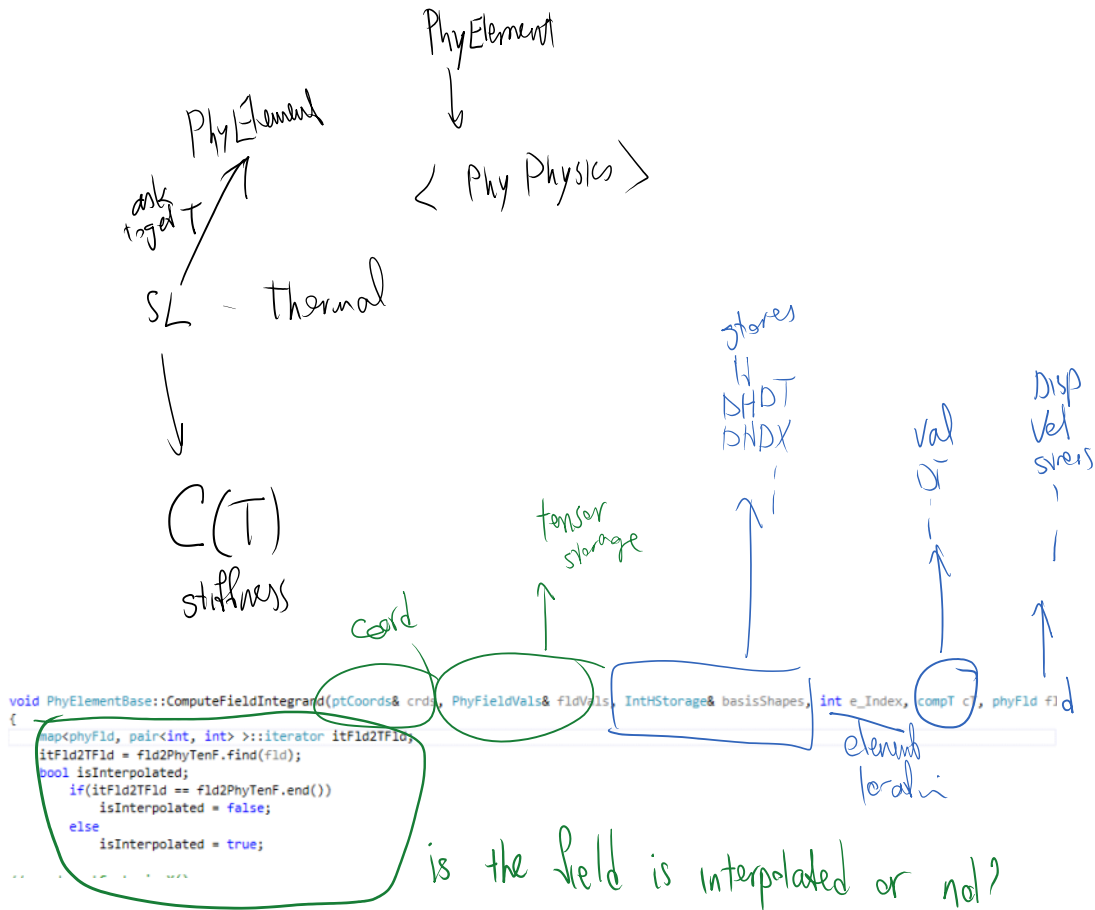


Computation of any type of tensor field needed in WR, error calculation etc.:



```
void PhyElementBase::ComputeFieldIntegrand(ptCoords& crds, PhyFieldVals& fldVals, IntHStorage& basisShapes, int e_Index, compT c, phyFld fld)
{
    map<phyFld, pair<int, int> >::iterator itFld2TFld;
    itFld2TFld = fld2PhyTenF.find(fld);
    bool isInterpolated;
    if(itFld2TFld == fld2PhyTenF.end())
        isInterpolated = false;
    else
        isInterpolated = true;
    ...
}
```

