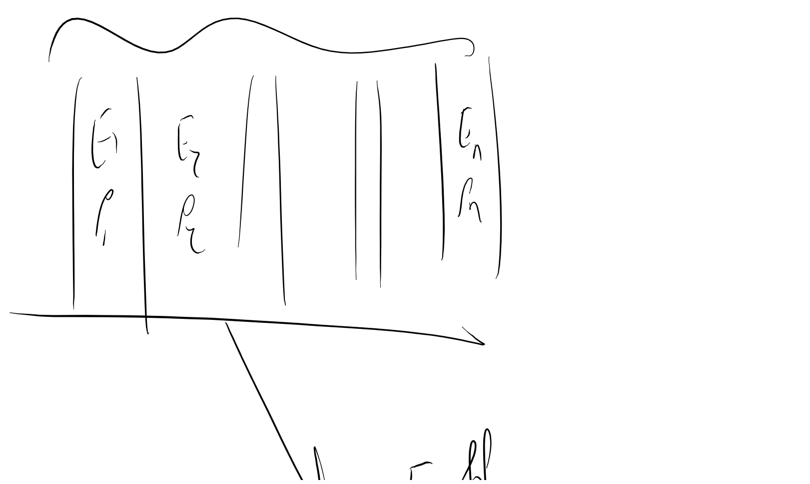
3D, 1D isotropic	σ_{11} ⟷	v_1 ⟷	$c_d\sqrt{\frac{\lambda+2\mu}{\rho}}$	Z_d	(E,ν)	ρ		
	σ_{12} ⟷	v_2	shear					
	σ_{13} ⟷	v_3	$c_s=\sqrt{\frac{\mu}{\rho}}$	Z_s Z_s	(λ,μ)			
Acoustic	γ	V	$\sqrt{\frac{K}{\rho}}$	$c\rho$	K_v bulk modulus	ρ		
EM	E_3	B_2	$\sqrt{\frac{1}{\varepsilon\mu}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$1/\varepsilon$ permittivity	μ permeability	$E_3=\frac{1}{\varepsilon}D_3$	$B_2=\mu H_2$

E, H electric & magnetic fields

D, B electric & magnetic fluxes

The uses of these coefficients:

