Computer Methods in Dynamics of Continua (ME/AE 599)

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Course description

This course is intended to serve as a sequel to an introductory finite element or computational mechanics courses. It is designed to deepen student's understanding of the characteristics of elliptic, parabolic, and hyperbolic partial differential equations (PDE) and get familiar with solution techniques for dynamic problems.

Course Objective

- Provide sufficient mathematical background to read the current literature and understand new developments in the field
- Familiarize the students with various numerical schemes for continuum dynamics
- Relate theory to practical applications in computational science and engineering.
- Develop the student's capabilities for technical communication and independent research in computational science and engineering.

Course Outline

1. PDE type classification and analytical methods

   (a) Various aspects for classifications of PDEs (order, number of unknowns, number of independent variables, *etc.*)
   Reading source: [Farlow, 2012] Lesson 1

   (b) Linear, semilinear, quasilinear, nonlinear PDEs
   Reading source: [Levandosky, 2002] Section 1. Introduction

   (c) Elliptic, parabolic, and hyperbolic PDEs

      i. Hyperbolic PDEs and characteristics
      Reading source: [Farlow, 2012] Lesson 27; [Levandosky, 2002] Section 2. First order PDEs and method of characteristics; [Loret, 2008] Chapter III. Classification of PDEs

      ii. Classification of a 2nd order PDE (scalar unknown) as a function of two or more independent parameters
      Reading source: [Levandosky, 2002] Section 4. Classification of second order PDEs; [Farlow, 2012] Lessons 23, 41; [Loret, 2008] Chapter III. Classification of PDEs

      iii. D'Alembert solution of the wave equation
      Reading source: [Farlow, 2012] Lesson 17;

      iv. Systems of PDEs (multiple unknowns) as a function of two or more independent parameters
      Reading source: [Loret, 2008] Chapter III. Classification of PDEs; [Levandosky, 2002] Section 8. hyperbolic systems of conservation laws; [LeVeque, 2002] sections 2.9, 2.11, 18 (particularly 18.5)

   (d) Method of characteristics: Shocks and rarefaction waves
   Reading source: [Farlow, 2012] Lessons 28 (scalar) and 29 (system); [Loret, 2008] Chapter IV. Method of characteristics and nonlinear PDEs; [Levandosky, 2002] Section 3. Conservation laws, jump conditions and rarefaction waves; [LeVeque, 2002] Chapter 11. Nonlinear scalar conservation laws; (and Chapter 13. Nonlinear systems of conservation laws).

   (e) Riemann solutions: linear and nonlinear hyperbolic PDEs
   Reading source: [LeVeque, 2002] Chapter 3: Characteristics and Riemann problems for linear hyperbolic PDEs; Section 9.9 Variable coefficient PDEs (for acoustic equations)

2. General solution schemes in space (or spacetime)

   (a) Finite Difference (FD)
   Reading source: [Strikwerda, 2004] §1.3

   (b) Finite Volume (FV)
   Reading source: [LeVeque, 2002] Chapter 4. FV methods; Chapter 15. FV methods for nonlinear systems.

   (c) Finite Element Method (FEM)

3. General solution schemes in time (or spacetime in item d)

   (a) Exact temporal integration

   (b) Time-marching schemes (discrete integration in time)

   (c) Modal superposition

      i. Mathematic preliminaries: Symmetric and generalized eigenvalue problems, Rayleigh's quotient.
         Reading source: [Bathe, 2006] §2.5, §2.6

      ii. Modal analysis with FEM method (no damping)
         Reading source: [Bathe, 2006] §9.3.1 and §9.3.2.

      iii. Modal analysis with FEM method (with damping)
         Reading source: [Bathe, 2006] §9.3.3l [Hughes, 2012] §7.2, "Viscous Damping"

      iv. Error estimates for eigenvalues and eigenvectors
         Reading source: [Hughes, 2012] §7.3.1

      v. Frequency spectra for different elements
         Reading source: [Hughes, 2012] §7.3.2 (Example 4)

   (d) Fully discretized methods in spacetime

4. A variety of time-marching schemes

   (a) Central, backward, forward difference methods, Wilson-θ, Newmark, collocation, and alpha methods
      Reading source: First order ODE: [Bathe, 2006] §9.6.1 Alpha methods; [Hughes, 2012] §8.1 Generalized trapezoidal method.
      Reading source: Central difference [Bathe, 2006] §9.2.1, Houbolt [Bathe, 2006] §9.2.2, Wilson-θ [Bathe, 2006] §9.2.3, Newmark [Bathe, 2006] §9.2.4, [Hughes, 2012] §9.1.1; Collocation method [Hughes, 2012] §9.3.3; HHR (alpha method) [Hughes, 2012] §9.3.3

   (b) Multi-step,-stage,-cycle time marching

      i. Multi-step methods (using $n > 1$ current and previous time step values)
         Reading source: [Hughes, 2012] §9.3

      ii. Multi-stage methods (*e.g.,* Runge-Kutta methods)
         Reading source: [Chapra and Canale, 2010] Chapter 25.

      iii. Multi-cycling (subcycling permits adjustment of local time steps based on element size)