is the field is interpolated or not?

```
if ((int)cT < (int)ctSource)
{
    if (isInterpolated) // the field is interpolated within the element
    {
        int phyIndex = itFld2TFld->second.first;
        int tenFldIndex = itFld2TFld->second.second;
        if (basisShapes.preComputed == true)
        {
            (physics[phyIndex]->pTFields[tenFldIndex]).PhyTFComputeIntegrand((int)cT)(crds, fldVals, basisShapes, e_Index, cVH, rT);
        }
        else
        {
            (physics[phyIndex]->pTFields[tenFldIndex]).PhyTFCompute((int)cT)(crds, fldVals, basisShapes, e_Index, cVH, rT);
        }
    }
    return;
}
```

eg. composition to SL

which field in solid (Disp 0 vel 1)

Disp → U, U, DNDX

Side note:

Idea of function pointer:

```
extern PhyTensorFieldcompPtr PhyTFComputeIntegrand[NUM_COMPT];
```

Indexed by computation type

- ctVal
- ctDT
- ctDX
- ...

a function of class this name is

a function of class

```
typedef void (PhyTensorField::*PhyTensorFieldCompPtr)
(ptCoords& , PhyFieldVals& , IntHStorage& , int, vsT, rotT);
```

Example:

A list of values of this array of function pointer must be specified:

For example for spatial derivative it's this function:

```
PhyTFComputeIntegrand[ctDX] = &PhyTensorField::ComputeDHDXIntegrand;
```

Other computation types:

```
void PhyElementBase::ComputeFieldIntegrand(ptCoords& crds
{
  map<phyFld, pair<int, int> >::iterator itFld2TFld;
  itFld2TFld = fld2PhyTenF.find(fld);
  bool isInterpolated;
  if(itFld2TFld == fld2PhyTenF.end())
    isInterpolated = false;
  else
    isInterpolated = true;

  // crds.setCartesianX();

  if ((int)ct < (int)ctSource)
  {
    if (isInterpolated) // the field is interpola
    {
      int phyIndex = itFld2TFld->second.first;
      int tenFldIndex = itFld2TFld->second.second;

```

Interpolated things

```
.....
}
}
}
```

Non Interpolated things
field name (U, V, S, ...)
Computation type

```
(physics[phyIndex]->*PhyCompute[ct][fld])(crds, fldVals, basisShapes, e_Index,
cVH, rT);
```

```
extern PhycompPtr PhyCompute[ NUM_COMPT ][ NUM_PHYFLD ];
```

```
typedef void (PhyPhysics::*PhycompPtr)(ptCoords& , PhyFieldVals& ,
    IntHStorage& , int, vsT, rotT);
```

For stress Value

```
PhyCompute[ctVal][pfStrsL] = &PhyPhysics::computeField_ctVal_pfStrsL;
```

The parent class (PhyPhysics has the declaration and empty implementation) and a derived class must provide the actual implementation
In PhyPhysics.h:

```
virtual void computeField_ctVal_pfStrsL(ptCoords& crds,
    PhyFieldVals& fldVals, IntHStorage& basisShapes, int
    e_Index, vsT cvH, rotT rT) {};
```

SLPhysics.cpp actual implementation

```
void SLPhysics::computeField_ctVal_pfStrsL(ptCoords&
    crds, PhyFieldVals& fldVals, IntHStorage& basisShapes,
    int e_Index, vsT cvH, rotT rT)
{
    ....
```

Now we can overview an integration routine:
Before that, there is an important enumeration:

```
typedef enum { intSInitialGuess, intSPreAssembly, intSAssembly,
    intSPostSol, intSPostProcess, intSFreePostProcess, intFreeFCFPrint }
    intStage;
```

Assembly

Initial guess

different post processing

error analysis for RUN TIME adaptivity

Integration function:

Integration of anything on an integration cell:
1.