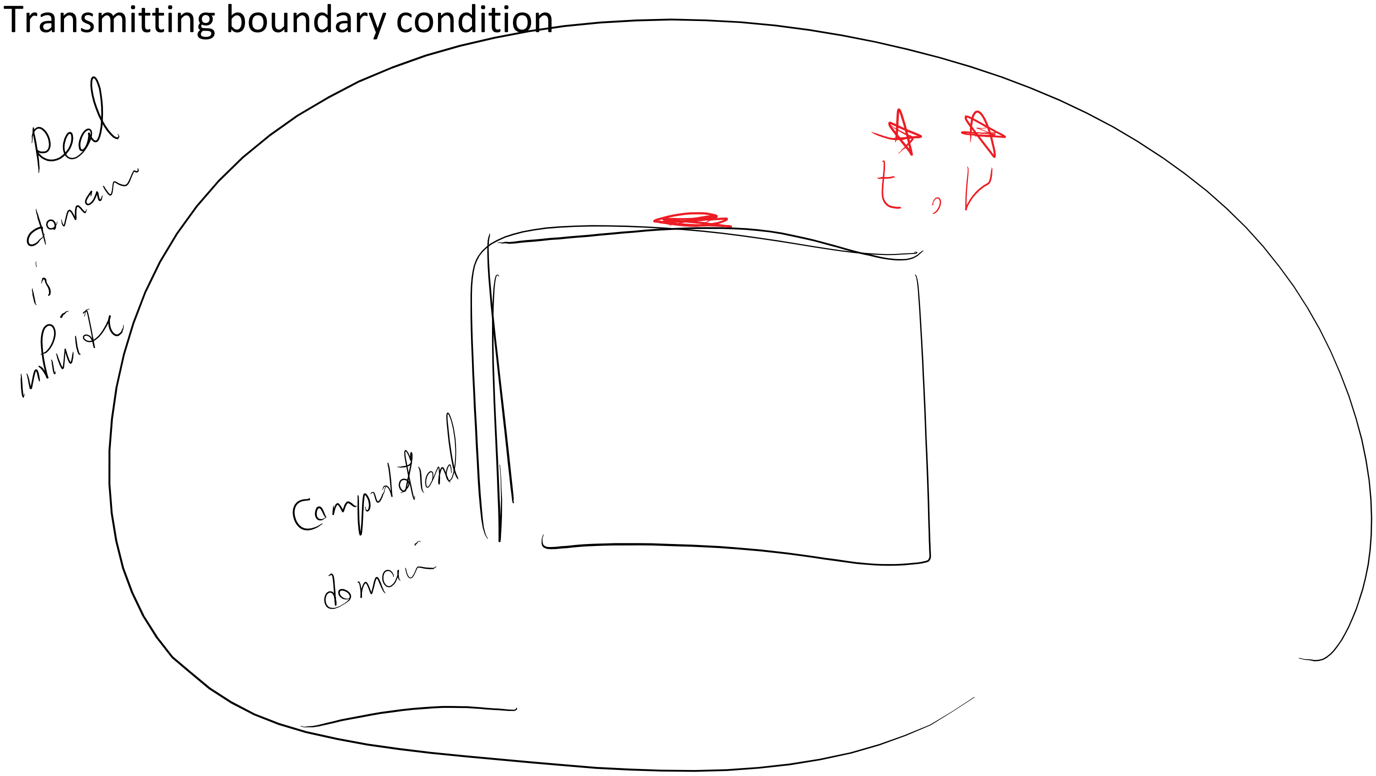
1. We can come up with transmission and reflection coefficients for more complex (1D) materials and characterize their effective properties (e.g. metamaterials).



