1. (**20 Points**) Classify the following PDEs as linear (L), semi-linear (SL), quasi-linear (QL), and fully-nonlinear (FN),

   (a) $u_t + x^2 u_x = 0$

   (b) $u_t + u_{xxx} + u u_x = 0$

   (c) $u_t + a(u) u_x = 0$

   (d) $u_x^2 + u_y^2 = 1$

   (e) $\text{div}\left(\frac{\nabla \mathbf{u}}{\sqrt{1+|\nabla \mathbf{u}|^2}}\right) = 0$

2. (**60 Points**) (a) Verify that the equation,

$$3u_{xx} + 7u_{xy} + 2u_{yy} = 0 \tag{1}$$

   is hyperbolic for all x and y, (b) find the new characteristic coordinates η, ξ, and (c) express the PDE in the canonical form (find Φ),

$$U_{\xi\eta} = \Phi(\xi, \eta, u, u_\xi, u_\eta) \tag{2}$$

3. (**10 Points**) Determine the type of Tricomi PDE based on the coordinate values (x, y) in terms of being hyperbolic, parabolic, and elliptic,

$$\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} = 0 \tag{3}$$

4. (**20 Points**) Determine the type (hyperbolic, parabolic, or elliptic) of the following constant-coefficient PDE,

$$u_{xx} + 2u_{xy} - 3u_{yz} + 5u_{zz} = 0 \tag{4}$$

5. (**60 Points**) Consider the initial value problem for the equation,

$$u_t + a u_x = f(x, t) \tag{5}$$

   with $u(0, x) = 0$ and

$$f(x,t) = \begin{cases} 1 & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

   Assume that a is positive. Show that the solution is given by,

$$u(x,t) = \begin{cases} 0 & x \leq 0 \\ \frac{x}{a} & x \geq 0 \text{ and } x - at \leq 0 \\ t & x \geq 0 \text{ and } x - at \geq 0 \end{cases} \tag{7}$$

6. (**80 Points**) Find a solution to the initial-value problem.

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_t + \begin{bmatrix} 1 & 4 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_x = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{8a}$$

$$\begin{bmatrix} u_1(x,0) \\ u_2(x,0) \end{bmatrix} = \begin{bmatrix} \text{sin}x \\ \text{cos}x \end{bmatrix} \tag{8b}$$

   Note: You need to express $u_1(x,t)$, and $u_2(x,t)$. For example, $u_1(x,t) = \frac{1}{2}\text{sin}(x - 5t) + \cdots$. Do not turn the system to a second order PDE. Solve it as a system of first order PDEs by deriving characteristics ω_1, ω_2, and finally expression the solution in terms of (x, t).