going from physics order &



physics order &
face (int, int, ext, body) interior
element



→ order

virtual
function

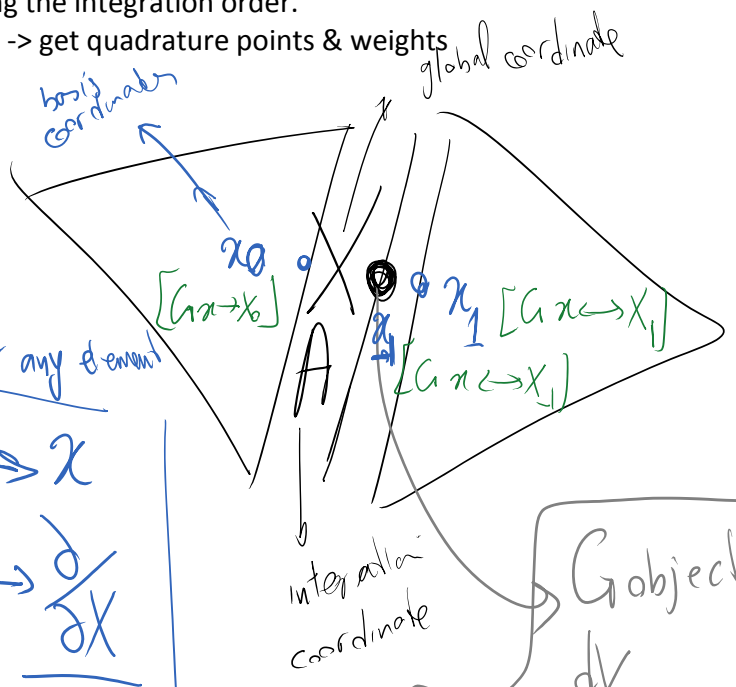
give
interior
order

SL 1F interior of element

$$\text{order} = 2p - 3$$

↓
polynomial order

1. Finding the integration order.
2. Order → get quadrature points & weights



for any element

$$\begin{pmatrix} X \\ A \end{pmatrix} \rightarrow \chi$$

$$\frac{d}{dx} \rightarrow \frac{d}{d\chi}$$

physical
der in \mathbb{R}^3

G object
dV
<face normals>
local coordinate

$$Q = \begin{bmatrix} e_1 \\ e_2 \\ c/\lambda \end{bmatrix}$$

line ^{int} order: p cell: $\frac{p+1}{2}$

$$Q = \left[\frac{\frac{v_1}{e_2}}{c_3} \right]$$

quad pts

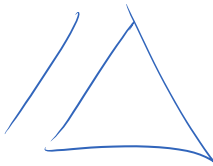


$$\Phi = \begin{bmatrix} 1 \\ \alpha_1 \\ \alpha_1^2 \\ \alpha_1^3 \\ \alpha_1^4 \end{bmatrix}$$

order 4

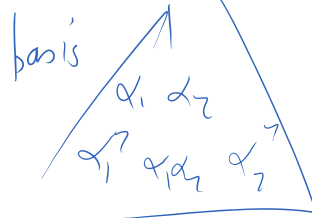
for pre calculation

geometry



integration order

pts are known



p=2 to

Tri

3. (we have all the quad points) ->

Looping over the quad points

a. Calculate all the shapes needed

$H_2, D1D1x, D1D1y, \dots$

IntHStorage

b. Tensor field storage

$(1, 1, 1, 2, 0) \dots (1, 1, 1, 2, 0) (P, val)$

intStorage

b. Tensor field storage
PhyFieldVals fldVals

$(C, Val), (V, Val), (S, Val)$

4. Now that all the tensor fields are calculated in fldVals we can go ahead and evaluate / part the integration stage computations.