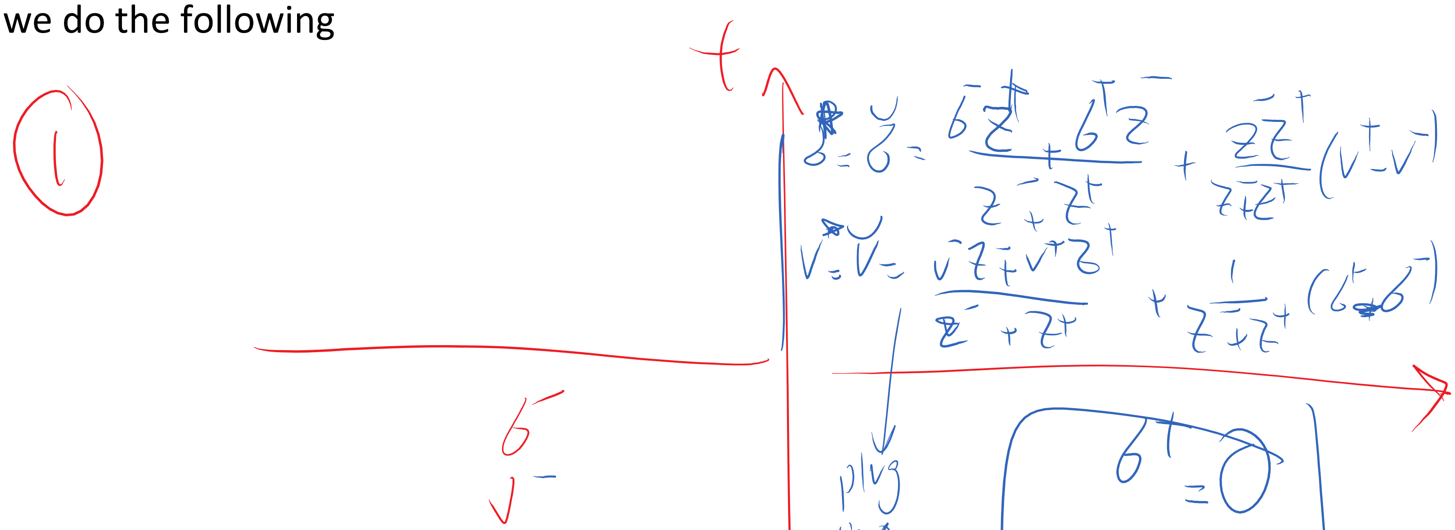
E_{off}

P_{off}

2. Transmitting boundary condition



For 1D (exact in 1D, not exact in 2D/3D) transmitting BC we do the following



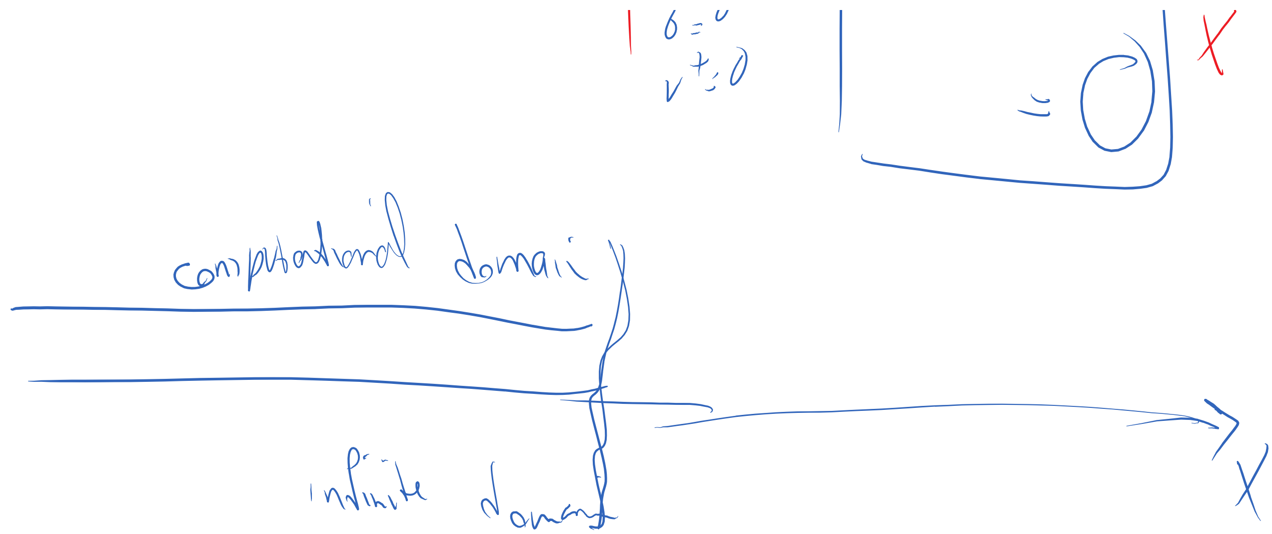
$$\sigma^* = \check{\sigma} = \frac{\sigma^- Z^+ + \sigma^+ Z^-}{Z^- + Z^+} + \frac{Z^- Z^+}{Z^- + Z^+}(v^+ - v^-)$$

$$v^* = \check{v} = \frac{v^- Z^- + v^+ Z^+}{Z^- + Z^+} + \frac{1}{Z^- + Z^+}(\sigma^+ - \sigma^-)$$

