2022/11/22

Tuesday, November 22, 2022 9:42 AM

Lorcher_2008_An explicit discontinuous Galerkin scheme with local time-stepping for general unsteady diffusion equations.pdf

To analyze the stability of the scheme, a periodic problem with a given spatial discretization is considered. As the problem (3.40) is linear, one can construct a matrix $\boldsymbol{W}$ such that

$$\hat{u}^{\text{new}} = \boldsymbol{W}\hat{u}^{\text{old}}, \tag{3.41}$$

$a^0 = W a^i$

where $\hat{u}^{\text{old}}$ denote DOF at a common time level t^{old} and $\hat{u}^{\text{new}}$ denote DOF at a common time level t^{new} with $t^{\text{new}} > t^{\text{old}}$. $\boldsymbol{W} = \boldsymbol{W}(\Delta t_i)$ depends on the time steps Δt_i of each element Q_i. Following the matrix method of stability analysis described in [14], the scheme is stable, if the spectral radius $\rho(\boldsymbol{W})$ is lower or equal to 1.

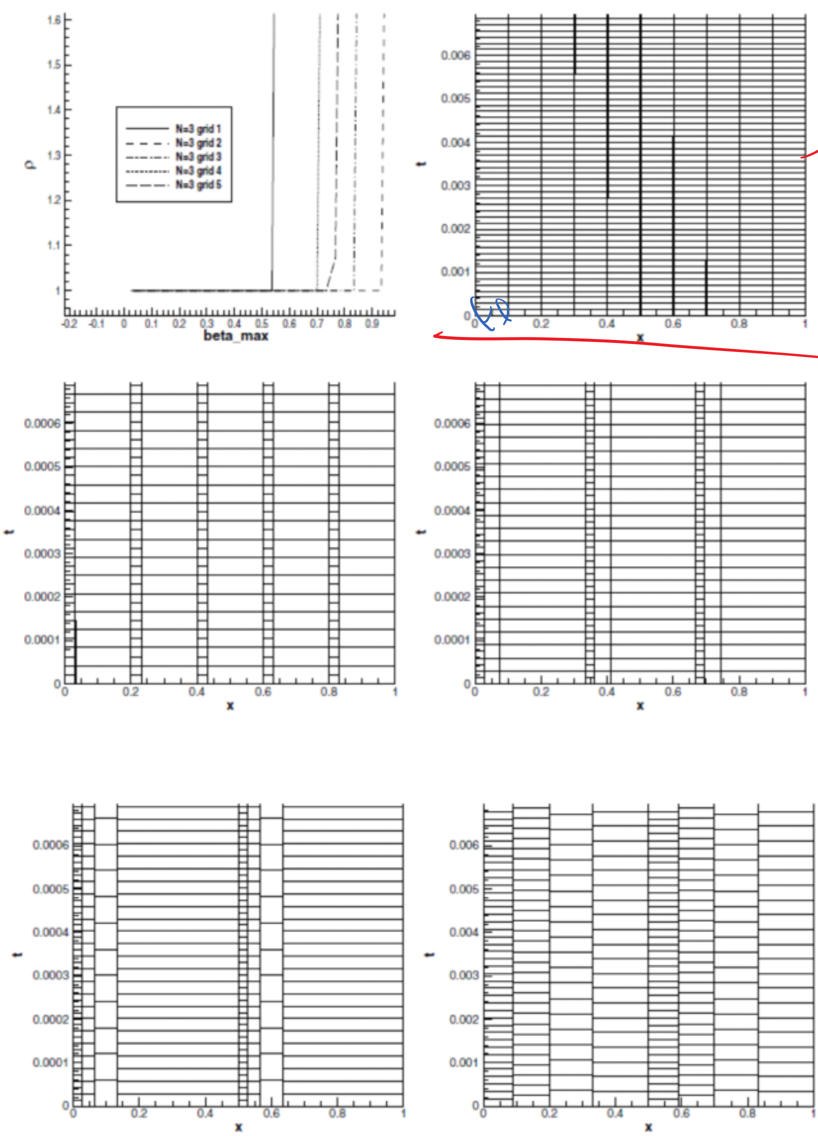
$$\Delta t \leqslant \beta(N)\frac{\Delta x^2}{(2N+1)^2\mu\sqrt{d}}, \tag{3.42}$$

$N = 1 \rightarrow N = 5$

10^4 reduction in time step

Table 1
Stability numbers of the STE-DG scheme

N	1	2	3	4	5	6	7
β	1.46	0.8	0.40	0.24	0.16	0.12	0.09



provides critical β

Fig. 3. $\rho(\boldsymbol{W})$ as function of $\beta_{\max}$ for space–time grids 1–5, space–time grids 1–5.