Overview of the code on what exactly is being done here

The part that we calculate order, quad points, and basis storage:

```

bool PhyIntCellBase::setAfterElementPolyOrderSetBeginningEachStage(intStage intS)
{
  bool b_time = ((intS == intSPreAssembly) || (intS == intSAssembly));
  if (b_time == true)
    intCellTimePtr->setStartgTimeIncremental();
  // don't want to recompute quad points if integration order does not change
  intNumQuadsTotal = intNumQuads.getBaseScalarValue();
  if (intNumQuadsTotal <= 0)
    intOrder.clear();
  intOrderPrevStep = intOrder;
  setIntegrationOrder(intS);
  // if (order NOT set previously OR the orders are not changed, no need to recompute quad points and shapes ....
  bool reComputeQuadPtVals = ((intOrderPrevStep.getSize() == 0) || (intOrderPrevStep != intOrder));
  if (reComputeQuadPtVals)
  {
    // setting all quad points needed for this int cell
    bool issueErrorIfPtsNotPrecomputed = true; // this can be reversed to false so no matter what the order of the element,
    bool useAlphaOrBetaOrder4MissingBeta = true; // if beta orders not provided, alpha orders will be used for them
    getQuadPointsWeightFunctions(intOrder, intCrds, intWeights, intT1, intNumQuads, issueErrorIfPtsNotPrecomputed, useAlpha
  }

  if ((intS != intSPreAssembly) &&
      (reComputeQuadPtVals || (intCellHStore.size() != intNumQuadsTotal)))
    PreComputeDistinctBaseShapes();
}

```

sets integration order

if # quad pts has changed (compute quad pts)

This place precalculates and sets ptCoords and Shapes

Now that ptCoords, and Shape storages are precomputed we can go to integration routine:

Actual integration routine:

integration stage

integration stage

Actual integration routine:

```
bool PhyIntCellBase::IntegrateIntegrationCell(intStage intS,
gTime *clockPreComputeFields, gTime *clockEvaluate)
{
  int facetT = (int)cellT.ct, intST = (int)intS;
```

interior, boundary, inflow, outflow

```
for (int quadPN = 0; quadPN < intNumQuadsTotal; ++quadPN)
{
  //DBOUT
  cout<<"quadPN "<<quadPN <<" / "<< intNumQuadsTotal <<endl;
  ptCoords *crdPtr = &intCrds(quadPN);
```

get the precomputed quad pt

Storage for all the tensors

```
// interior integration
PhyFieldVals fldVals(this, crdPtr); // storage for shapes and values
// Computing values before going into integrands
bool successfulPreCompute = (fldVals.*ComputeIntegrand[intST][cellT.ct])
(*crdPtr, intCellHStore[quadPN]);
```

precomputed shape storage

Evaluation stage:

Again we use function pointers

here we have all tensor fields & process the integration stage

```
if (bInteriorInt == true)
{
  pe = interiorIntBase.eh.getPhyE();
  if ((pe->*InteriorIntegrands[intST])(factor, eInteriorI, *crdPtr, this, quadPN, fldVals) == false)
    exit(0); //parallel edit
  //return false;
}
for (int facet = 0; facet < numFacetInActivePatch; ++facet)
{
  pe = facetIntBase[facet].eh.getPhyE();
  if ((pe->*FacetIntegrands[intST][facetT])(factor, facet, *crdPtr, this, quadPN, fldVals) == false)
    exit(0); //parallel edit
  //return false;
}
```

face

face type for faces

Example

```
// intSAssembly
bool PhyElementBase::IntegrandInterior_intSAssembly(double factor, int e_Index, ptCoords& crds, PhyIntCellBase*
{
  for (int phy = 0; phy < num_physics; ++phy)
  {
    PD4 ( db << "INTERIOR_ASSEMBLY_phy" << phy << " / " << num_physics << '\n');
    if (physics[phy]->IntegrandInterior_intSAssembly(factor, e_Index, crds, pic, quadPN, fldVals) == false)
      return false;
  }
}
```

```

// intSAssembly
bool PhyElementBase::IntegrandInterior_intSAssembly(double factor, int e_Index, ptCoords& crds, PhyIntCellBase*
{
    for (int phy = 0; phy < num_physics; ++phy)
    {
        PD4 ( db << "INTERIOR_ASSMBLY_phy" << phy << " / " << num_physics << '\n');
        if (physics[phy]->IntegrandInterior_intSAssembly(factor, e_Index, crds, pic, quadPN, fldVals) == false)
            return false;
        }
    }
    return true;
}