plug $\sigma^+ = 0$, $v^+ = 0$

$\sigma^+ = 0$

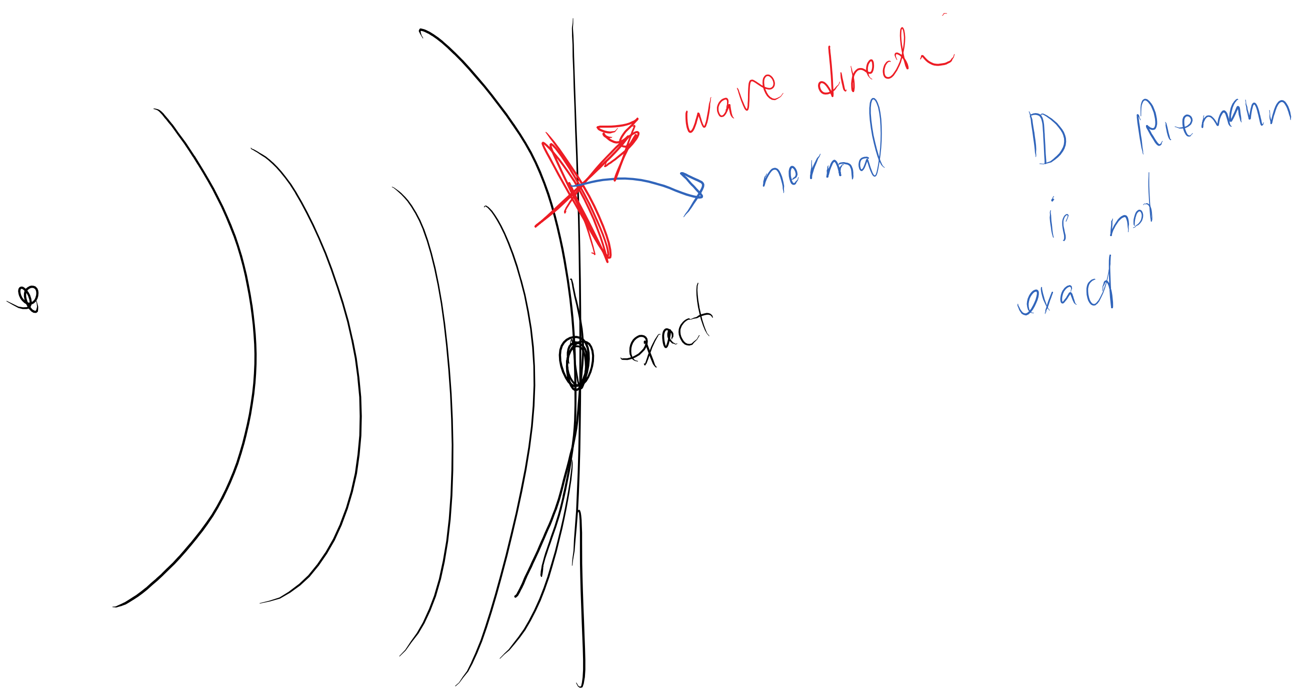
$v^+ = 0$



Silver-Muller Boundary condition (It's a 1D Riemann solution)

Not exact in 2D & 3D

Boundary of the domain



This is decent but can give you wrong results in 2D / 3D.

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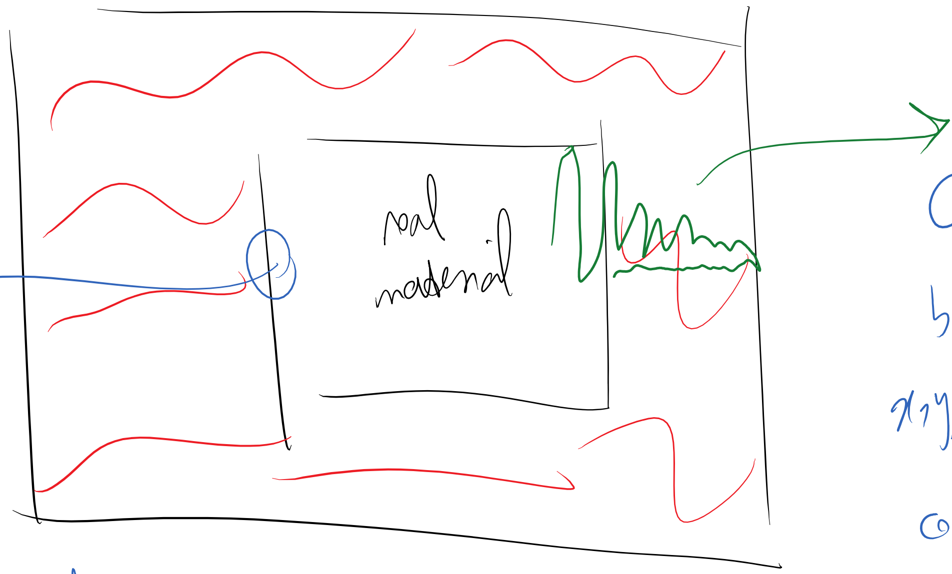
Perfectly Matched Layer (a layer around the material with the same impedance)