   (c) Comparison of implicit and explicit methods

      i. Need to solve a global matrix system for implicit / explicit methods
         Reading source: Relevant information: [Hughes, 2012] Element-by-element (EBE) implicit methods.

      ii. Linear vs. nonlinear PDEs

      iii. Structural dynamics vs. wave propagation problems
         Reading source: [Bathe, 2006] §9.4.4

    iv. Lumped and consistent mass matrices
    Reading source: [Hughes, 2012] §7.3.2; [Bathe, 2006] Example 9.14, Table E9.14

  (d) High order integration in time

5. Analysis of direct integration schemes

  (a) Stability, accuracy, and convergence concept
  Reading source: FD scheme: [Strikwerda, 2004] §1.4, §1.5; [Hughes, 2012] §8.2.3: consistency

  (b) von Neumann (Fourier) analysis for linear PDEs solved by finite difference (FD) methods
  Reading source: [Strikwerda, 2004] §2.1, §2.2(, and §2.3)

  (c) Order of convergence
  Reading source: FD: [Strikwerda, 2004] §3.1, §3.2; FEM: [Bathe, 2006] §4.3 (particularly 4.3.5)

  (d) Measures of accuracy such as numerical dissipation and dispersion

  (e) Single-dof (SDOF) to multi-dof (MDOF) system analysis

    i. Using modal decomposition to show how SDOF stability limits are used (for generalized trapezoidal method).
    Reading source: [Hughes, 2012] §8.2

    ii. Natural frequency of various element types ⇒ their effect on stable time step (explicit methods)
    Reading source: [Hughes, 2012] §9.2; [Bathe, 2006] Table 9.5

  (f) Stability analysis of various time-stepping methods

    i. Generalized trapezoidal, Houbolt, central difference, Wilson-θ, & Newark methods
    Reading source: [Bathe, 2006] §9.4.1 Expression of new time step values in terms of previous time step in matrix form ($\mathbf{A}$).
    Reading source: Stability analysis using amplification matrix $\mathbf{A}$: [Bathe, 2006] §9.4.2; [Hughes, 2012] §9.1.2
    Reading source: [Bathe, 2006] §9.4.3 Accuracy analysis.

  (g) Practical considerations

    i. Handling of high-frequency oscillations
    Reading source: [Hughes, 2012] §9.1.2 ("High frequency behavior" and "viscous damping" discussions.)

    ii. Overshoot and undershoot
    Reading source: [Hughes, 2012] §9.3.3 ("Overshoot" discussion)

  (h) More advanced topics

    i. Different types of stability (Absolutely stable, A-stable, stiffly stable, etc.)
    Reading source: [Hughes, 2012] §9.3 (discussions on stability)

6. Adaptivity and multiscale discretizations for dynamic problems

   (a) Sources of stiffness (geometric from mesh adaptivity or complex geometry; multiscale problems); PDE mode transition

   (b) Hybrid implicit-explicit methods (IMEX): Mesh partitioning and operator splitting
   Reading source: [Hughes, 2012] §9.4 (mostly §9.4.1)

   (c) Multistep methods

   (d) Adaptivity in time

7. Nonlinear problem

   (a) Explicit methods
   Reading source: [Bathe, 2006] §9.5.1

   (b) Implicit methods
   Reading source: [Bathe, 2006] §9.5.2

   (c) Modal superposition
   Reading source: [Bathe, 2006] §9.5.3

8. Brief references to elastodynamics, electromagnetics, heat conduction, acoustic equations, and fluid mechanics

Color symbols: Not discussed in detail   Not covered in this course but a relevant topic

Books: Some references are shared with you in the beginning of the semester. Hughes and Farlow books can be purchased for relatively low prices ($10-$20). Farlow's book is written in very simple language and covers a variety of topic related to PDEs. It is only used in a few places in this course. Strikwerda is a very good book on the analysis of finite difference methods. The first 80 pages can be found on google books.

References

[Bathe, 2006] Bathe, K.-J. (2006). *Finite element procedures*. Klaus-Jurgen Bathe.

[Chapra and Canale, 2010] Chapra, S. C. and Canale, R. P. (2010). *Numerical methods for engineers*, volume 2. McGraw-Hill. 6th edition.

[Farlow, 2012] Farlow, S. J. (2012). *Partial differential equations for scientists and engineers*. Courier Corporation.

[Hughes, 2012] Hughes, T. J. (2012). *The finite element method: linear static and dynamic finite element analysis*. Courier Corporation.

[Levandosky, 2002] Levandosky, J. (2002). Math 220A, partial differential equations of applied mathematics, Stanford university. `http://web.stanford.edu/class/math220a/lecturenotes.html`.

[LeVeque, 2002] LeVeque, R. L. (2002). *Finite Volume Methods for Hyperbolic Problems.* Cambridge University Press.

[Loret, 2008] Loret, B. (2008). Notes partial differential equations PDEs, institut national polytechnique de grenoble (inpg). http://geo.hmg.inpg.fr/loret/enseee/maths/loret_maths-EEE.html#TOP.

[Strikwerda, 2004] Strikwerda, J. C. (2004). *Finite difference schemes and partial differential equations.* SIAM.