```

Examples of function pointers for "processing of the interior OR faces attached to PIC:

```

extern PhyInteriorIntPtr InteriorIntegrands[NumIntStage];
extern PhyFacetIntPtr FacetIntegrands[NumIntStage][NumPhyCell_t];

```

```

typedef bool (PhyElementBase::*PhyInteriorIntPtr)(double, int, ptCoords&, PhyIntCellBase*, int, PhyFieldVals&);
typedef bool (PhyElementBase::*PhyFacetIntPtr)(double, int, ptCoords&, PhyIntCellBase*, int, PhyFieldVals&);

```

...
The definition of these function pointers

```

....
//intSAssembly
InteriorIntegrands[intSAssembly] = &PhyElementBase::IntegrandInterior_intSAssembly;

FacetIntegrands[intSAssembly][noneCellT] = NULL;
FacetIntegrands[intSAssembly][inflowT] = &PhyElementBase::IntegrandFacet_intSAssembly_inflowT;
FacetIntegrands[intSAssembly][outflowT] = &PhyElementBase::IntegrandFacet_intSAssembly_outflowT;
FacetIntegrands[intSAssembly][interiorT] = &PhyElementBase::IntegrandFacet_intSAssembly_interiorT;
FacetIntegrands[intSAssembly][boundaryT] = &PhyElementBase::IntegrandFacet_intSAssembly_boundaryT;

....
bool PhyElementBase::IntegrandFacet_intSAssembly_interiorT(double factor, int e_Index, ptCoords& crds, PhyIntCellBase* pic, int quadPN, PhyFieldVals& fldV
{
    for (int phy = 0; phy < num_physics; ++phy)
    {
        PD4 ( db << "INTERIOR_FACET_ASSMBLY_phy" << phy << " / " << num_physics << '\n');
        if (physics[phy]->IntegrandFacet_intSAssembly_interiorT(factor, e_Index, crds, pic, quadPN, fldVals) == false)
            return false;
        }
    }
    return true;
}

....
bool PhyPhysics::IntegrandFacet_intSAssembly_interiorT(double factor, int e_Index, ptCoords& crds, PhyIntCellBase* pic, int quadPN, PhyFieldVals& fldVals)
{
    double factorTerm; // The factor in front of the integration term
    vsT cVH = patch->patchFlags.vs_intSAssemblyFldKRUupdate;
    FEM_Terms* femTerms = getFEM_AssemblyTerms();
    AssemblyIntegrandTerm* siTerm;

#ifdef PRINT_DEBUG4
    VECTOR XCoord;
    crds.setCartesianX();
    crds.XC.getXX(XCoord);
    db << "ASMBLY_INT_FACE_quadPN\t" << quadPN << "\tcrd\t" << XCoord << "\tdesc\t" << pic->descFacet[e_Index] << "\tdecInterior\t" << pic->descInterior <<
    db << "Number_interior_sdx terms\t" << femTerms->num_weightField_Assembly_Facet_sdx_interiorT<< '\n';
#endif

    /**dx part
    for (int term = 0; term < femTerms->num_weightField_Assembly_Facet_sdx_interiorT; ++term)