Beranger 94, 95



same Z no reflection

Chew 94 by stretching x, y, z in complex plane

1. Z doesn't change
2. Material can become super damping

Riemann solutions for 2D, 3D problems.

$$f_{t,t} + \nabla \cdot f_x = S$$

f_x [P | φ | ρ |

$$f_{t,t} + \quad f_{x} = \quad)$$

$$f_x = \left[f_1 \,\middle|\, f_2 \,\middle|\, f_3 \right]$$

$$f_{t,t} + f_{1,1} + f_{2,2} + f_{3,3} = s$$

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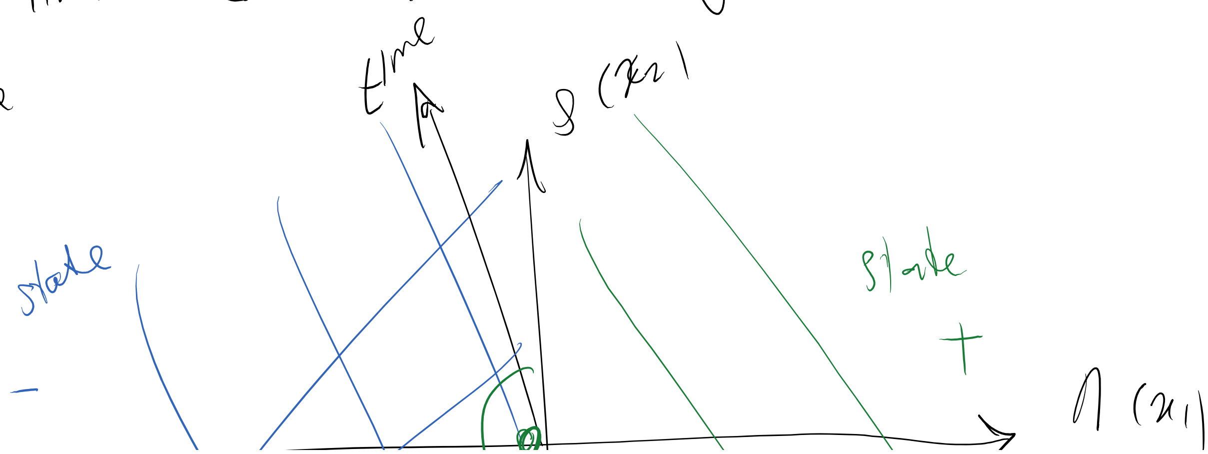
for a linear problem