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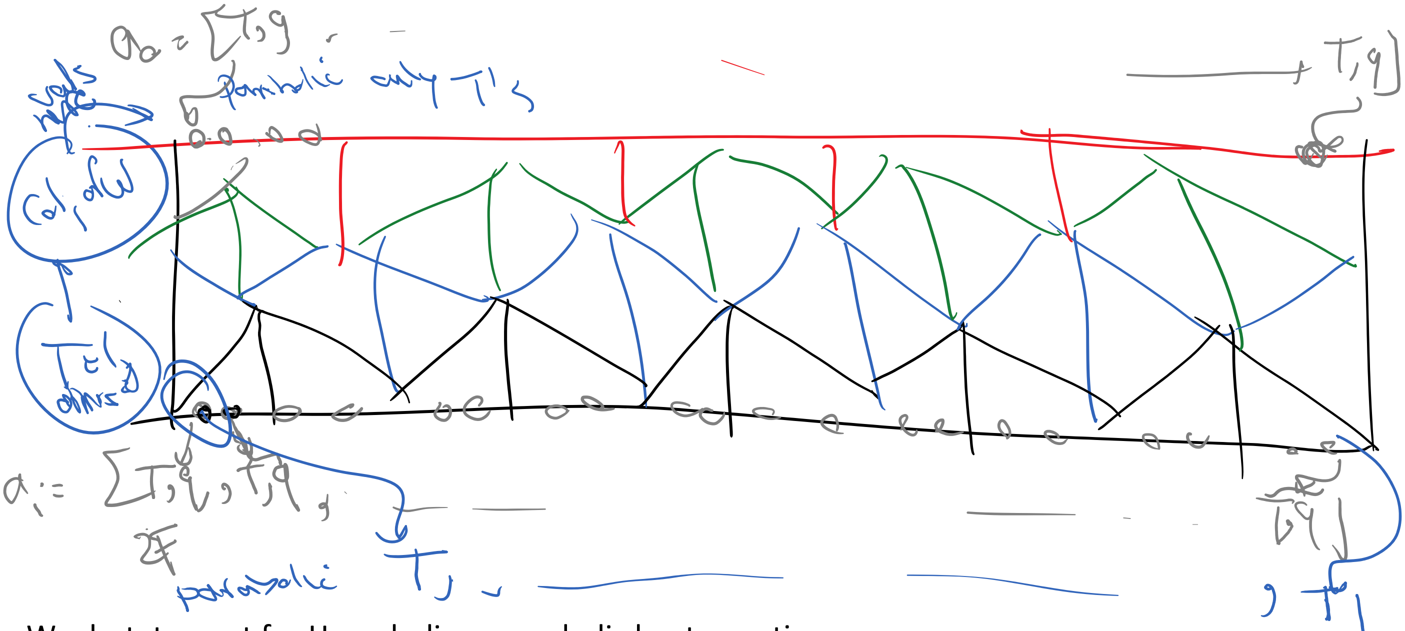
The uniform mesh has the most restrictive beta

How to get $\rho(W)$



Weak statement for Hyperbolic or parabolic heat equation

$$\int_e \Big(-\hat{T}_{,t} C T - \hat{T}_{,x} q - \hat{T} Q - \kappa^{-1}\tau \hat{q}_{,t} q - \hat{q}_{,x} T + \kappa^{-1}\eta \hat{q} q \Big) \mathrm{d}V +$$

$$\int_{\partial e} \Big(\hat{T}\big[C T^* n_t + \cancel{q^*_n} \big] + \hat{q}\big[\cancel{\kappa^{-1}\tau q^* n_t} + \cancel{T^* n_x} \big] \Big) \mathrm{d}S = 0$$

$\tau = 0$

parabolic $\tau = 0$

parabolic we only have initial T^*

IC

$= T_{ini}$

$J_{n_t = -1}$
$n_x = 0$

$a_o = W a_i$

$$a_i = \begin{bmatrix} 0 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

just position

$W a_i = W e_i$ = column i of the matrix

$$W a_j = W e_j = (\text{column})_j \text{ of the matrix}$$

There are a lot of methods that don't need the explicit form of a matrix to do the certain matrix calculation (very useful in solving PDEs, because as you see above getting these matrices is extremely expensive)

$Ax = b$ I don't need A!

$\max(|\lambda_i|)$ of $A = \rho(A)$ " → GMRES

Also gives eigen value

x given → need to be able to get Ax

p → Ap

$a_i \longrightarrow a_0$ easy!

IC → final solution

Power iteration

random x_0

x_0, $x_{i+1} = \underline{A x_i}$

that's

$$\left(x_i = A^n x_0\right)$$

$$x_0 = \sum_{i=1}^{m} \lambda_i u_i$$

eigenvalues, eigenvectors

$$|\lambda_1| \geq |\lambda_2| \ldots$$

$$x_n = \sum_{i=1}^{m} \lambda_i^n u_i \longrightarrow \lambda_1^n u_1$$

$\|x\|$

$$\boxed{\frac{|x_{n+1}|}{|x_n|} \longrightarrow \rho(A)}$$