```



```

siTerm = &femTerms->weightField_Assembly_Facet_sdx_interiorT[term];
if (siTerm ->hasConstantFactor == true)
    factorTerm = siTerm->termConstantFactor;
else
    factorTerm = siTerm->termConstantFactor * IntegrandFacet_intSAssembly_sdx_interiorT_ComponentFactor(term, e_Index, crds, pic, quadPN, fldVals);

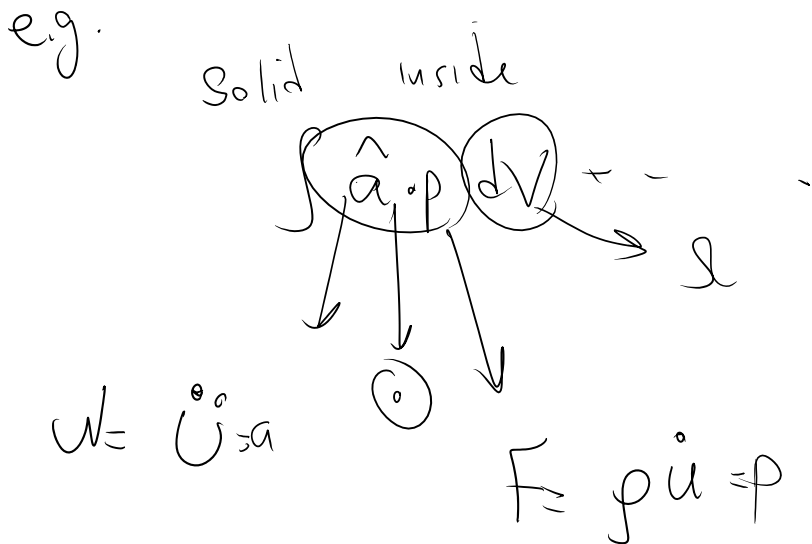
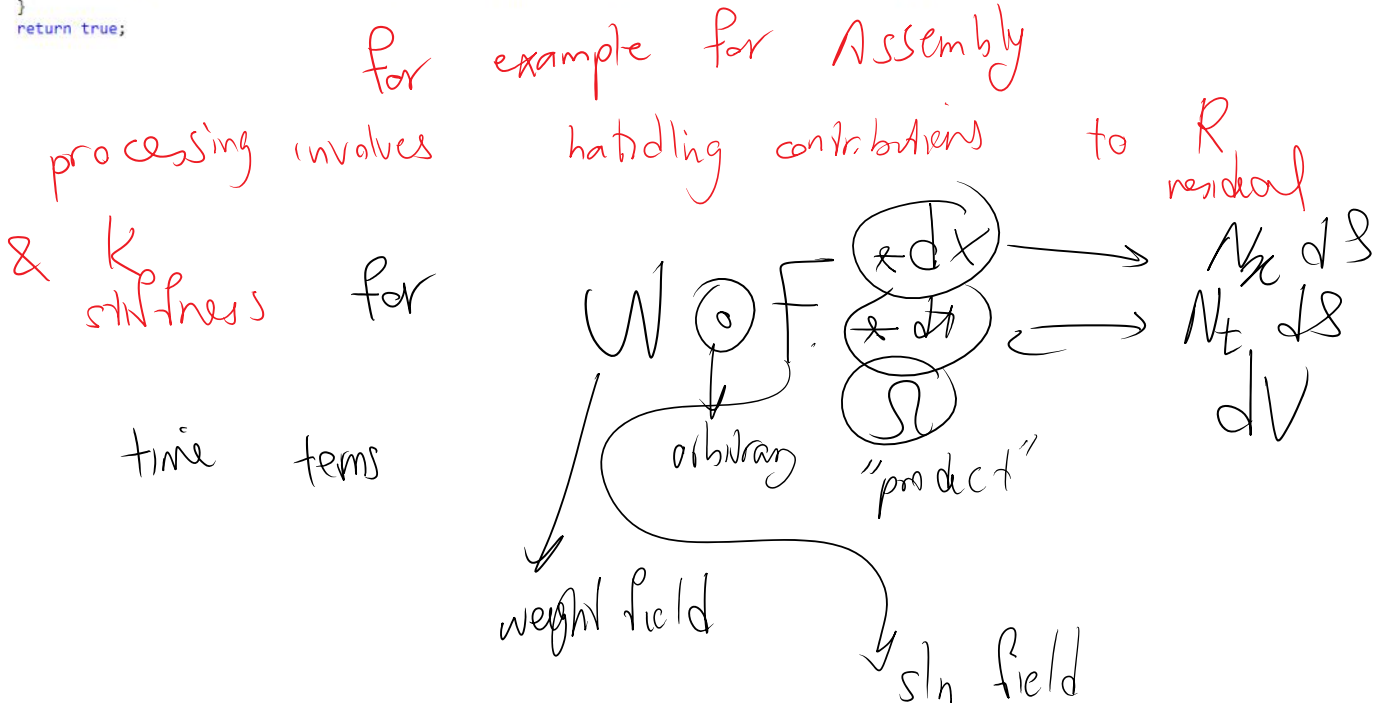
PD4 ( db << "term_INT_sdx" << term << '\n' << siTerm << "\tconstFact\t" << siTerm->termConstantFactor << "\tfactorTerm\t" << factorTerm << '\n';)
#pragma omp critical(PhyPhysicsCE3)
fldVals.updateAssemblyStiffnessReidual_Facet_sdx_interiorTboundaryT(pic, e_Index, *siTerm, factorTerm, factor, cVH);
}
fCompMode fcMode = fldVals.pfv_getFaceComputationMode();
/**dt part
PD4 (db << "Number_interior_sdt terms\t" << femTerms->num_weightField_Assembly_Facet_sdt_interiorT<< '\n');
PD4(db << "fcMode\t" << fcMode << '\n');
if (fcMode == dcm_only_sdxNZ)
return true;
//dt = 0 for vertical facet
for (int term = 0; term < femTerms->num_weightField_Assembly_Facet_sdt_interiorT; ++term)
{