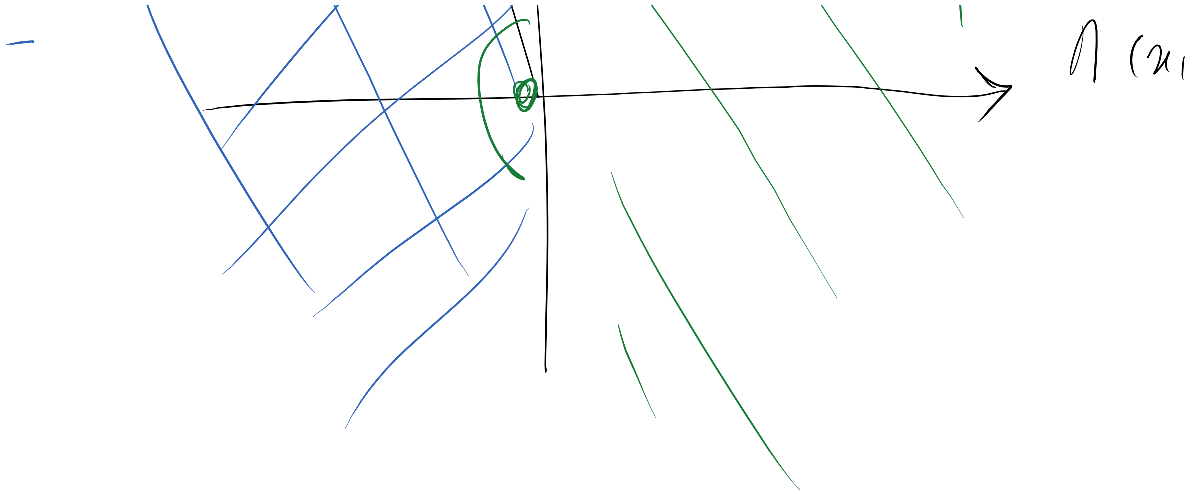
$$\left.\begin{aligned} f_t &= q \\ f_1 &= A_1 q \\ f_2 &= A_2 q \\ f_3 &= A_3 q \end{aligned}\right\} \longrightarrow$$

(3) $$\boxed{\dot{q} + A_1 q_{,1} + A_2 q_{,2} + A_3 q_{,3} = s}$$

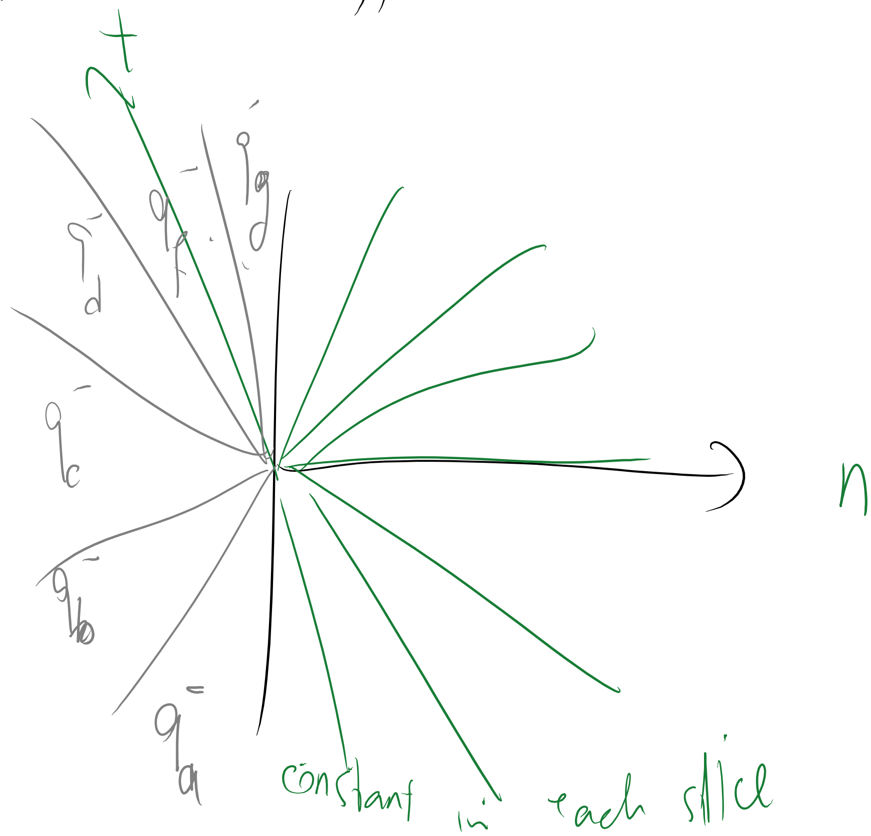
3D linear conservation law strong form

2D x time





Approximation (math community)



The least accurate is if we assume ONLY 1 state on each side. We'll solve that problem the next time.