```

```

siTerm = &femTerms->weightField_Assembly_Facet_sdt_interiorT[term];
if (siTerm ->hasConstantFactor == true)
    factorTerm = siTerm->termConstantFactor;
else
    factorTerm = siTerm->termConstantFactor * IntegrandFacet_intSAssembly_sdt_interiorT_ComponentFactor(term, e_Index, crds, pic, quadPN, fldVals);
PD4 ( db << "term_INT_sdt" << term << '\n' << siTerm << "\tconstFact\t" << siTerm->termConstantFactor << "\tfactorTerm\t" << factorTerm << '\n';)
#pragma omp critical(PhyPhysicsCE3)
fldVals.updateAssemblyStiffnessReidual_Facet_sdt_interiorTboundaryT(pic, e_Index, *siTerm, factorTerm, factor, cVH);
}
return true;
}

```



facos : solid

$$\int_{\omega} u (\delta^A - \delta) N_x d\Omega$$

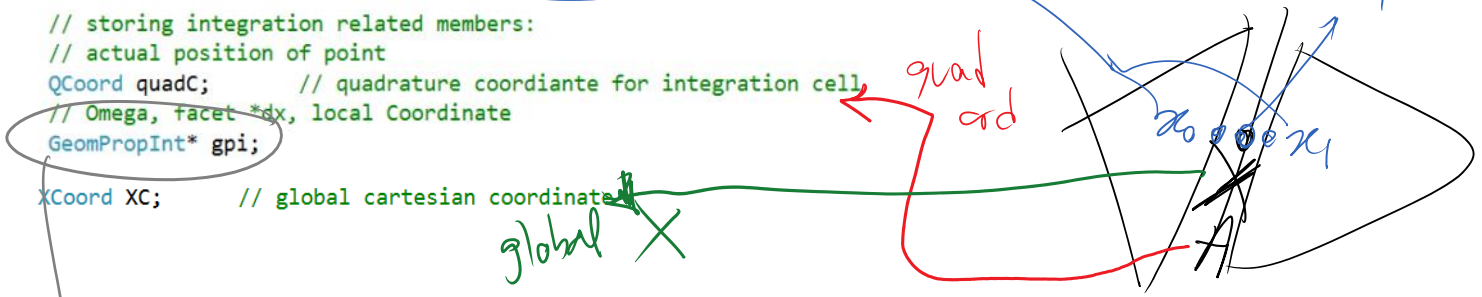
geometry part

⋮

Please see the coordinate classes:
physics\PhyCoord.h

```
class ptCoords
{
    eCoord eCrdInterior; // the coordinate for element on interiorIntegration cell
    vector<eCoord> eCrdFacet; // the vector of coordinates for elements on facetIntegration cells

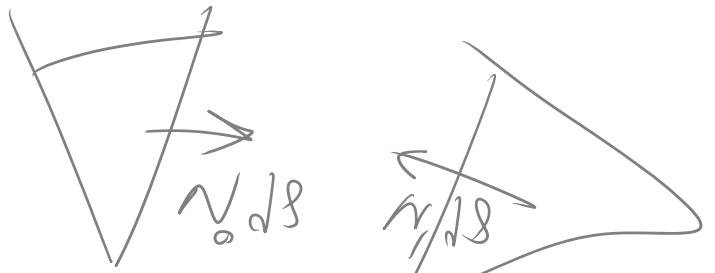
    // storing integration related members:
    // actual position of point
    QCoord quadC; // quadrature coordinate for integration cell
    // Omega, facet *dx, local Coordinate
    GeomPropInt* gpi;
    XCoord XC; // global cartesian coordinate
}
```



geometry object storing

2 n.d.s

per elements
with facets



with facts
on pi

$\sqrt{Nd8}$ $\sqrt{Nd8}$

($Nd8 \approx dx$ in differential forms notation)

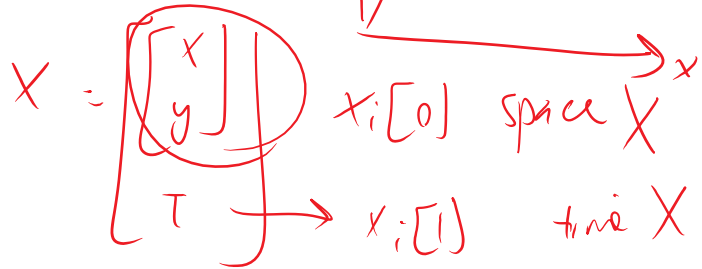
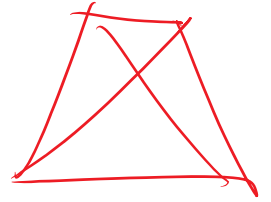
```
class XCoord
```



```
{
  // global coordinate at once
  vector<double> X;
  // break-down of the global coordinate determined by CoordManager
  vector< vector< double> > Xi;
```

global Cartesian X

break down of X (eg.



```
class QCoord : public GQuadCoord
```



```
{
  GeomPropBase int gpb;
  GQuadIntOrder int_order;
  GQuadNumber int_number;
  XCoord *XC;
```

↔ A quadrature coordinate
geom Object capable of
transfers between A & X
(quad) (global)

these objects (integration order
& number of quad pt)

can be used in employing precalculated
shape functions

κ

basis coordinate

```

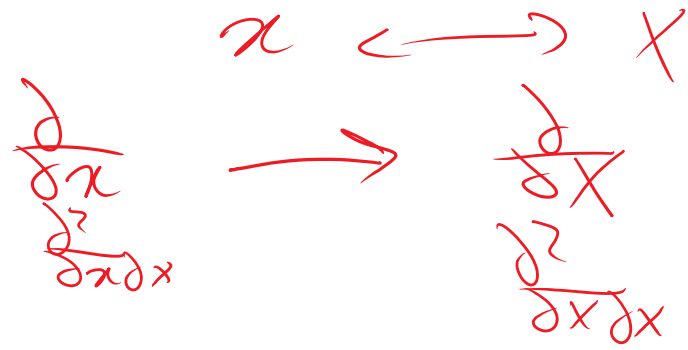
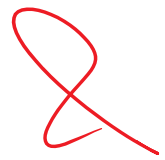
class eCoord // coordinate class for element
{
  GeomPropBase e_gpb;
  PhyInt2EBasePtr* i2ePtr;
  BasisCoord basisC;
  QCoord *quadC;
  XCoord *XC; // global ca
  PhyElementBaseInterior* pei;
}

```

this is actually the basis coordinate "vector"

geometry class capable of

(basis) global



coordinate transformations

basis derivatives of shapes

to global derivatives